Lecture #11
MAT.123

● Midterm Review:
   Part II

① $\sin 3x = \frac{1}{2}$ $0 < x < \pi$

$3x = 30°$
$3x = 30°, 150°, 390°,$
     $510°,$

$\underbrace{\qquad\qquad\qquad}$

$\Downarrow$ ‖

$x = 10°, 50°, 130°, 170°$ ⟶ ※ note we need to find all values of x that are less than 180°
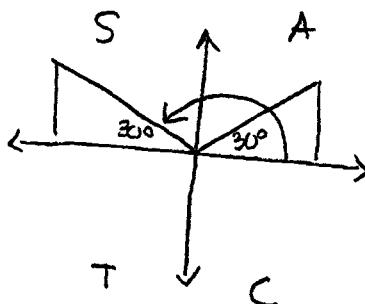
"where is sine $\frac{1}{2}$?
   $30°, 150°, \ldots$"

Now convert to radians by multiplying by $\frac{\pi}{180}$:

$$10° \times \frac{\pi}{180} = \frac{10\pi}{180} = \frac{\pi}{18}$$

$$30° \times \frac{\pi}{180} = \frac{30\pi}{180} = \frac{\pi}{6}$$

$$150° \times \frac{\pi}{180} = \frac{150\pi}{180} = \frac{5\pi}{6}$$

$$170° \times \frac{\pi}{180} = \frac{170\pi}{180} = \frac{17\pi}{18}$$

Answer:
$\frac{\pi}{18}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{18}$

② $\cos 3x = \frac{\sqrt{3}}{2}$ $0 \leq x \leq 2\pi$

"where is cosine equal to $\frac{\sqrt{3}}{2}$?"
$3x = 30°, 330°, 390°, 690°, 750°, 1050°$
$x = 10°, 110°, 130°, 230°, 250°, 350°$

※ need to find all x-values between 0° and 360°

now convert to radians:

$x = \frac{10\pi}{180}, \frac{110\pi}{180}, \frac{130\pi}{180}, \frac{230\pi}{180}, \frac{250\pi}{180}, \frac{350\pi}{180}$

$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$ ⟶ Answer
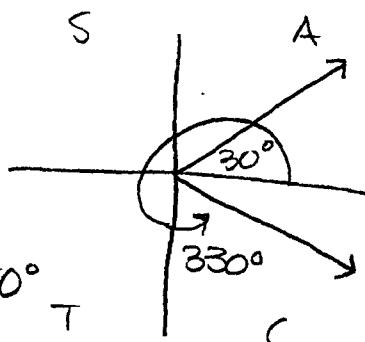
③ Graphing:

$$y = \textcircled{4}\sin\left(x - \boxed{\tfrac{\pi}{6}}\right) \textcircled{+2} \longrightarrow \text{shift up 2}$$

stretch by 4 ↗   ↑ shift right by $\pi/6$

midline $y = 2$

$\tfrac{\pi}{6}$ $\tfrac{7\pi}{6}$ $2\pi + \tfrac{\pi}{6} = \tfrac{13\pi}{6}$
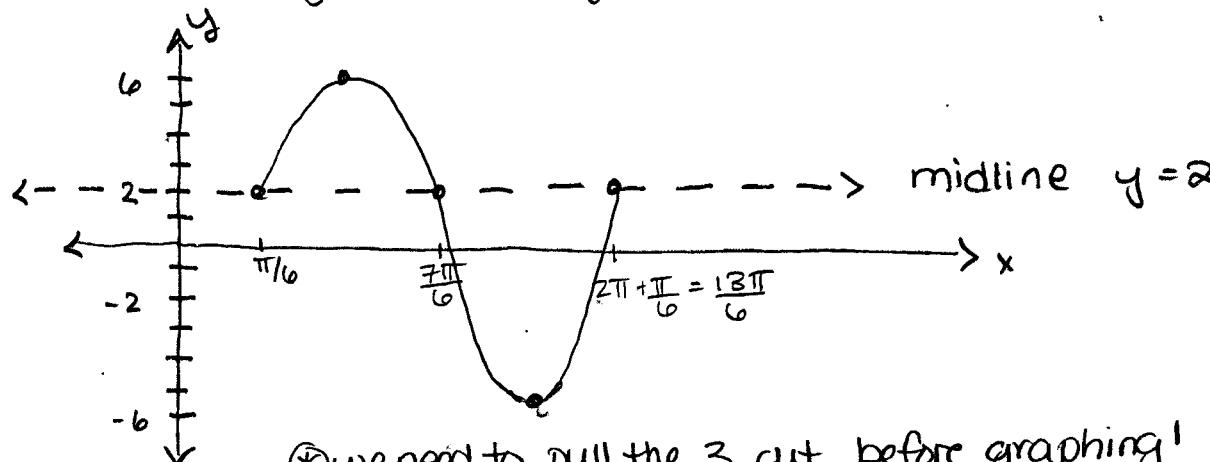
✱ we need to pull the 3 out before graphing!

④ $y = 4\sin(3x - \pi) + 2$

$= \textcircled{4}\sin\boxed{3}\left(x - \tfrac{\pi}{3}\right)\textcircled{+2} \longrightarrow$ shift up 2

stretch by 4   ↳ shift right by $\pi/3$

→ compress by 3

midline $y = 2$

$\tfrac{\pi}{3}$ $\tfrac{4\pi}{3}$ $2\pi + \pi/3 = \tfrac{7\pi}{6}$

$\boxed{\pi/9}$ $\boxed{4\pi/9}$ $\boxed{7\pi/9}$
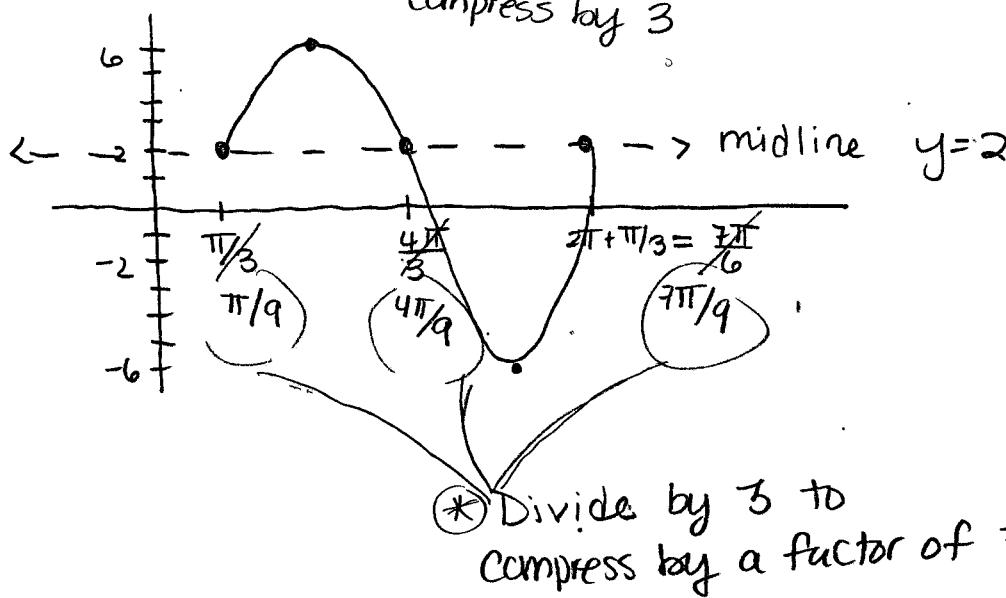
✱ Divide by 3 to compress by a factor of 3

⑤ A person is driving 50 miles/hour for 2 hours. How far did they travel?

$$D = r \cdot t \quad (\text{"Distance equals rate times time"})$$

$$D = 50 \, m/hr \cdot 2hr = \boxed{100 \, hrs.} \; \text{Answer}.$$

⑥ A person is driving on a long road trip. For the first 2 hours they drive 50 miles/hr. Then they slow down for the following two hours and only go 30 miles/hour. Then he stops for an hour, to eat. He gets back on the road and travels at 60 miles/hour for the remaining -90 miles.

$$f(t) = \begin{cases} 50t \,; & 0 \leq t < 2 \\ 100 + 30(t-2) \,; & 2 \leq t < 4 \\ 160 \,; & 4 \leq t < 5 \\ 160 + 65(t-5) \,; & 5 \leq t \leq \frac{90}{65} + 5 \end{cases}$$

$$r \cdot t = D$$
$$65t = 90$$
$$t = \frac{90}{65}$$

this shows us the distance that has been traveled based on the time that has passed.

⑦ Power Company Problem

cost   hour

$$C(h) = \begin{cases} .24h \,; & 0 \leq h < 250 \\ 60 + .26(h-250) \,; & 250 \leq h < 750 \\ 190 + .28(h-750) \,; & 750 \leq h \end{cases}$$

this function gives us the cost/hour charged for power.

⑧ If $f(x) = \dfrac{4x^3 - 1}{7}$, find $f^{-1}(x)$.

$$y = \frac{4x^3 - 1}{7}$$

$$7 \cdot x = \frac{4y^3 - 1}{7} \cdot 7$$

$$7x = 4y^3 - 1$$
$$ +1 \qquad +1$$

$$\frac{7x+1}{4} = \frac{4y^3}{4}$$

$$\sqrt[3]{\frac{7x+1}{4}} \overset{3}{=} \sqrt[3]{y^3}$$

$$y = \sqrt[3]{\frac{7x+1}{4}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{7x+1}{4}}$$ Answer.

⑨ $f(x) = \dfrac{2x+11}{3x-7}$ find $f^{-1}(x)$

$y = \dfrac{2x+11}{3x-7}$

$\dfrac{x}{1} \bcancel{\dfrac{}{}} \dfrac{2y+11}{3y-7}$

$2y+11 = x(3y-7)$

$2y+11 = 3xy - 7x$

$2y - 3xy = -11 - 7x$

$y(2-3x) = -11 - 7x$

$y = \dfrac{-11-7x}{2-3x}$

Answer

$f^{-1}(x) = \dfrac{-11-7x}{2-3x} = \dfrac{7x+11}{3x-2}$

⑩ a. $f(x) = \sqrt{3-5x}$, find the Domain.

$3 - 5x \geq 0$

$-5x \geq -3$

$\dfrac{-5x}{-5} \geq \dfrac{-3}{-5}$

Answer

$x \leq \sqrt[3]{5}$ or $(-\infty, \sqrt[3]{5})$

※ you cannot take the square root of a negative number.

b. $f(x) = \sqrt[3]{3-5x}$, find the Domain.

Answer $(-\infty, \infty)$ or all real numbers $(\mathbb{R})$

※ you can take the cube root of any number

c. $f(x) = \dfrac{\sqrt{2x-1}}{x-10}$, find the Domain.

$2x - 1 \geq 0$ $x - 10 \neq 0$

$x \geq \frac{1}{2}$ and $x \neq 10$

Answer

$[\frac{1}{2}, 10) \cup (10, \infty)$

⑪ If $\sin x = \dfrac{8}{9}$, $\dfrac{\pi}{2} \leq x \leq \pi$, find $\tan x$.
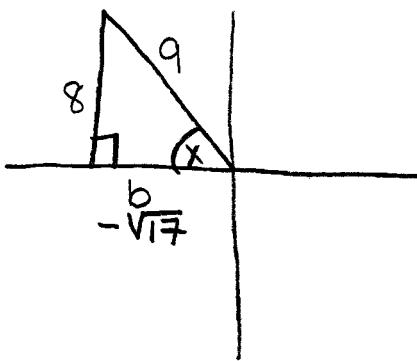
$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 9^2$$
$$b^2 = 17$$
$$b = \pm\sqrt{17}$$

$$\tan x = \dfrac{-8}{\sqrt{17}}$$

⑫ $f(x) = \sqrt{x^3 - 1}$ $\qquad g(x) = \dfrac{2}{x}$

a. find $f \circ g(x)$

$$\sqrt{\left(\tfrac{2}{x}\right)^3 - 1}$$

* take $g(x)$ and plug it in for x in $f(x)$

b. find $g \circ f(x)$

$$\dfrac{2}{\sqrt{x^3 - 1}}$$

* take $f(x)$ and plug it in for x in $g(x)$.

Good Luck!!!