Lecture #8
MAT 123

Piecewise Functions, graph transformations

- Piecewise Function: A function defined in pieces

ex: $f(x) = \begin{cases} x+2 \; ; & x<0 \\ x-2 \; ; & x \geq 0 \end{cases}$ Domain

example: "Word Problem" Labor costs \$10/hour for the first hour and \$12/hour for one to five hours, and \$14/hour after that.

Write a piecewise function to model this information.

$$\boxed{L(x) = \begin{cases} 10x \; ; & 0 < x \leq 1 \\ 12(x-1)+10 \; ; & 1 < x \leq 5 \\ 14(x-5)+58 \; ; & x > 5 \end{cases}}$$

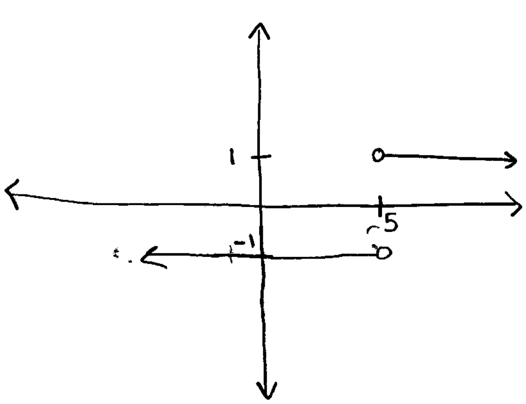
- example: absolute value function

$f(x) = |x| = \begin{cases} x \; ; & x \geq 0 \\ -x \; ; & x < 0 \end{cases}$

example:

$f(x) = \dfrac{|x-5|}{x-5} = \begin{cases} \dfrac{x-5}{x-5} \; ; & x>5 \\ \dfrac{-(x+5)}{x-5} \; ; & x<5 \end{cases}$ * notice $x \neq 5$

$= \begin{cases} 1 \; ; & x>5 \\ -1 \; ; & x<5 \end{cases}$



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example:

A pool fills with 12 gallons/hr of water for the first 3 hours and 10 gallons/hr of water after that. Express the amount of water in the pool as a piecewise function.

$$f(x) = \begin{cases} 12x \; ; & 0 \le x \le 3 \\ 10(x-3)+36 \; ; & x > 3 \end{cases}$$

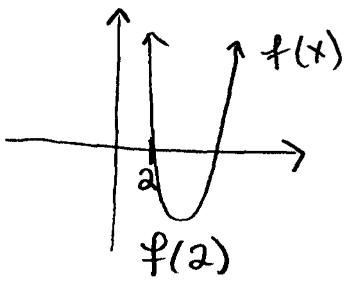
x = hours
$f(x)$ = amt. of water in x hours

How much water is in the pool after 10 hours?

$$f(10) = 10(10-3)+36$$
$$= 100 - 30 + 36 = \boxed{106 \text{ gallons of water}}$$

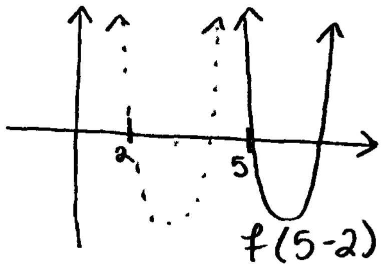
Transformations of Functions

"changing a function in someway"



$f(x)$

moved to the right.



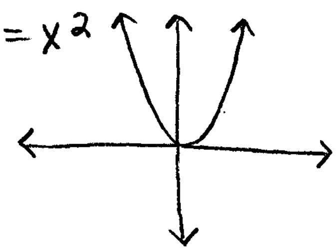
$f(x-3)$

shifts the curve 3 units to the right.

Rules:

① To shift a function to the right by "h" units use $f(x-h)$

ex: $f(x) = x^2$

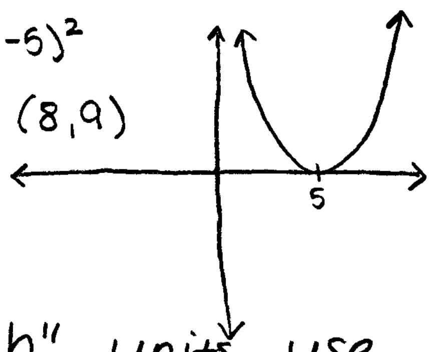


(3,9)

shifted 5 units to the right.

$f(x) = f(x-5)^2$

(8,9)

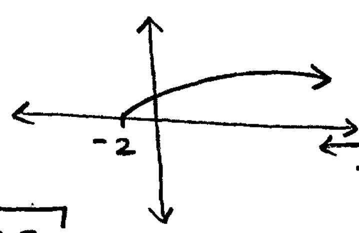


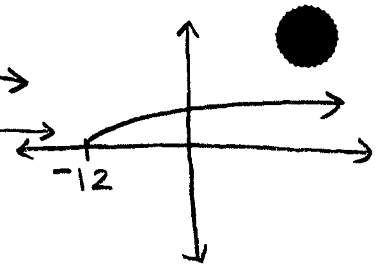
② To shift a function to the left by "h" units use $f(x+h)$

ex: $f(x) = \sqrt{x+2}$

shift $f(x)$ 10 units to the left $\Rightarrow$

$f(x+10) = \sqrt{(x+10)+2} = \sqrt{x+12}$

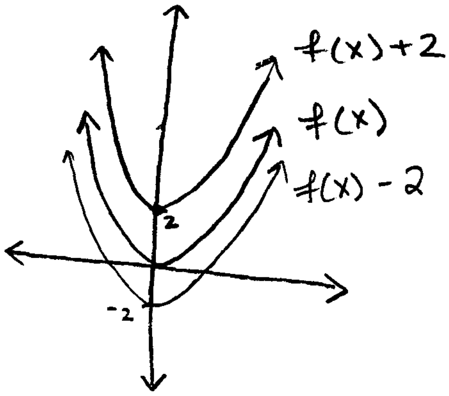




(3) To shift a function "K" units up use
$f(x) + K$

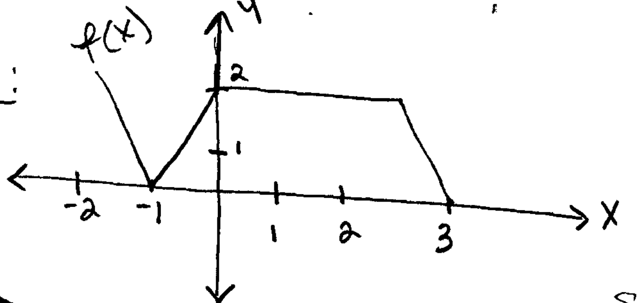
(4) To shift a function "K" units down use
$f(x) - K$

- ex: $f(x) = x^2$
$f(x) + 2 = x^2 + 2$
$f(x) - 2 = x^2 - 2$

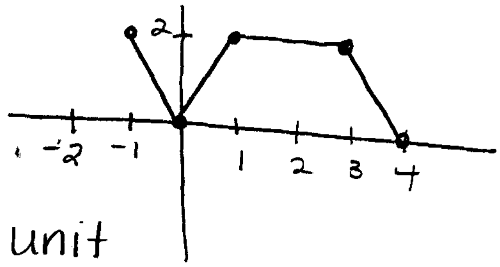


(*) notice: • horizontal shifts add/subtract to the x inside the parentheses
• vertical shifts add/subtract outside the parentheses

ex:



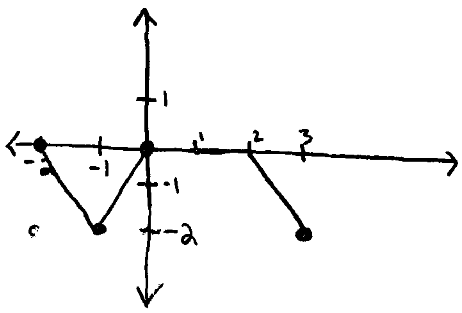
$f(x-1)$
$\longrightarrow$



shif 1 unit to the right.

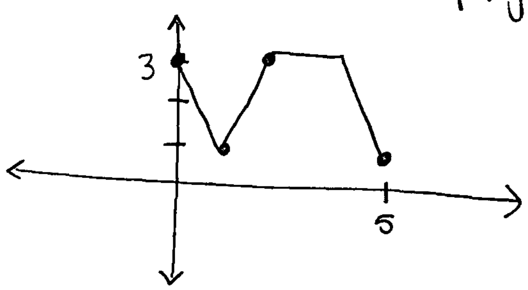
- $\downarrow$

shift by 2 units down $f(x) - 2$



$f(x-2) + 1$

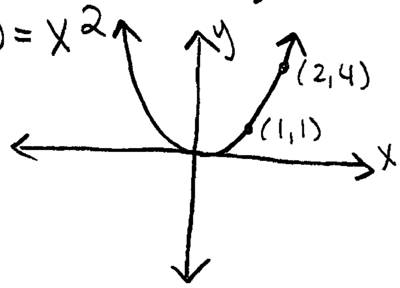
shift 2 units right AND 1 unit up.



(5) To stretch vertically by A units use
$A \cdot f(x)$

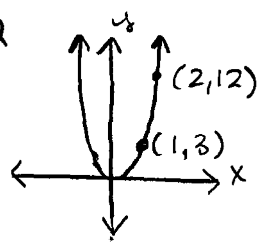
(6) To shrink horizontally by B units use $\rightarrow$ "compress"
$f(Bx)$

- ex: $f(x) = x^2$

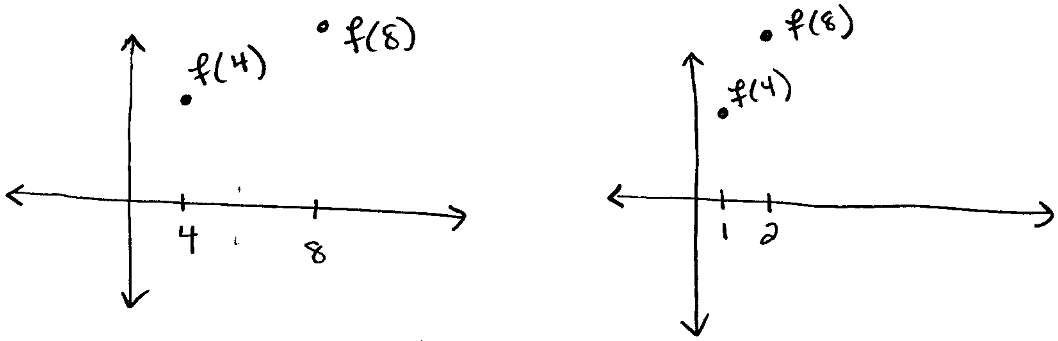


$f(x) = 3x^2$

"stretch vertically"

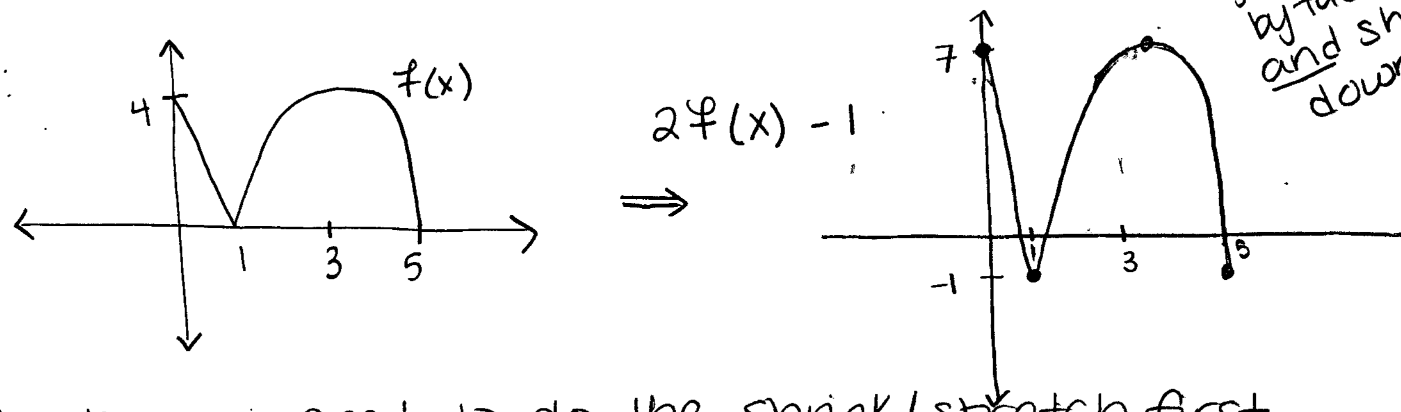


$f(x)$ $\quad$ $f(4x)$



"horizontal compression"

ex:

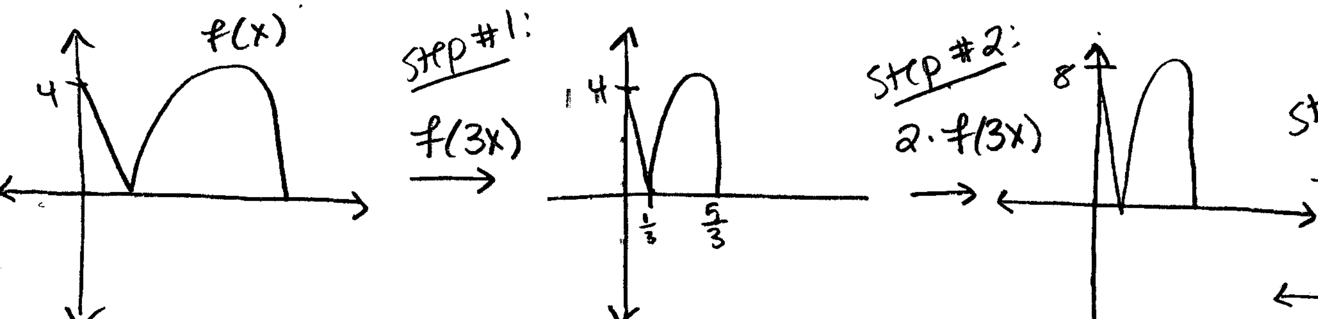


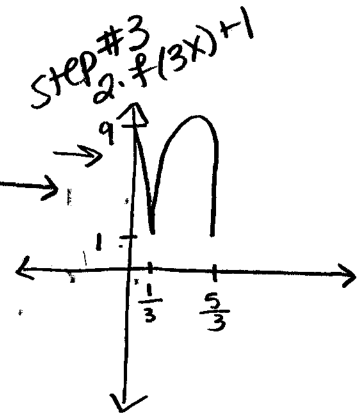
stretch vertically by factor of 2 and shift down 1 unit

(*) note you need to do the shrink/stretch first then the vertical shift b/c we have to do multiplication Before subtraction (PEMDAS)

Graph:

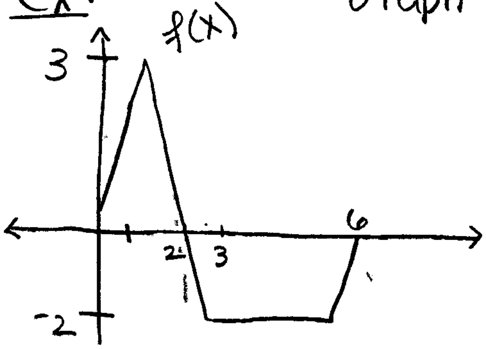
$2 \cdot f(3x) + 1$



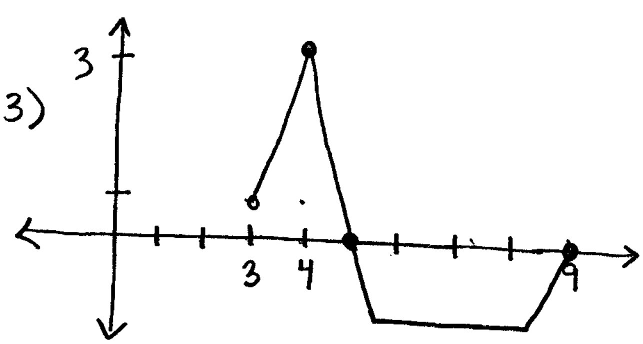


(*) Doing these problems in steps is helpful! make sure to follow PEMDAS

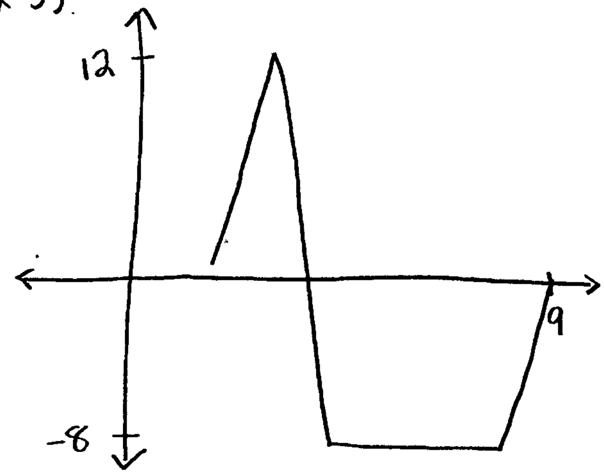
ex: $\quad$ Graph: $4 \cdot f(x-3) - 1$



$f(x-3)$ $\rightarrow$



4. $f(x-3)$ →



4. $f(x-3)-1$ →