Lecture #13
MAT 123

Exponents, ellipses, polynomial division

Exponents

$x^5 + x^8 = x^5 + x^8$

$x^5 \cdot x^8 = x^{5+8} = x^{13} \Longrightarrow (x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) = x^{13}$

$(x^5)^8 = x^{5 \cdot 8} = x^{40} \Longrightarrow \underbrace{(x^5 \cdot x^5 \cdot x^5 \cdots x^5)}_{8} = x^{40}$

Basic Exponent Rules:

$x^a \cdot x^b = x^{a+b}$

$(x^a)^b = x^{a \cdot b}$

$\frac{x^a}{x^b} = x^{a-b}$

* make sure you know these Rules!

example: $\frac{x^5}{x^5} = x^{5-5} = x^0 = 1$

$x^0 = 1$ always (except when $x=0$)

example: $x^{-a} = x^{0-a} = \frac{x^0}{x^a} = \frac{1}{x^a}$

$x^{-a} = \frac{1}{x^a}$

example: $x^{\frac{1}{a}} = \sqrt[a]{x}$

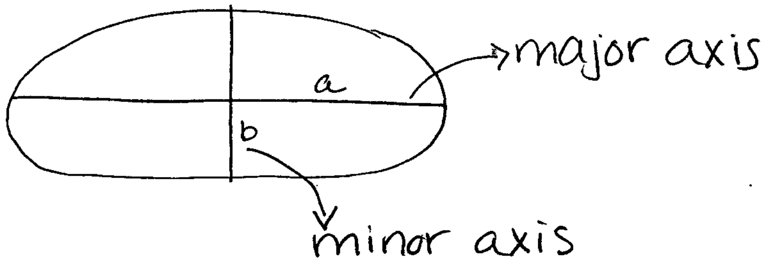
* Rules of Exponents *

1. $x^a \cdot x^b = x^{a+b}$
2. $(x^a)^b = x^{a \cdot b}$
3. $\frac{x^a}{x^b} = x^{a-b}$
4. $x^0 = 1$
5. $x^{-a} = \frac{1}{x^a}$
6. $x^{\frac{1}{a}} = \sqrt[a]{x}$

Need to know these Rules!

Ellipses

- "stretched out circle"



"a, b are like radii in a circle."

$x^2 + y^2 = r^2$ (Circle)

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ ellipse centered at $(0,0)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$ ellipse centered at (h,k)

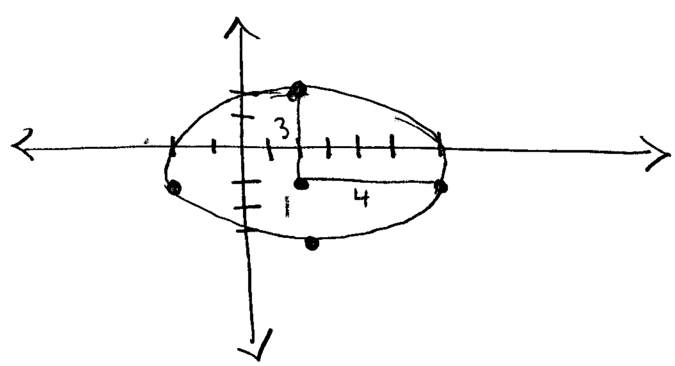
example: $$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$$

Graph this

$\sqrt{16} = a = 4$

$\sqrt{9} = b = 3$

center $= (2,-1)$

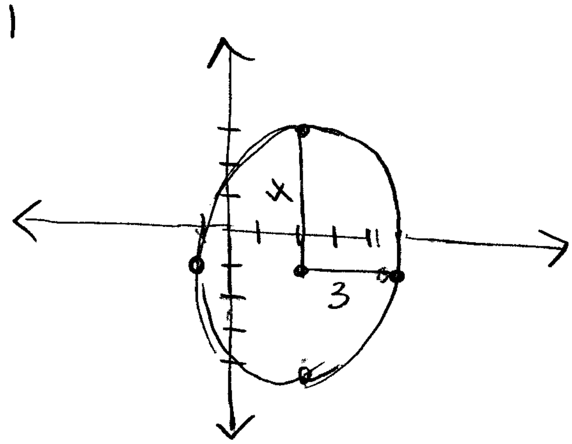


example: $$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$$

$a = 3$

$b = 4$

center $= (2,-1)$



Polynomial long division

- $$\frac{x^3+5x^2+7x+3}{x+1} = (x+1)(\_\_\_\_\_)$$

$$\begin{array}{r|l}
 & x^2+4x+3 \\ \hline
x+1 & \cancel{x^3}+5x^2+7x+3 \\
 & -(\cancel{x^3}+x^2) \downarrow \\ \hline
 & 4\cancel{x^2}+7x \\
 & -(4x^2+4x) \downarrow \\ \hline
 & 3\cancel{x}+3 \\
 & -(\cancel{3x}+3) \\ \hline
 & 0
\end{array}$$

look at the 1^{st} term
" x times what is x^3?"
x^2
notice that is our first term
" x times what is $4x^2$?"
$4x$
2nd term
" x times what is $3x$?"
3
last term

Thus
$$\frac{x^3+5x^2+7x+3}{x+1} = (x+1)(x^2+4x+3)$$
:)

- **example:**

$$\frac{x^3-3x^2-8x+4}{x+2} = ?$$

$$\begin{array}{r|l}
 & x^2-5x+2 \\ \hline
x+2 & \cancel{x^3}-3x^2-8x+4 \\
 & -(\cancel{x^3}+2x^2) \downarrow \\ \hline
 & -5\cancel{x^2}-8x \\
 & -(-\cancel{5}x^2-10x) \downarrow \\ \hline
 & 2\cancel{x}+4 \\
 & -(2\cancel{x}+4) \\ \hline
 & 0
\end{array}$$

so $$\frac{x^3-3x^2-8x+4}{x+2} = (x+2)(x^2-5x+2)$$

-

example:

$$
\begin{array}{r}
x^3-5x^2+4x+1 \\
x+3\,\overline{)\,\cancel{x^4}-2x^3-11x^2+13x+3} \\
-(\cancel{x^3}+3x^3)\qquad\qquad\qquad \\
\hline
-5\cancel{x^3}-11x^2\qquad\qquad \\
-(-5\cancel{x^3}-15x^2)\qquad\qquad \\
\hline
4\cancel{x^2}+13x\qquad \\
-(\cancel{4x^2}+12x)\qquad \\
\hline
\cancel{x}+3 \\
-(\cancel{x}+3) \\
\hline
0
\end{array}
$$

So $(x+3)(x^3-5x^2+4x+1) = x^4-2x^3-11x^2+13x+3$

or

$$\frac{x^4-2x^3-11x^2+13x+3}{x+3} = x^3-5x^2+4x+1$$

example:

$$
\begin{array}{r}
x^3-4x^2+3x+1 \\
x-5\,\overline{)\,\cancel{x^4}-9x^3+23x^2-14x-5} \\
-(\cancel{x^4}-5x^3)\qquad\qquad\qquad \\
\hline
-4\cancel{x^3}+23x^2\qquad\qquad \\
-(-\cancel{4}x^3+20x^2)\qquad\qquad \\
\hline
3x\cancel{^2}-14x\qquad \\
-(\cancel{3}x^2-15x)\qquad \\
\hline
\cancel{x}-5 \\
-(\cancel{x}-5) \\
\hline
0
\end{array}
$$

$$\frac{x^4-9x^3+23x^2-14x-5}{x-5} = x^3-4x^2+3x+1$$

OR

$$(x-5)(x^3-4x^2+3x+1) = x^4-9x^3+23x^2-14x-5$$

What happens if it doesn't divide evenly?

o o o

$\Longrightarrow$

example:

$$\begin{array}{r l}
 & x^2-3x-11 \quad + \quad \dfrac{-56}{x-5} \\
x-5 \; \big) & x^3-8x^2+4x-1 \\
 & -(\cancel{x^3}-5x^2) \\ \hline
 & -3\cancel{x^2}+4x \\
 & -(\cancel{-3x^2}+15x) \\ \hline
 & -11\cancel{x}-1 \\
 & -(\cancel{-11x}+55) \\ \hline
 & -56. \leftarrow \text{Remainder}
\end{array}$$

So $\dfrac{x^3-8x^2+4x-1}{x-5} = x^2-3x-11+\left(\dfrac{-56}{x-5}\right)$

example:

$$\begin{array}{r l}
 & x^3+x^2-x+3 \\
x^2+4 \; \big) & x^5+x^4+3x^3+7x^2-4x+12 \\
 & -(x^5++4x^3 \\ \hline
 & \cancel{x^4}-x^3+7x^2 \\
 & -(\cancel{x^4}+___+4x^2) \\ \hline
 & -\cancel{x^3}+3x^2-4x \\
 & -(\cancel{-x^3}+___-4x) \\ \hline
 & 3\cancel{x^2}+0x+12 \\
 & -(\cancel{3x^2}+___+12) \\ \hline
 & 0
\end{array}$$

(*) Notice

x^2+4 is NOT a linear term so we need to make sure we line up the terms correctly when doing our long division.