Lecture #10
MAT 123

Midterm Review

(1) $f(x)$ "stuff"
- Domain
- Inverse
- composition
- $f(x+h) - f(x)$

(2) Trigonometry
- special angles
- six trig functions
- word problems

} Topics on the midterm!

Practice Questions

① $f(x) = 5x^3 - 4x + 1$

$g(x) = \frac{2}{x-6}$

Find: a. $g(f(2))$
b. $f(g(8))$
c. $f(g(x))$ and $g(f(x))$

a. $g(f(2)) = ?$

$f(2) = 5(2)^3 - 4(2) + 1$
$= 5(8) - 8 + 1$
$= 33$

$g(f(2)) = g(33)$

$g(33) = \frac{2}{33-6} = \boxed{\frac{2}{27}}$

b. $f(g(8)) = ?$

$g(8) = \frac{2}{8-6} = \frac{2}{2} = 1$

$f(g(8)) = f(1)$

$f(1) = 5(1)^3 - 4(1) + 1$
$= 5 - 4 + 1 = \boxed{2}$

c. $f(g(x)) = ?$

$= \boxed{5\left(\frac{2}{x-6}\right)^3 - 4\left(\frac{2}{x-6}\right) + 1}$

$g(f(x)) = ?$

$= \frac{2}{5x^3 - 4x + 1 - 6}$

$= \boxed{\frac{2}{5x^3 - 4x - 5}}$

② $f(x) = 5x^3 - 4$
Find $f^{-1}(x)$

① $y = 5x^3 - 4$

② $x = 5y^3 - 4$

③ $x + 4 = 5y^3$

$\frac{x+4}{5} = y^3$

$\sqrt[3]{\frac{x+4}{5}} = y$

④ $\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+4}{5}}}$

Steps to find the inverse function ($f^{-1}(x)$)

① change $f(x)$ to y.
② switch x and y
③ solve for y
④ change y back to $f^{-1}(x)$

③ Find the Domain of:

a. $f(x) = \sqrt{8x-2}$

b. $f(x) = \frac{1}{5x+1}$

c. $f(x) = \frac{\sqrt{8x+2}}{5x+1}$

d. $f(x) = \frac{5x+1}{\sqrt{8x+2}}$

When finding the Domain of a function look for the following things:

① you cannot have a negative under a square root
② you cannot have 0 in the denominator
③ if you have a square root in the denominator you have to set what is under the radical greater than 0 (bc we can't have 0 or a negative)

a. $8x - 2 \geq 0$

$8x \geq 2$

$\boxed{x \geq 1/4}$ or $[1/4, \infty)$

b. $5x + 1 \neq 0$

$5x \neq -1$

$\boxed{x \neq \frac{-1}{5}}$ or $(-\infty, -1/5) \cup (-1/5, \infty)$

c. $8x + 2 \geq 0$ AND $5x + 1 \neq 0$

$8x \geq -2$ $\quad x \neq \frac{-1}{5}$

$x \geq -1/4$

$\boxed{x \geq -1/4 \text{ and } x \neq \frac{-1}{5}}$

d. $8x + 2 > 0$

$8x > -2$

$\boxed{x > -1/4}$

④ $f(x) = 6x^2 + 4$

Find $f(x+h) - f(x)$

$f(x+h) = 6(x+h)^2 + 4$

$= 6(x+h)(x+h) + 4$

$= 6(x^2 + 2xh + h^2) + 4$

$= 6x^2 + 12xh + 6h^2 + 4$

(*) DON'T forget the parenthesis

$f(x+h) - f(x) = 6x^2 + 12xh + 6h^2 + 4 - (6x^2 + 4)$

$= 6x^2 + 12xh + 6h^2 + 4 - 6x^2 - 4$

$= \boxed{12xh + 6h^2}$

⑤ $f(x) = 4x^3 - 3x + 5$

Find $f(x+h) - f(x)$

$f(x+h) = 4(x+h)^3 - 3(x+h) + 5$

$= 4(x^3 + 3x^2h + 3xh^2 + h^3) - 3x - 3h + 5$

$= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 3x - 3h + 5$

$f(x+h) - f(x) = 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 3x - 3h + 5 - (4x^3 - 3x + 5)$

$= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 3x - 3h + 5 - 4x^3 + 3x - 5$

$= \boxed{12x^2h + 12xh^2 + 4h^3 - 3h}$