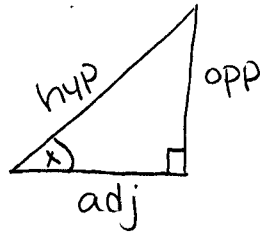
Lecture #20
MAT 123

Trigonometry Review / Law of Sines

Right Δ Trig



$$\sin x = \frac{opp}{hyp}$$

$$\cos x = \frac{adj}{hyp}$$

$$\tan x = \frac{opp}{adj}$$

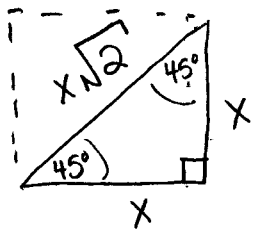
sin — hyp — adj — tan

"SohCahToa"

opp — cos — hyp — opp — adj

Special Δ's

(1) 45-45-90

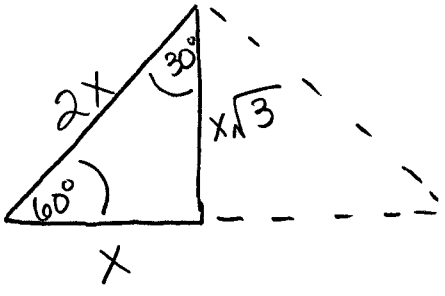


$$\sin 45° = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{x}{x} = 1$$

(2) 30-60-90



$$\sin 30° = \frac{x}{2x} = \frac{1}{2} \qquad \sin 60° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{x}{2x} = \frac{1}{2}$$

$$\tan 30° = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \frac{x\sqrt{3}}{x} = \sqrt{3}$$

notice: $\sin 30° = \cos 60°$
$\cos 30° = \sin 60°$

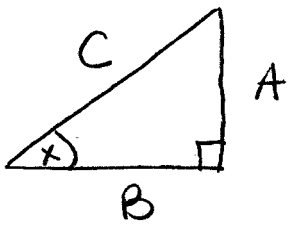
sine and cosine are complementary functions!

$30 + 60 = 90$

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$\tan x = \frac{\sin x}{\cos x}$

you NEED to know these exact values for the exam!



$* \frac{A^2}{C^2} = \left(\frac{A}{C}\right)^2$

$\frac{A^2 + B^2 = C^2}{C^2 \quad C^2 \quad C^2}$

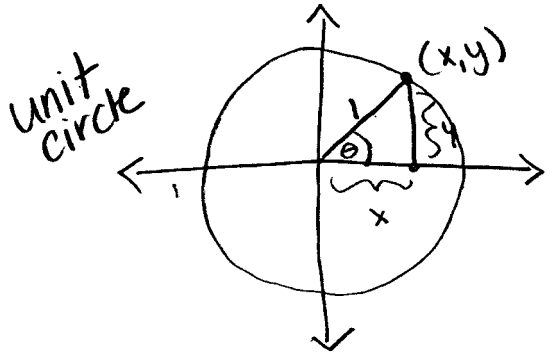
$\left(\frac{A^2}{C^2}\right) + \left(\frac{B^2}{C^2}\right) = 1$

$\sin^2 x + \cos^2 x = 1$

note:
$\sin^2 x = (\sin x)^2$
notation

$\tan x = \frac{A}{B} = \frac{\frac{A}{C}}{\frac{B}{C}} = \frac{\sin x}{\cos x}$

Extending Trig to all angles:



$\sin\theta = \frac{y}{1} = y$

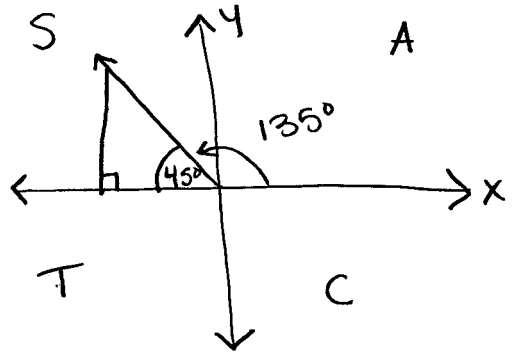
$\cos\theta = \frac{x}{1} = x$

$(x,y) \Longleftrightarrow (\cos\theta, \sin\theta)$

Quad I: (x,y) sin and cos both (+)
Quad II: $(-x,y)$ sin(+) cos(-)
Quad III: $(-x,-y)$ sin and cos both (-)
Quad IV: $(x,-y)$ sin(-) cos(+)

"All Students Take Calculus"
All → Quad I, all (+)
Students → Quad II, sin (+)
Take → Quad III, tan (+)
Calculus → Quad IV, cos (+)

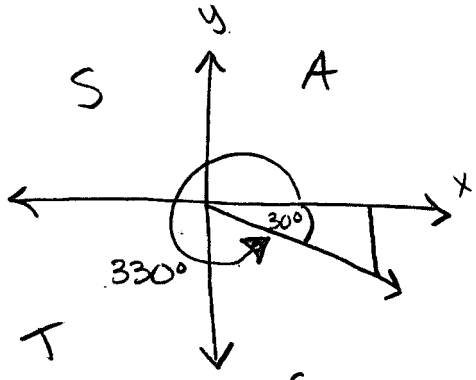
① $\sin(135)^\circ = ?$
② $\tan(330^\circ) = ?$
③ $\cos(225^\circ) = ?$
④ $\sin(210^\circ) = ?$
⑤ $\cos(240^\circ) = ?$

① 

$180 - 135 = 45$

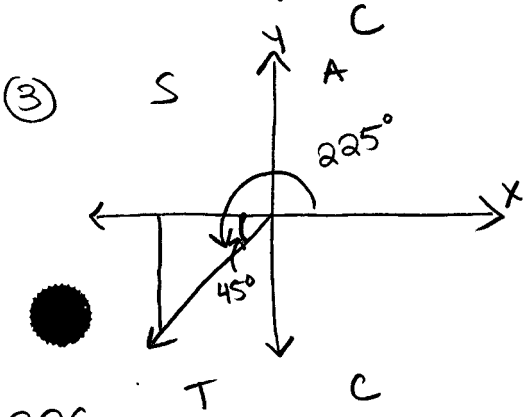
$\sin 135^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 135^\circ = \frac{\sqrt{3}}{2}$

② 

$\tan(330^\circ) = -\tan(30^\circ) = -\frac{1}{\sqrt{3}}$

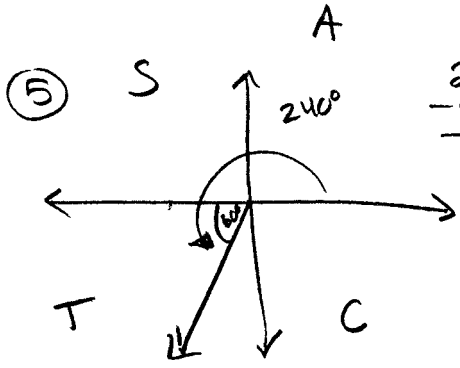
$\tan 330^\circ = -\frac{1}{\sqrt{3}}$

③ 

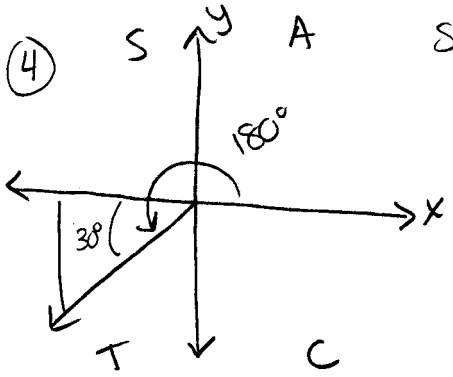
$\cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

$\cos 225^\circ = -\frac{\sqrt{2}}{2}$

$225 - 180 = 45$

⑤ 

$240 - 180 = 60$

④ 

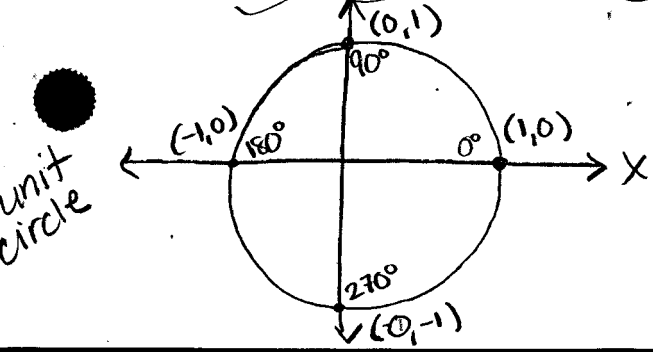
$\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$

$\sin 210^\circ = -\frac{1}{2}$

$\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$

$\cos 240^\circ = -\frac{1}{2}$

OTHER SPECIAL ANGLES



unit circle

$0^\circ = (1,0) = (\cos 0^\circ, \sin 0^\circ)$

$\cos 0^\circ = 1, \sin 0^\circ = 0$

$90^\circ = (0,1)$ $\cos 90^\circ = 0, \sin 90^\circ = 1$

$180^\circ = (-1,0)$ $\cos 180^\circ = -1, \sin 180^\circ = 0$

$270^\circ = (0,-1)$ $\cos 270^\circ = 0, \sin 270^\circ = -1$

	0°	90°	180°	270°
sin	0	1	0	-1
cos	1	0	-1	0
tan	0	und	0	und

undefined b/c you cannot divide by 0

Radians

$360° = 2\pi$ radians

$180° = \pi$ radians

π conversion factor!

To convert from degrees to radians:

$\theta \cdot \frac{\pi}{180°}$

To convert from radians to degrees:

$\theta \cdot \frac{180°}{\pi}$

Convert the following:

① $135°$

$135° \times \frac{\pi}{180°} = \frac{135\pi}{180} = \boxed{\frac{3\pi}{4}}$

② $330°$

$330° \times \frac{\pi}{180°} = \frac{330\pi}{180} = \boxed{\frac{11\pi}{6}}$

③ $225°$

$225° \times \frac{\pi}{180°} = \frac{225\pi}{180} = \boxed{\frac{5\pi}{4}}$

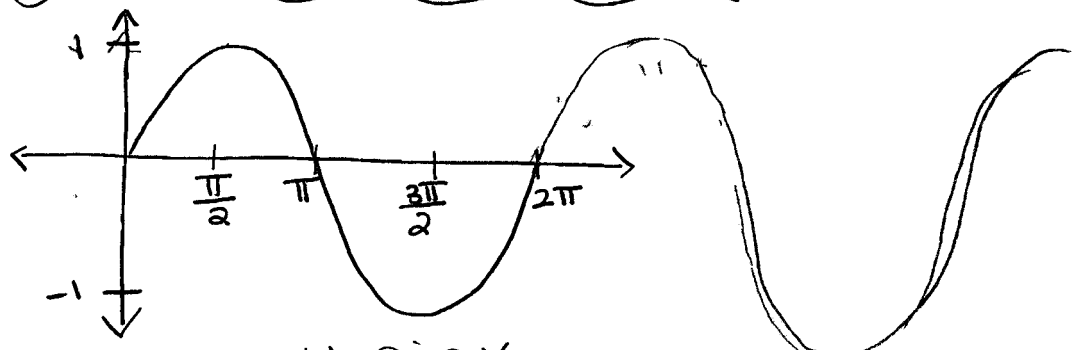
④ $210°$

$210° \times \frac{\pi}{180°} = \frac{210\pi}{180} = \boxed{\frac{7\pi}{6}}$

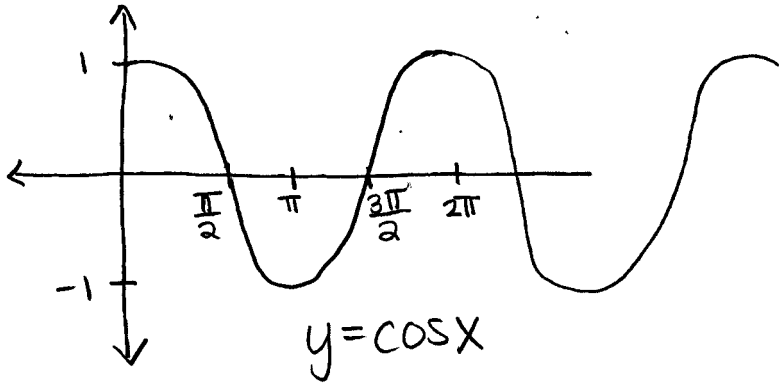
⑤ $240°$

$240° \times \frac{\pi}{180°} = \frac{240\pi}{180} = \boxed{\frac{4\pi}{3}}$

Graphing Trig Functions



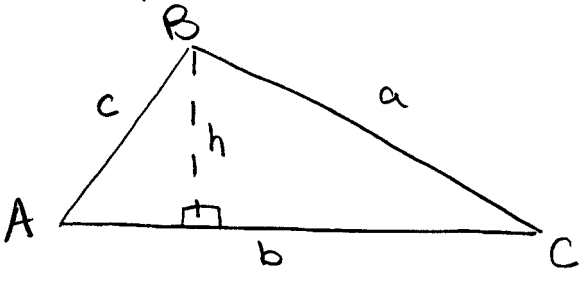
$y = \sin x$



$y = \cos x$

notice:
$y = \cos x$ is the same graph as $y = \sin x$ just shifted by 90°

What happens when we don't have a right Δ???



$\sin A = \frac{h}{c}$ $\sin C = \frac{h}{a}$

$c \cdot \sin A = h$ $a \cdot \sin C = h$

$c \cdot \sin A = a \cdot \sin C$

$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$

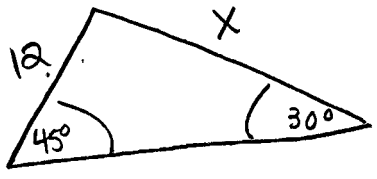
* We can use this identity for any Δ, it doesn't have to be a right Δ!

A, B, C are angles
a, b, c are side lengths.

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Problem: Find x



$\frac{\sin 45°}{x} = \frac{\sin 30°}{12}$

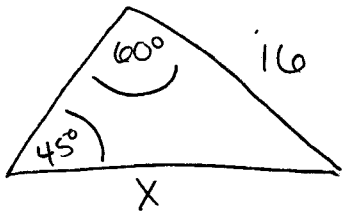
$\frac{12 \cdot \sin 45°}{\sin 30°} = \frac{x \cdot \sin 30°}{\sin 30°}$

$x = \frac{12 \cdot \sin 45°}{\sin 30°}$

$x = \frac{(12)(\frac{\sqrt{2}}{2})}{\frac{1}{2}}$

$x = 12\sqrt{2}$

ex.



Find X

$$\frac{\sin 45^\circ}{16} = \frac{\sin 60^\circ}{X}$$

$$\frac{X \cdot \sin 45^\circ}{\sin 45^\circ} = \frac{16 \cdot \sin 60^\circ}{\sin 45^\circ}$$

$$X = \frac{16 \cdot \sin 60^\circ}{\sin 45^\circ}$$

$$X = \frac{16\left(\frac{\sqrt{3}}{2}\right)}{\frac{\sqrt{2}}{2}} = \frac{16\sqrt{3}}{2} \cdot \frac{2}{\sqrt{2}} = \frac{16\sqrt{3}}{\sqrt{2}}$$

$$\boxed{X = \frac{16\sqrt{3}}{\sqrt{2}}}$$