Lecture #12
MAT 123

Lines, Circles, and Parabolas

Lines:

- consistent slope
- we can find the equation of a line if we are given two points on the line.

↳ ex: find the equation of the line through (10, 3) and (6, 11).

Slope $= \dfrac{y_1 - y_2}{x_1 - x_2}$ Slope $= \dfrac{11-3}{6-10} = \dfrac{8}{-4} = -2$

Point-slope formula: $y - y_1 = m(x - x_1)$

m = slope
(x_1, y_1) = point on the line

$$y - 3 = -2(x - 10)$$
$$y - 3 = -2x + 20$$
$$y = -2x + 20 + 3$$
$$\boxed{y = -2x + 23}$$

→ this is the equation of the line through (10,3) and (6,11)

ex: find the equation of the line through (6,5) and (2,−8)

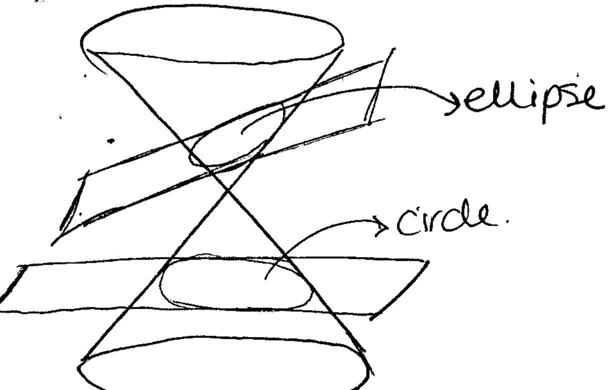
$$y - y_1 = m(x - x_1)$$

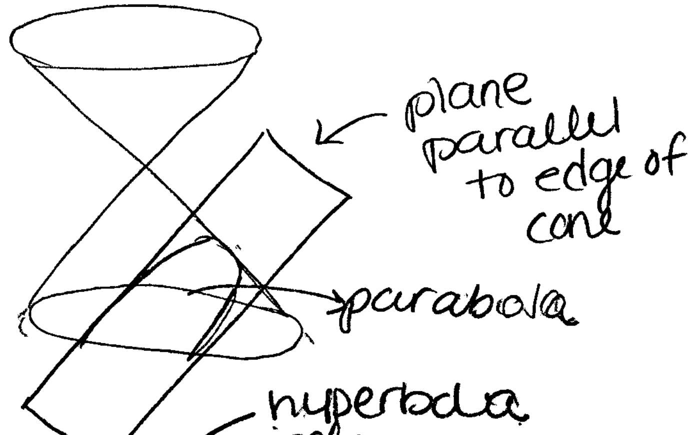
$$m = \frac{5-(-8)}{6-2} = \frac{5+8}{4} = \frac{13}{4}$$

$$\boxed{y - 5 = \frac{13}{4}(x - 6)}$$

$$\boxed{y = \frac{13}{4}x - \frac{78}{4} + 5}$$

both of these are acceptable answers!

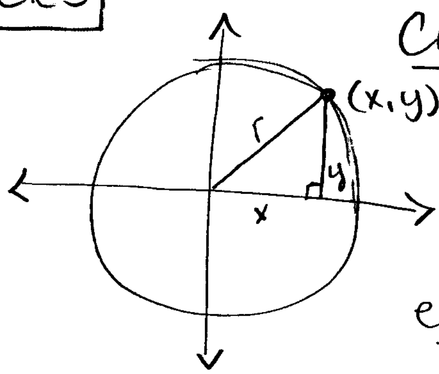




"Conic Sections" — hyperbola, circle, ellipse, parabola

* In this course we will talk about parabolas, circles, and ellipses.

Circles



Circle: r = radius.

$x^2 + y^2 = r^2$ Equation of a circle centered at the origin w/ radius r

example: $x^2 + y^2 = 25$
this is a circle centered at the origin with a radius of 5

what if we have a circle NOT centered at the origin?

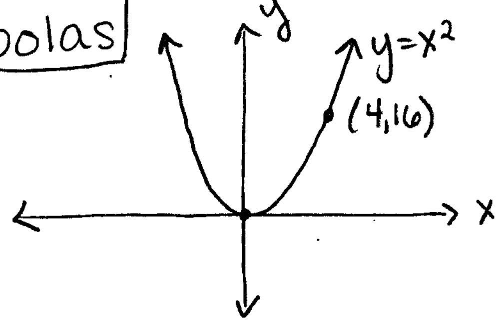
* $(x-h)^2 + (y-K)^2 = R^2$ Equation of a circle centered at (h, K) with radius R

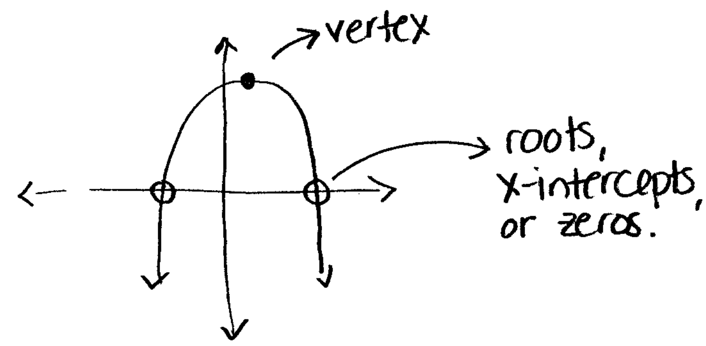
Important! make sure you know this!

example: Find the equation of a circle centered at $(5, 11)$ with a radius of 4.

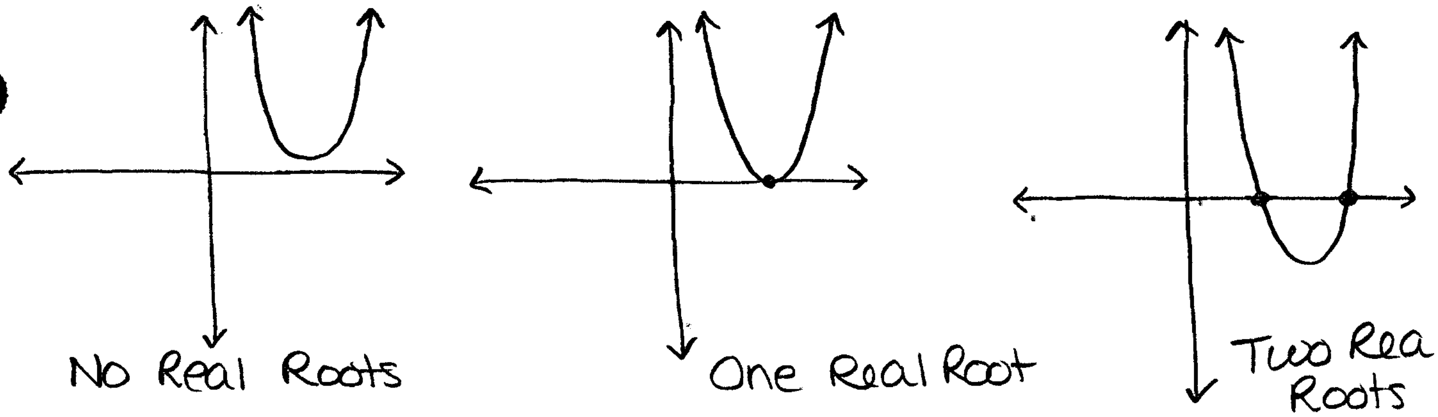
$(x-5)^2 + (y-11)^2 = 4^2$ → Answer

Parabolas





Three Different Parabolas



Given: $y = x^2 + 12x - 20$

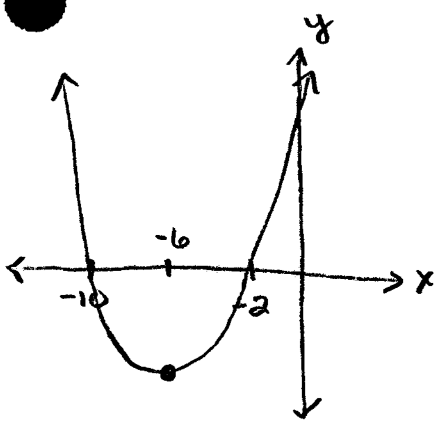
How do we find the roots and vertex?

Factor: $y = x^2 + 12x - 20$

$(x+10)(x+2) = 0$

$x + 10 = 0$	$x + 2 = 0$
$x = -10$	$x = -2$

the vertex is half way between the two real roots. @ $x = -6$



roots or zeros or where the parabola crosses the x-axis

the vertex is a point so we have the x-value. To find the y-value use $x = -6$ in the original equation.

$y = x^2 + 12x - 20$

$y = (-6)^2 + 12(-6) - 20$

$y = 36 - 72 - 20 = -56$

vertex $= (-6, -56)$

Another way we can find the vertex is by completing the square.

$x^2 + 12x + 20 = 0$

$x^2 + 12x = 20 \qquad \left(\frac{12}{2}\right)^2$

$x^2 + 12x + 6^2 = 20 + 6^2$

$(\cancel{x}^2 + 6)^2 = -20 + 36$

$(x+6)^2 = 16 \implies (x+6)^2 - 16 = 0$

Practice Completing the Square:

$$x^2+8x+\cancel{5}=0 \quad -5$$
$$\quad\quad\quad -5$$

$$x^2+8x=-5$$
$$x^2+8x+16=-5+16$$
$$(x+4)^2=11$$
$$(x+4)^2-11=0$$

to solve for the roots:

$$(x+4)^2\cancel{-11}=0 \quad +11$$
$$\quad\quad +11$$

$$\sqrt{(x+4)^2}=\sqrt{11}$$
$$(x+4)=\pm\sqrt{11}$$
$$x=\pm\sqrt{11}-4$$
$$x=4\pm\sqrt{11}$$

ex: $x^2+10x-7=0$
$$x^2+10x=7$$
$$x^2+10x+25=7+25$$
$$(x+5)^2=32$$
$$x+5=\pm\sqrt{32}$$
$$x=-5\pm\sqrt{32}$$

General Case:
$$ax^2+bx+c=0$$
$$ax^2+bx=-c$$
$$x^2+\frac{b}{a}x=\frac{-c}{a}$$
$$x+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

How to Complete the Square:

Step 1: move constant term over

Step 2: add $\left(\frac{b}{2}\right)^2$ to both sides of the equation

Step 3: rewrite the left side as $\left(x+\frac{b}{2}\right)^2$

Step 4: Solve

Note we can only complete the square when the x^2 term has a coefficient of 1!

ex: $3x^2+12x-5=0$
$$\frac{3x^2+12x=5}{3}$$
$$x^2+4x=\frac{5}{3}$$
$$x^2+4x+4=\frac{5}{3}+4$$
$$(x+2)^2=\frac{17}{3}$$
$$x+2=\pm\sqrt{\frac{17}{3}}$$
$$x=-2\pm\sqrt{\frac{17}{3}}$$

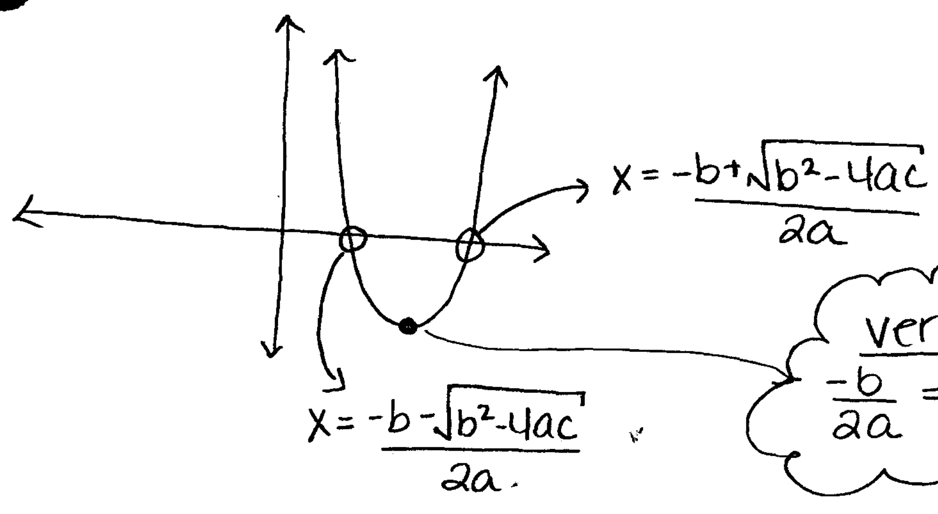
$$\left(x+\frac{b}{2a}\right)=\frac{\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Quadratic Formula!

↓

Another way to find the roots of a quadratic equation

Quadratic Formula $X = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ → This will give you where the parabola crosses the x-axis



vertex: $\dfrac{-b}{2a} = x$ → * this is called the axis of symmetry or the x-coordinate of the vertex

if $b^2-4ac=0$ => then there is only one root!
if $b^2-4ac<0$ => then there are no real roots!
if $b^2-4ac>0$ => then you have two real roots!

example: $y = 2x^2 - 14x + 24$
How can we graph this?

$X = \dfrac{-b}{2a} = \dfrac{14}{2(2)} = \dfrac{14}{4} = \dfrac{7}{2}$ → x-coordinate of the vertex

$y = 2\left(\frac{7}{2}\right)^2 - 14\left(\frac{7}{2}\right) + 24$

$= 2\left(\frac{14}{4}\right) - \frac{14 \cdot 7}{2} + 24$

$= \frac{2 \cdot 49}{4} - \frac{14 \cdot 7}{2} + 24 = 24.5 - 49 + 24 = -\frac{1}{2}$

vertex $= \left(\frac{7}{2}, -\frac{1}{2}\right)$

$X = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

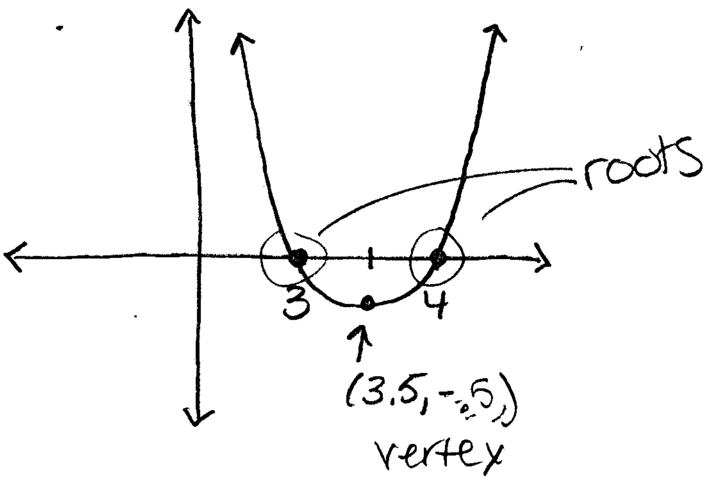
$X = \dfrac{14 \pm \sqrt{14^2-4(2)(24)}}{2(2)}$

$X = \dfrac{14 \pm \sqrt{4}}{4} = \dfrac{14 \pm 2}{4}$

$x = \dfrac{14+2}{4} = 4$

$x = \dfrac{14-2}{4} = 3$

roots: $X=4$ and $X=3$



y-intercept:
Plug in 0 for x and solve for y.

$y = 2x^2 - 14x + 24$

$y = 2(0)^2 - 14(0) + 24$

$y = 24$

$(0, 24) = $ y-intercept

example: $y = x^2 - 16x - 36$

$x = \dfrac{16 \pm \sqrt{(16)^2 - 4(1)(-36)}}{2(1)}$

$x = \dfrac{16 \pm \sqrt{400}}{2}$

$x = \dfrac{16 + \sqrt{400}}{2} = 18$

$x = \dfrac{16 - \sqrt{400}}{2} = -2$

roots:
$x = 18$
$x = -2$
$(18, 0)$
$(-2, 0)$

vertex: $x = \dfrac{-b}{2a}$

$x = \dfrac{16}{2(1)} = 8$

$y = (8)^2 - 16(8) - 36$

$y = 64 - 128 - 36$

$y = -100$

vertex $= (8, -100)$

sketch: