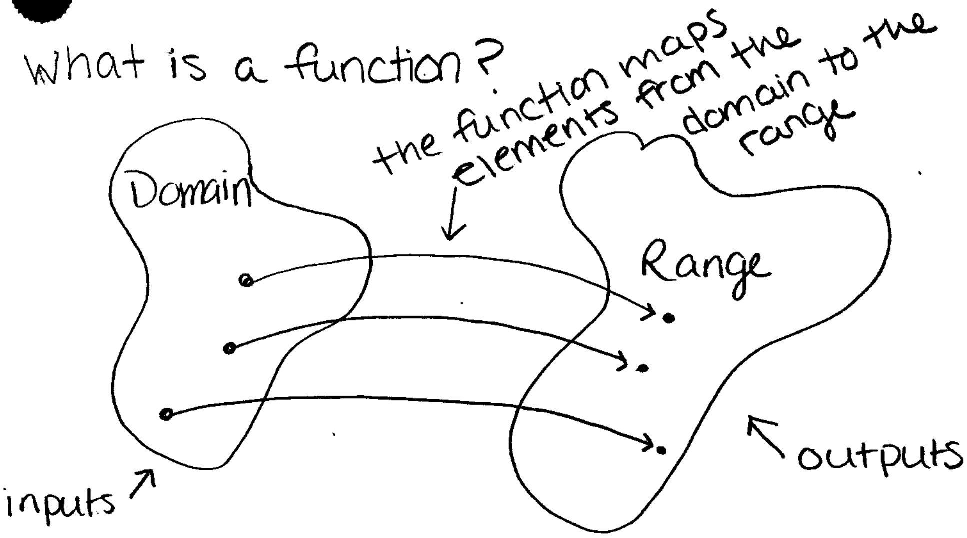
Lecture #6
MAT 123

Functions, Domain, and Range

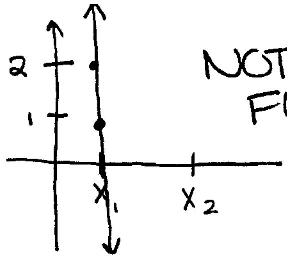
What is a function?



we can think of a function being like a machine that when you put something in you get something else out

Definition: A <u>function</u> is a relation between a set of inputs and a set of outputs in which each input is related to <u>exactly one</u> output.

* for each element in the domain there is only one element in the range.

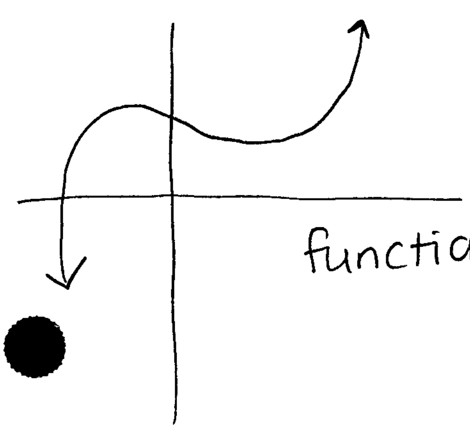


NOT A FUNCTION

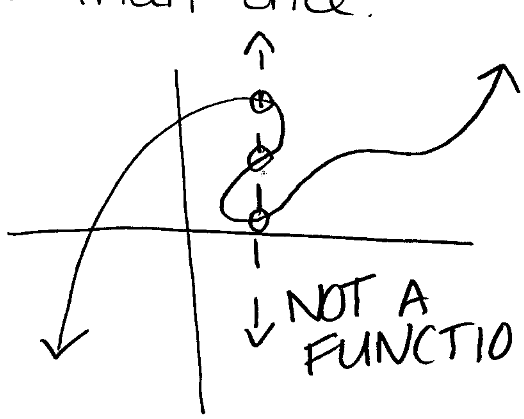
when we are looking at a graph we use the <u>verticle line test</u> to determine if it is a function.

To pass the verticle line test there is no place on the graph that you can draw a verticle line and hit the graph more than once.

<u>examples</u>:



function ✓



NOT A FUNCTION

Domain: what you are allowed to put into the function
Range: what you get out of a function

What is the domain of:
$\sin(x) \rightarrow$ All real numbers

Notation:

$f(x)$ "A function of x"
$f(\theta)$ "A function of θ"

* whatever is inside the parathensis is the Domain.

ex: $f(x) = x^2$
Domain = all real numbers
↳ we can square ANY number

Types of Numbers:
① Counting numbers / Natural Numbers → 1, 2, 3, 4, ...
② Integers → whole #s (NOT fractions) → ..., -3, -2, -1, 0, 1, 2, ...
③ Rational Numbers → a # that can be represented by a ratio of an integer and another integer
$\frac{p}{q}$ where p, q are integers, $q \neq 0$
④ Irrational Numbers → #s that cannot be represented as a ratio of two numbers
this is what we deal with in this course → ⑤ Real Numbers → Rational and Irrational #'s

ex: $f(x) = \sqrt{x}$
Domain = "positive real #'s and 0"
$= x \geq 0$
= x is non-negative reals

different ways to say the same thing.

Interval Notation:

[include

(does not include

ex: $[4, 6)$
all #'s between 4 and 6 including 4 and not including 6.
$4 \leq x < 6$

ex: $[0, 10]$
$0 \leq x \leq 10$

ex: $(0, 10)$
$0 < x < 10$

ex: $[0, 10)$
$0 \leq x < 10$

note: we can Never include infinity (∞) because we can never actually "get to" ∞

- ex: $[0, \infty)$ $x \geq 0$

ex: $(-\infty, 0]$ $x \leq 0$

ex: All reals except $x=5$
in interval notation: $(-\infty, 5) \cup (5, \infty)$ "union"

* WebAssign makes you use interval notation!

ex: $f(x) = \sqrt{x-3}$
what is the domain?
$x - 3 \geq 0$
$\boxed{x \geq 3}$ OR $\boxed{[3, \infty)}$

to find the domain of a square root, you take what is under the radical and set it greater than or equal to 0!

- ex: $f(x) = \sqrt{5-4x}$
Domain is?
$5 - 4x \geq 0$
$-4x \geq -5$
$\overline{-4} \quad \overline{-4}$
$\boxed{x \leq 5/4}$ OR $\boxed{(-\infty, 5/4]}$

Notation:
All Real #'s = $\mathbb{R}$

ex: $f(x) = \sqrt[3]{x-3}$
"cube root"
What is the Domain?
$\boxed{\text{All real \#s.}}$
OR
$\boxed{(-\infty, \infty)}$

note:
you can only take the even root of a positive number, but you can take the odd root of all real #s.
$f(x) = \sqrt[n]{x}$
if n = positive, even
Domain = $[0, \infty)$
if n = positive, odd
Domain = $(-\infty, \infty)$

ex: $f(x) = \dfrac{5}{x-2}$
Domain = all reals except $x=2$
$(-\infty, 2) \cup (2, \infty)$

- because we can't have "0" in the denominator!
$x - 2 \neq 0$
$x \neq 2$

ex: $f(x) = \dfrac{5}{\sqrt{x-2}}$ ← we can't have 0 in the denominator AND you can't take the square root of a negative number.

$x - 2 > 0$

Domain = $\boxed{x > 2}$

OR

$\boxed{(2, \infty)}$

ex: $f(x) = \dfrac{\sqrt{x-2}}{5}$

Domain → $x - 2 \geq 0$

$\boxed{x \geq 2}$

OR

$\boxed{[2, \infty)}$

ex: $f(x) = \dfrac{x-5}{\sqrt{x-3}}$ ← the numerator can be ANYTHING!

Domain→

$x - 3 > 0$

$\boxed{x > 3}$ OR $\boxed{(3, \infty)}$

ex: $f(x) = \dfrac{\sqrt{x-3}}{x-5}$

$x - 5 \neq 0$

Domain $x \neq 5$ AND $x - 3 \geq 0$

$x \geq 3$

"x can be any # greater than 3 except 5"

$\boxed{x \geq 3,\ x \neq 5}$ OR $\boxed{[3,5) \cup (5, \infty)}$

notation:
"Domain ="
D: { }

ex: $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

what is the Domain?

$\cos x \neq 0$

$x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$

OR

$x \neq \frac{\pi}{2} + n\pi$ where n = integer.

ex: $f(x) = \dfrac{5}{\sqrt{x-\pi}}$

what is the Domain?

$x - \pi > 0$

$x > \pi$

$\boxed{D: \{x > \pi\}}$ OR $\boxed{(\pi, \infty)}$

ex: $f(x) = \dfrac{\sqrt{x}}{x^2-9}$

$x \geq 0$ AND $x^2 - 9 \neq 0$

$x^2 \neq 9$

$x \neq \pm 3$

we don't need to worry about $x \neq -3$ since $x \geq 0$ already.

$\boxed{D: \{x \geq 0, x \neq 3\}}$

OR

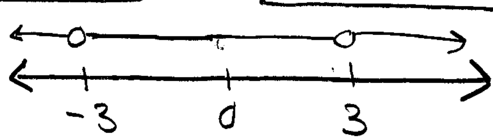
$\boxed{D: [0, 3) \cup (3, \infty)}$

ex: $f(x) = \dfrac{x}{x^2-9}$

$x^2 - 9 \neq 0$

$x^2 \neq 9$

$\boxed{D: \{x \neq \pm 3\}}$ OR $\boxed{(-\infty, -3) \cup (-3, 3) \cup (3, \infty)}$

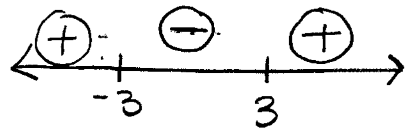


ex: $f(x) = \dfrac{5}{\sqrt{x^2-9}}$

what is the Domain?

$x^2 - 9 > 0$ $\longrightarrow$ *factor!

$(x+3)(x-3) > 0$



test #'s in each interval to see if the function is positive or negative.

so $\boxed{D: \{\, x < -3 \text{ or } x > 3 \,\}}$

OR

$\boxed{(-\infty, -3) \cup (3, \infty)}$