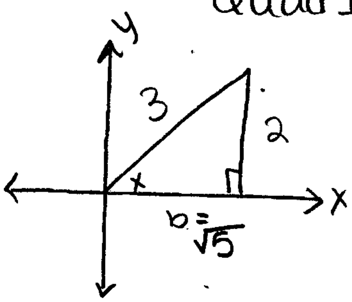
Lecture #22
MAT 123

Inverse Trig Functions

$\tan(\underbrace{\sin^{-1}(\tfrac{2}{3})}_{\text{Quad I}}) = ?$ → "what angle has a sine of $\frac{2}{3}$?" angle x!



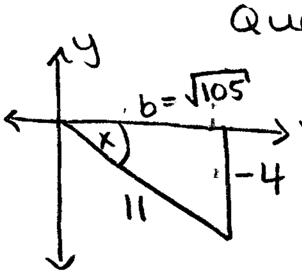
$3^2 = 2^2 + b^2$
$9 = 4 + b^2$
$5 = b^2$
$b = \sqrt{5}$

$\tan x = \boxed{\dfrac{2}{\sqrt{5}}}$

Recall:

	+	−
$\sin^{-1}$	I	IV
$\cos^{-1}$	I	II
$\tan^{-1}$	I	IV

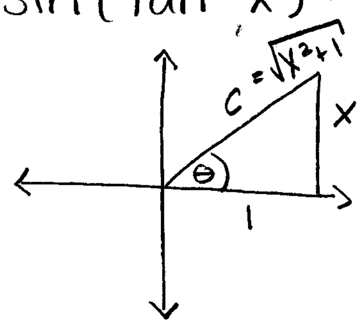
$\cos(\underbrace{\sin^{-1}(\tfrac{-4}{11})}_{\text{Quad IV}}) = ?$



$(-4)^2 + b^2 = 11^2$
$16 + b^2 = 121$
$b^2 = 105$
$b = \sqrt{105}$

$\cos x = \boxed{\dfrac{\sqrt{105}}{11}}$

$\sin(\tan^{-1} x) = ?$

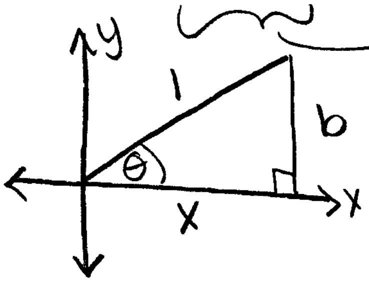


$c^2 = x^2 + 1^2$
$c^2 = x^2 + 1$
$c = \sqrt{x^2+1}$

$\sin\theta = \boxed{\dfrac{x}{\sqrt{x^2+1}}}$

<u>note</u>: $\tan\theta = \dfrac{x}{1} = \dfrac{\text{opp}}{\text{adj}}$

ex: $\sin(\underbrace{\cos^{-1} x}) = ?$ $\rightarrow \cos\theta = \frac{x}{1}$



$c^2 = a^2 + b^2$

$1^2 = x^2 + b^2$

$1 - x^2 = b^2$

$b = \sqrt{1-x^2}$

$\sin\theta = \frac{\sqrt{1-x^2}}{1}$

$= \boxed{\sqrt{1-x^2}}$

ex: $\underbrace{\sin}(\underbrace{\sin^{-1}(\tfrac{1}{2})}) = ?$

angle whose sin is $\frac{1}{2}$

take the sine of that angle

$\sin(\sin^{-1}(\tfrac{1}{2})) = \sin x = \boxed{\frac{1}{2}}$

sin and $\sin^{-1}$ are inverses of each other so they sort of just cancel each other out.

ex: $\sin^{-1}(\sin \frac{\pi}{6}) = ?$

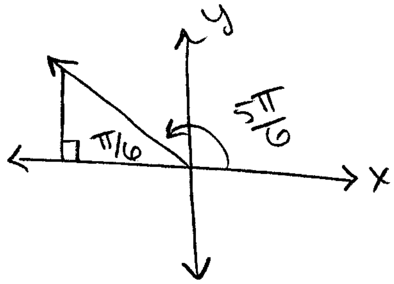
$\boxed{\frac{\pi}{6}}$

$\sin^{-1}(\sin\frac{\pi}{6}) = \underbrace{\sin^{-1}(\tfrac{1}{2})} = \frac{\pi}{6}$

"what angle has a sine equal to $\frac{1}{2}$?"

ex: $\sin^{-1}(\sin(\frac{5\pi}{6})) = ?$

$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$



ex: $\sin^{-1}(\sin(\frac{\pi}{4})) = ?$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

ex: $\sin^{-1}(\sin \frac{9\pi}{4}) = ?$

$\frac{\pi}{4}$ reference angle

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

ex: $\cos^{-1}(\cos\frac{\pi}{3}) = ?$

$\cos^{-1}(\frac{1}{2}) = \boxed{\frac{\pi}{3}}$

ex: $\cos^{-1}(\cos\frac{5\pi}{3}) = ?$

$\cos^{-1}(\frac{1}{2}) = \boxed{\frac{\pi}{3}}$

$\frac{5\pi}{3} = 300°$

$\frac{\pi}{3}$ = reference ∡

(*) if you don't like radians you can convert to degrees!

ex: $\cos^{-1}(\cos\frac{2\pi}{3}) = ?$

$\cos^{-1}(-\frac{1}{2}) = \boxed{\frac{2\pi}{3}}$

ex: $\cos^{-1}(\cos(\frac{5\pi}{6})) = ?$

$\cos^{-1}(-\frac{\sqrt{3}}{2}) = \boxed{\frac{5\pi}{6}}$

ex: $\sin^{-1}(\sin\frac{\pi}{3}) = ?$

$\sin^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\frac{\pi}{3}}$

ex: $\sin^{-1}(\sin 60°) = \sin^{-1}(\frac{\sqrt{3}}{2}) = \boxed{60°}$

ex: $\sin^{-1}(\sin 420°) = \sin^{-1}(\frac{\sqrt{3}}{2}) = \boxed{60°}$

ex: $\sin^{-1}(\sin\frac{4\pi}{3}) = ?$

$\sin^{-1}(-\frac{\sqrt{3}}{2}) = \boxed{-\frac{\pi}{3}}$

ex: $\cos^{-1}(\cos\frac{\pi}{3}) = ?$

$\cos^{-1}(\frac{1}{2}) = \boxed{\frac{\pi}{3}}$

ex: $\sin^{-1}(\sin\frac{2\pi}{3}) = ?$

$\sin^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\frac{\pi}{3}}$

ex: $\cos^{-1}(\cos(\frac{7\pi}{6})) = ?$

$\cos^{-1}(\cos 210°)$

$\cos^{-1}(-\frac{\sqrt{3}}{2}) = \boxed{\frac{5\pi}{6}}$

ex: $\cos^{-1}(\cos\frac{11\pi}{6}) = ?$

$\cos^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\frac{\pi}{6}}$

ex: $\tan^{-1}(\tan\frac{3\pi}{4}) = ?$

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

ex: $\tan^{-1}(\tan(\frac{\pi}{4})) = ?$

$\boxed{\frac{\pi}{4}}$

ex: $\sin^{-1}(\sin\frac{7\pi}{6}) = ?$

$\sin^{-1}(-\frac{1}{2}) = \boxed{-\frac{\pi}{6}}$

ex: $\sin^{-1}(\sin 390°) = 30°$

ex: $\sin^{-1}(\sin 210°) = -30°$

ex: $\cos^{-1}(\cos 750°) = 30°$

ex: $\tan^{-1}\left(\tan \frac{15\pi}{4}\right) = -\frac{\pi}{4}$