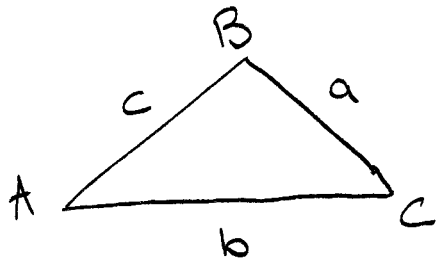
Lecture #21
MAT 123

Law of Sines, Law of Cosines, & Inverse Trig functions

Recall:

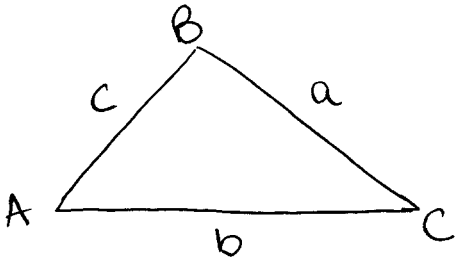
Law of Sines (ASA, SAA)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

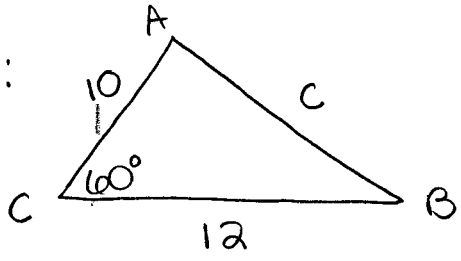


Law of Cosines (SSS, SAS)

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

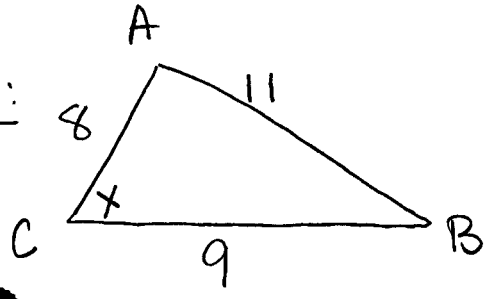


ex:



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 12^2 + 10^2 - 2(12)(10)\cos(60°)$$
$$c^2 = 144 + 100 - 2(12)(10)\cos(60°)$$
$$c^2 = 244 - 240 \cdot \cos(60°)$$
$$c^2 = 244 - 240\left(\tfrac{1}{2}\right)$$
$$c^2 = 244 - 120 = 124$$
$$\sqrt{c^2} = \sqrt{124}$$
$$\boxed{c = \sqrt{124}}$$

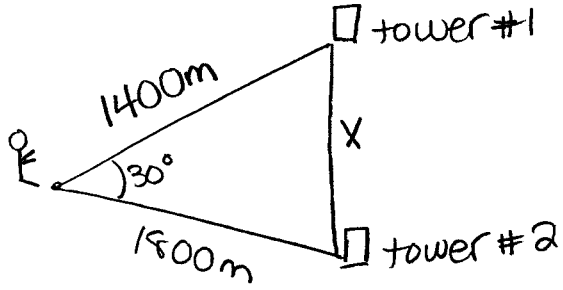
ex:



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$11^2 = 9^2 + 8^2 - 2(9)(8)\cos C$$
$$121 = 81 + 64 - 144\cdot\cos C$$
$$121 = 145 - 144\cos C$$
$$-145 \quad -145$$
$$\frac{-24}{-144} = \frac{-144\cos C}{-144}$$
$$\frac{24}{144} = \cos C$$
$$\frac{1}{6} = \cos C$$
$$C = \cos^{-1}\left(\frac{1}{6}\right)$$

Word problem

You measure the distance to a tower to be 1400m. You turn through an angle of 30° and measure the distance to a different tower to be 1800m. How far are the towers?



*need a calculator!

$$X^2 = 1400^2 + 1800^2 - 2(1400)(1800)\cos(30°)$$

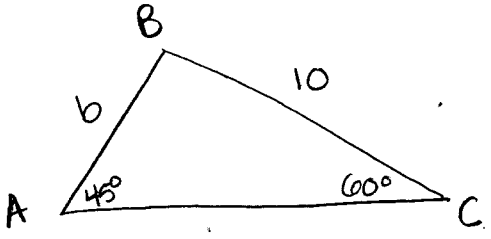
$$X^2 = 5200000 - 5040000\left(\frac{\sqrt{3}}{2}\right)$$

$$X^2 = 5200000 - 2520000\sqrt{3}$$

$$X = \sqrt{5200000 - 2520000\sqrt{3}}$$

$$\boxed{X = 913.91\text{m}}$$

ex:



find b

$$\frac{\sin 45°}{10} = \frac{\sin 60°}{b}$$

$$\frac{b \cdot \sin 45°}{\sin 45°} = \frac{10 \cdot \sin 60°}{\sin 45°}$$

$$b = \frac{10 \cdot \sin 60°}{\sin 45°} = \frac{10\left(\frac{\sqrt{3}}{2}\right)}{\frac{\sqrt{2}}{2}}$$

$$b = 5\sqrt{3} \cdot \frac{2}{\sqrt{2}} = \frac{10\sqrt{3}}{\sqrt{2}}$$

$$\boxed{b = \frac{10\sqrt{3}}{\sqrt{2}}}$$

Inverse Trig Functions

$$\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \arcsin\left(\frac{1}{2}\right)$$

just notation

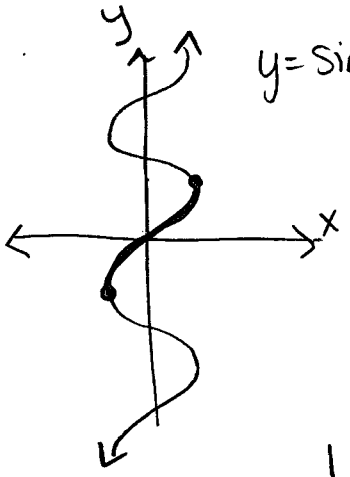
$$\sin X = A \iff X = \sin^{-1}(A)$$

(*) notice for Inverse Trig functions our input is the ratio of sides and our output is an angle!

$\sin\frac{\pi}{6} = \frac{1}{2}$

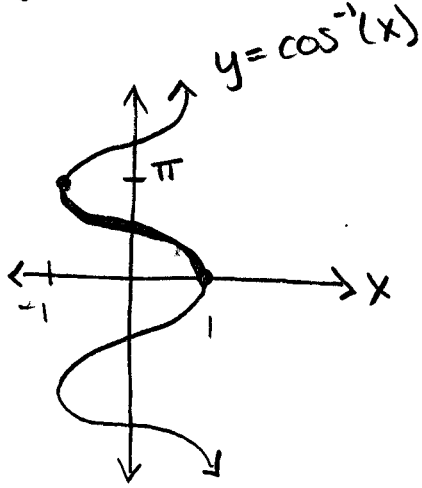
$\sin^{-1}\left(\frac{1}{2}\right) = \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$ infinite # of solutions

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ → this is called the principal angle.



$y = \sin^{-1}(x)$

Not a function!
We must restrict our domain to $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

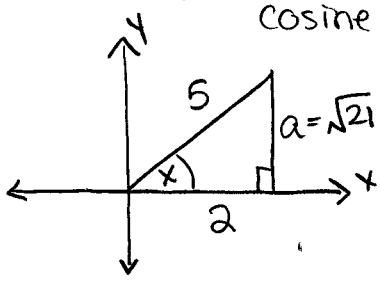


	Domain	Range
$\sin x$	all reals	$-1 \le y \le 1$
$\sin^{-1} x$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ (IV, I)
$\cos x$	all reals	$-1 \le y \le 1$
$\cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$

Switch the domain and range, but we must restrict

ex: $\sin\left(\cos^{-1}\left(\frac{2}{5}\right)\right) = ?$

cosine of some angle (x) is $\frac{2}{5}$



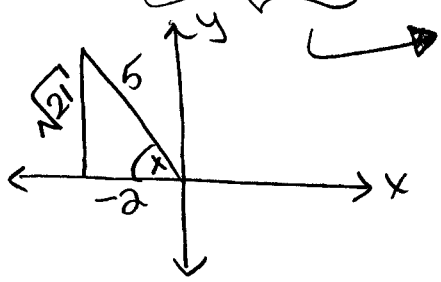
$a^2 + b^2 = c^2$
$a^2 + 2^2 = 5^2$
$a^2 = 25 - 4$
$a = \sqrt{21}$

Now use of Δ to find $\sin(x)$

$\sin x = \boxed{\frac{\sqrt{21}}{5}}$

ex: $\sin\left(\cos^{-1}\left(-\frac{2}{5}\right)\right) = ?$

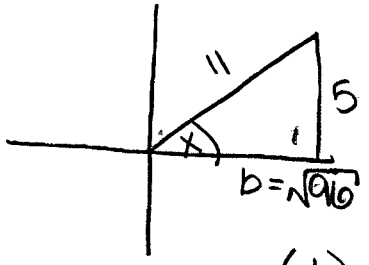
use this part to draw a △



$\sin x = \boxed{\frac{\sqrt{21}}{5}}$

Since $\cos^{-1}$ is (-) we are in Quad II

ex: $\cos\left(\sin^{-1}\left(\frac{5}{11}\right)\right) = ?$



$a^2+b^2=c^2$

$5^2+b^2=11^2$

$b^2 = 121-25$

$b=\sqrt{96}$

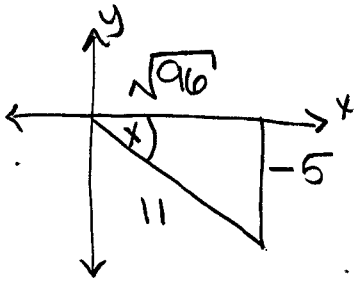
$\cos x = \boxed{\frac{\sqrt{96}}{11}}$

function	(+) $x \geq 0$	(-) $x < 0$
$\sin^{-1} x$	I	IV
$\cos^{-1} x$	I	II
$\tan^{-1} x$	I	IV

Never use III Quad! for inverse trig functions!

ex: $\cos\left(\sin^{-1}\left(\frac{-5}{11}\right)\right) = ?$

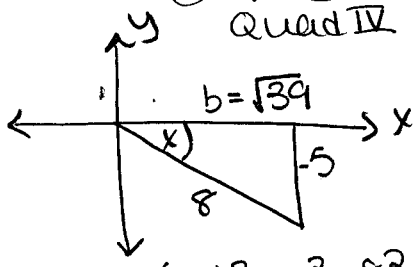
Quad IV



$\cos x = \boxed{\frac{\sqrt{96}}{11}}$

ex: $\cos\left(\sin^{-1}\left(\frac{-5}{8}\right)\right) = ?$

Quad IV



$(-5)^2+b^2=8^2$

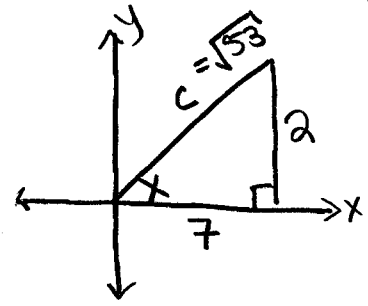
$b^2 = 39$

$b=\sqrt{39}$

$\cos x = \boxed{\frac{\sqrt{39}}{8}}$

ex: $\sin\left(\tan^{-1}\left(\frac{2}{7}\right)\right) = ?$

Quad I



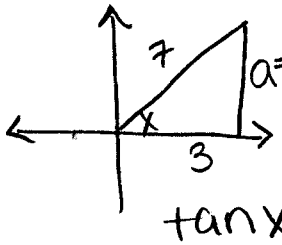
$2^2+7^2=c^2$

$53=c^2$

$c=\sqrt{53}$

$\sin x = \boxed{\frac{2}{\sqrt{53}}}$

ex: $\tan\left(\cos^{-1}\left(\frac{3}{7}\right)\right) = ?$



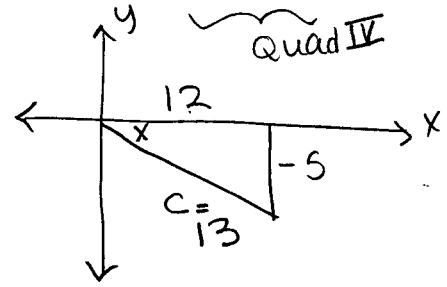
$a^2 = 7^2-3^2$

$a^2 = 40$

$a=\sqrt{40}$

$\tan x = \boxed{\frac{\sqrt{40}}{3}}$

ex: $\sin(\tan^{-1}(\frac{-5}{12}))=?$



$c^2=12^2+(-5)^2$

$c^2=169$

$c=\sqrt{169}=13$

$\sin x=\boxed{\frac{-5}{13}}$.