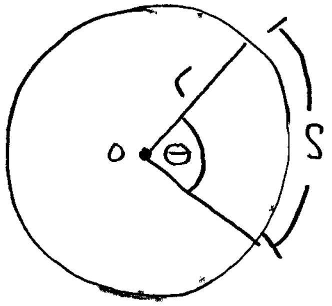
Lecture #5
MAT 123

How do we find the length of an arc of a sector of a circle?



$$\boxed{\frac{\theta}{360^\circ} = \frac{s}{2\pi r}}$$

→ use this formula when θ is in degrees

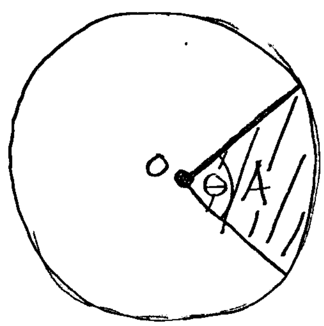
OR

$$\frac{\theta}{\cancel{2\pi}} = \frac{s}{\cancel{2\pi} r}$$

$$\theta = \frac{s}{r} \rightarrow \boxed{s = r \cdot \theta}$$

→ use this formula when θ is in radians

How do we find the area of a sector of a circle?



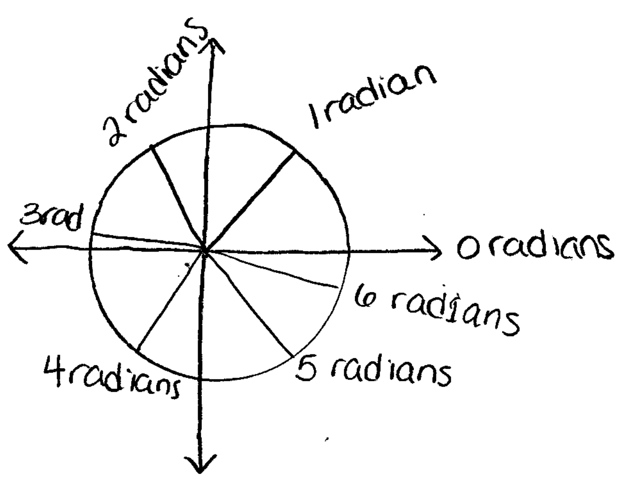
$$\boxed{\frac{\theta}{360^\circ} = \frac{A}{\pi r^2}}$$

→ when θ is in degrees

$$\frac{\theta}{2\cancel{\pi}} = \frac{A}{\cancel{\pi} r^2}$$

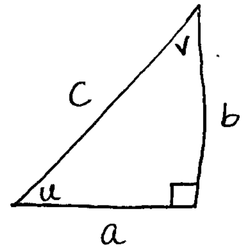
$$\boxed{A = \frac{1}{2} r^2 \theta}$$

→ when θ is in radians.



<u>note</u>: $2\pi \approx 6.3$ radians.

problem:



Suppose $b=4$ and $\sin v = \frac{3}{13}$. find a.

$$a^2+b^2=c^2$$
$$a^2+4^2=c^2$$
$$a^2+16=c^2$$

$$\sin v = \frac{a}{c} = \frac{3}{13}$$
$$13a=3c$$

plug in to solve for a.

$$c=\frac{13a}{3}$$

$$a^2+16=\left(\frac{13a}{3}\right)^2$$
$$9\times\left(a^2+16=\frac{169a^2}{9}\right)$$
$$9a^2+144=169a^2$$
$$144=160a^2$$
$$a^2=\frac{144}{160} \Rightarrow \boxed{a=\sqrt{\frac{144}{160}}}$$

Answer.

More Trig Functions

Reciprocal Trig Functions:

① $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Why do we need more trig functions??

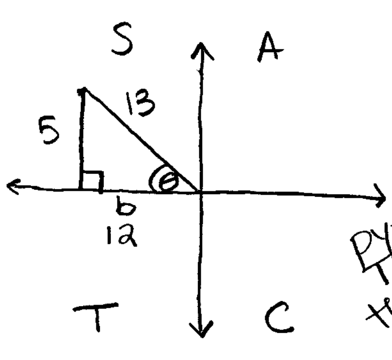
② $\sec\theta = \frac{1}{\cos\theta}$

ex: $\frac{\tan\theta}{\sin\theta} = \tan\theta \cdot \csc\theta$

③ $\csc\theta = \frac{1}{\sin\theta}$

θ is in quadrant 2

example: Suppose $\frac{\pi}{2} < \theta < \pi$ and $\sin\theta = \frac{5}{13}$. Find the other 5 trig functions of θ.



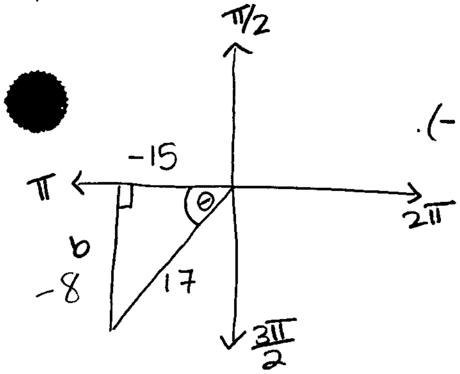
$$5^2+b^2=13^2$$
$$25+b^2=169$$
$$b=\pm 12$$

use Pythagorean Thm to find the missing side

Answers

$\sin\theta=\frac{5}{13}$	$\csc\theta=\frac{13}{5}$
$\cos\theta=\frac{-12}{13}$	$\sec\theta=\frac{-13}{12}$
$\tan\theta=\frac{-5}{12}$	$\cot\theta=-\frac{12}{5}$

example: Suppose $\pi < \theta < \frac{3\pi}{2}$ and $\cos\theta = \frac{-15}{17}$. find the other 5 trig functions of θ.



$a^2+b^2=c^2$

$(-15)^2+b^2=(17)^2$

$225+b^2=289$

$b^2=64$

$b=\pm\sqrt{64}=\pm 8$

$\sin\theta=\frac{-8}{17}$	$\csc\theta=\frac{-17}{8}$
$\cos\theta=\frac{-15}{17}$	$\sec\theta=\frac{-17}{15}$
$\tan\theta=\frac{8}{15}$	$\cot\theta=\frac{15}{8}$

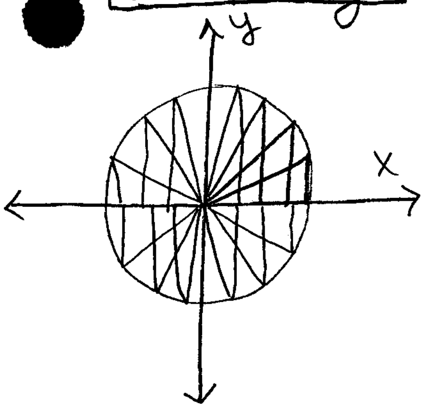
↑ Answers ↑

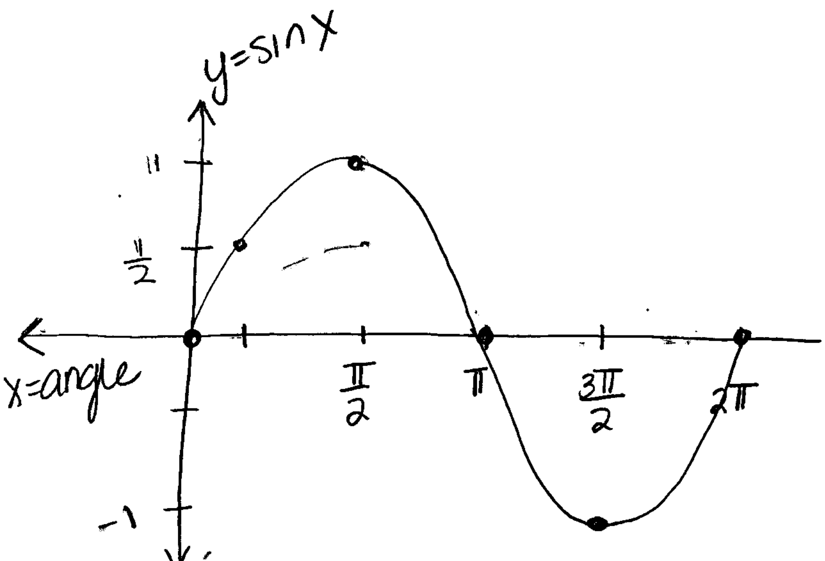
∗ notice → draw a picture to do these problems!
→ make sure your Δ is in the correct quadrant!
→ be careful with the signs (+,-) use "ASTC"

∗ note → if $\sin\theta=+$ then $\csc\theta=+$
if $\cos\theta=+$ then $\sec\theta=+$
if $\tan\theta=+$ then $\cot\theta=+$

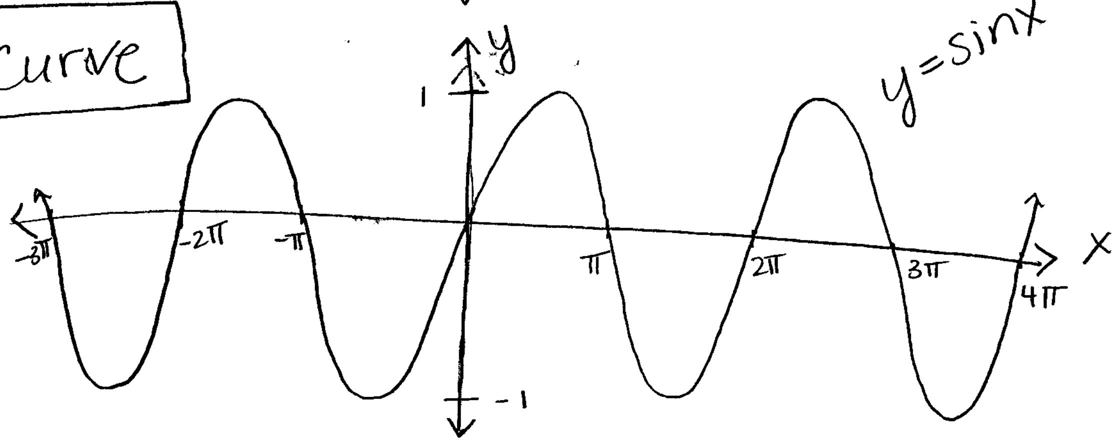
same for negative!

Graphing



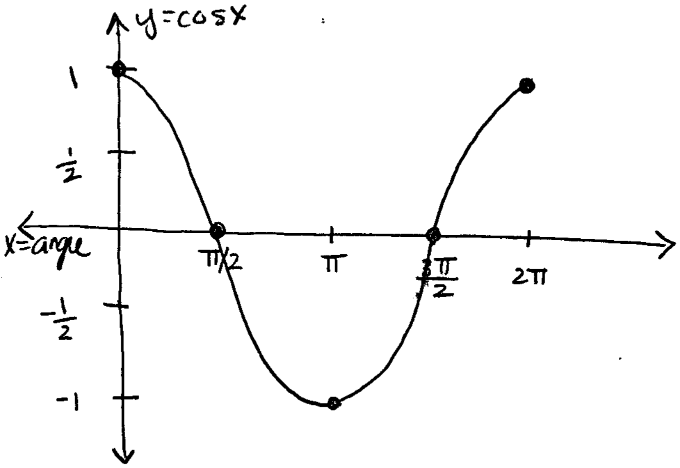


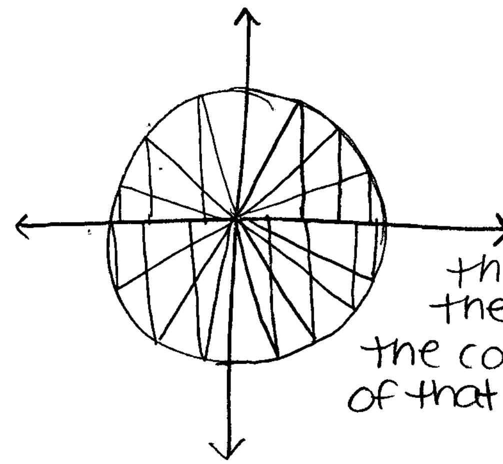
Sine Curve



this curve goes on forever in both the positive and negative x direction.

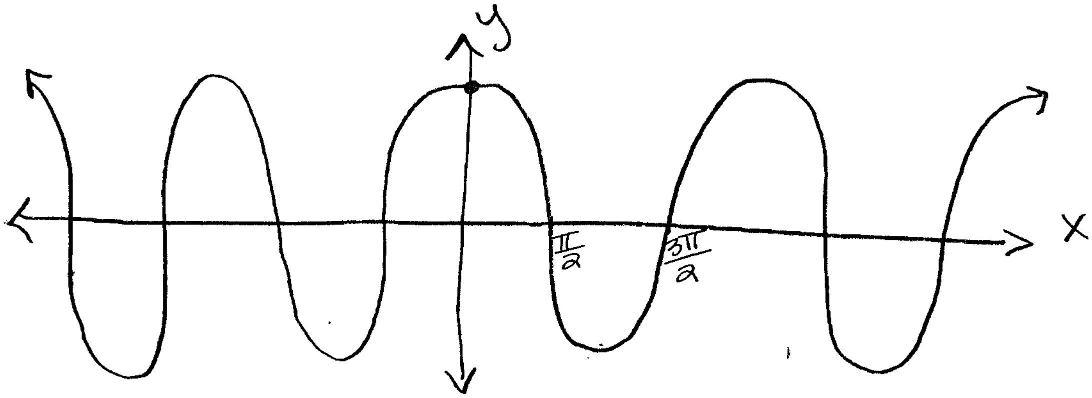
Cosine curve



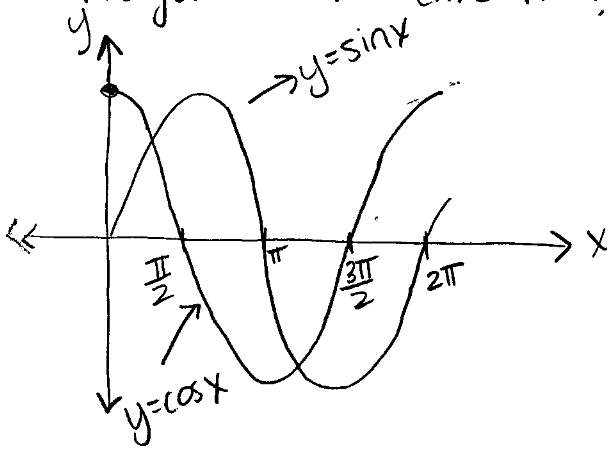


the length on the x-axis is the cosine value of that angle.

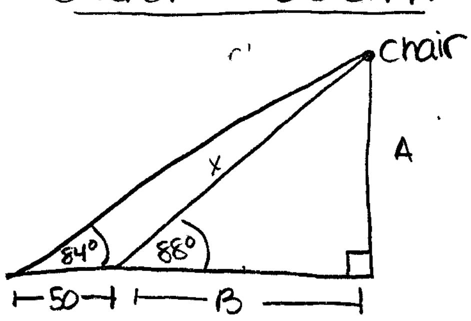
*note: the cosine curve is the same as the sine curve just shifted by 90° (or π/2) to the right



the curve goes on forever in both the positive and negative x direction.



Older Problem:



$\tan 84° = \frac{A}{50+B}$

$(\tan 84°)(50+B) = A$

$\tan 88° = \frac{A}{B}$

$\frac{\tan 88°}{1} = \frac{(\tan 84°)(50+B)}{B}$

$B \cdot \tan 88° = 50 \cdot \tan 84° + B \cdot \tan 84°$

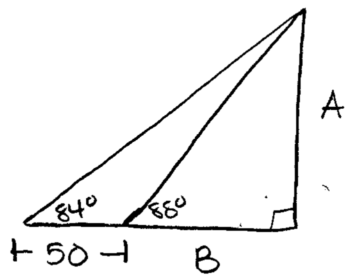
$B \cdot \tan 88° - B(\tan 84°) = 50 \cdot \tan 84°$

$B(\tan 88° - \tan 84°) = 50 \cdot \tan 84°$

$$\frac{B(\tan 88^\circ - \cancel{\tan 84^\circ})}{\tan 88^\circ - \cancel{\tan 84^\circ}} = \frac{50 \cdot \tan 84^\circ}{\tan 88^\circ - \tan 84^\circ}$$

$$B = \frac{50 \cdot \tan 84^\circ}{\tan 88^\circ - \tan 84^\circ} = 24.8782$$

↑ need to use calculator!



$$\tan 88^\circ = \frac{A}{B} \Rightarrow \tan 88^\circ = \frac{A}{24.8782}$$

$$A = 24.878 \cdot \tan 88^\circ$$

$$A = 712.41844$$

$$a^2 + b^2 = c^2$$

$$\sqrt{(712.41844)^2 + (24.8782)^2} = c$$