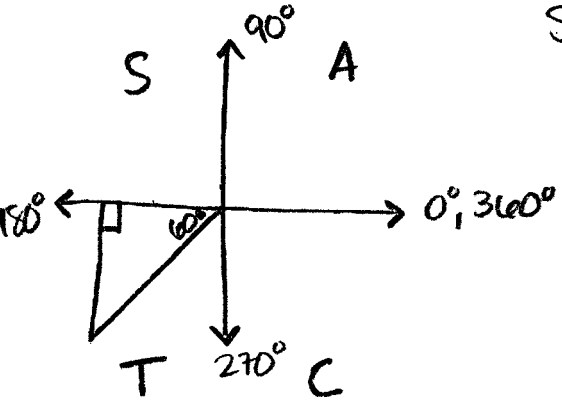
Lecture #3
MAT 123

Radian Measure and Unit Circle Trigonometry

Recall:

$\sin 240° = ?$

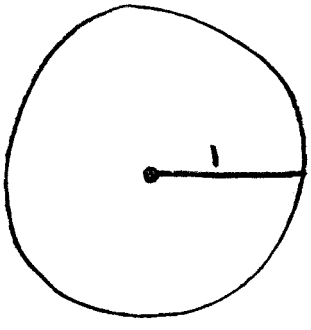
Steps:
Find angle with x-axis ← (reference angle)
Figure out positive or negative
Find value



$\sin 240° = -\sin 60°$

$= \boxed{-\frac{\sqrt{3}}{2}}$

Radians



$C = 2\pi(1) = 360°$

a.k.a. it is 2π radians to get around the entire circle.

(*) 2π radians $= 360°$

$360° = 2\pi$

$180° = \pi$ radians

$1° = \frac{\pi}{180°} \Rightarrow 1 = \frac{180°}{\pi}$

(*) radians have no units.

conversion factors.

to convert from degrees to radians → multiply by $\frac{\pi}{180}$

to convert from radians to degrees → multiply by $\frac{180}{\pi}$

examples: ⊛ Note this are important angles to know for this course.

Degrees	Radians
0°	0
90°	$\frac{\pi}{2}$
180°	π
270°	$\frac{3\pi}{2}$
360°	2π

$90° \times \frac{\pi}{180°} = \frac{90°\pi}{180°} = \frac{\pi}{2}$

$180° \times \frac{\pi}{180°} = \frac{180°\pi}{180°} = \pi$

$270° \times \frac{\pi}{180°} = \frac{270°\pi}{180°} = \frac{3\pi}{2}$

$360° \times \frac{\pi}{180°} = \frac{360°\pi}{180°} = 2\pi$

Degrees	Radians
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$

$30° \times \frac{\pi}{180°} = \frac{\pi}{6}$

$60° \times \frac{\pi}{180°} = \frac{\pi}{3}$

$45° \times \frac{\pi}{180} = \frac{\pi}{4}$

example: convert $\frac{7\pi}{6}$ to degrees.

$\frac{7\pi}{6} \times \frac{180}{\pi} = \frac{7 \cdot 180}{6} = 210°$ ⊛ notice the π cancelled, this will always happen when going from radians to degrees.

example:

$310° = x$ radians

$310° \times \frac{\pi}{180°} = \frac{31\pi}{18}$

example:

$\frac{11\pi}{3} = x$ degrees

$\frac{11\pi}{3} \times \frac{180}{\pi} = 660°$

example:

$\frac{4\pi}{15}$ = x degrees

$\frac{4\pi}{15} \times \frac{180}{\pi} = 48°$

example:

570° = x radians.

$570° \times \frac{\pi}{180°} = \frac{57\pi}{18} = \frac{19\pi}{6}$

Degrees → Radians : $\frac{\pi}{180}$

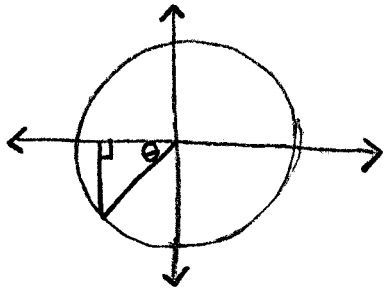
Radians → Degrees : $\frac{180}{\pi}$

Need to know these conversion factors!!!

Practice:

① Find $\sin \frac{7\pi}{6} = ?$

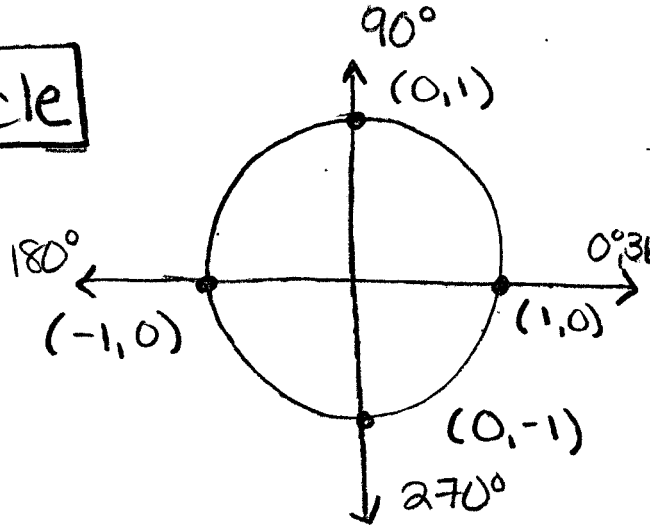
$$\sin \frac{7\pi}{6} = \sin\left(\frac{7\pi}{6}\right) \times \frac{180}{\pi} = \sin 210°$$



$\theta = 210° - 180° = 30°$

$\sin 210° = -\sin 30° = -\frac{1}{2}$

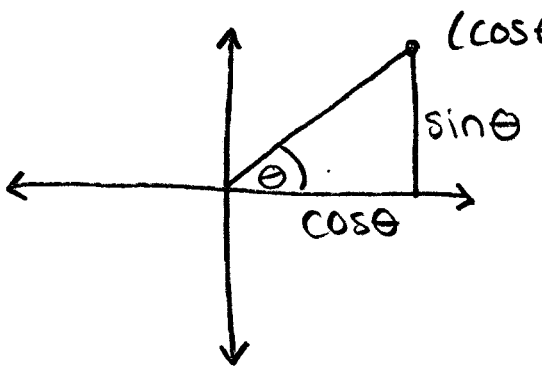
Unit Circle



radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
degrees	0°	90°	180°	270°
sin	0	1	0	-1
cos	1	0	-1	0
tan	0	und	0	und

$\cos 0° = 1$, $\sin 0° = 0$ ← look at the coordinates

$(x, y) \rightarrow (\cos\theta, \sin\theta)$

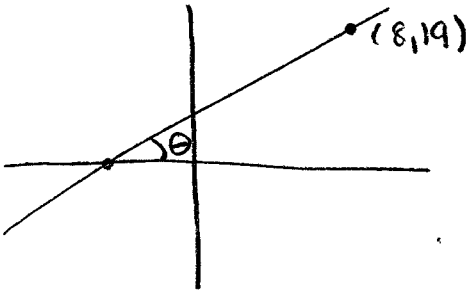


$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta$ = slope of line

Problems:

① A line goes through (8, 19) and makes an angle of 60° with the x-axis. Find the equation of the line.



$\tan 60° =$ slope of that line

$m = \tan 60° = \sqrt{3}$

$y = mx + b$

$y = \sqrt{3}x + 8$ ← Answer

OR

Point slope formula:

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) = given point

Answer → $y - 18 = \sqrt{3}(x - 8)$

② A line makes an angle of $\frac{\pi}{5}$ with the x-axis and goes through (3, 23). Find the equation of the line.

$m = \tan\left(\frac{\pi}{5}\right)$ need to use the calculator!

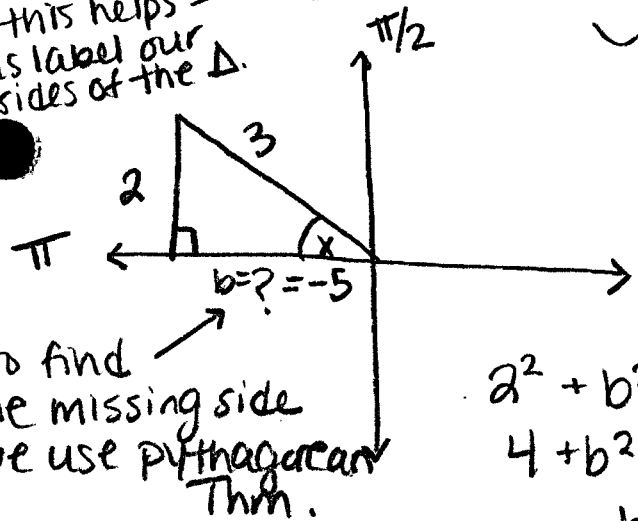
$y - y_1 = m(x - x_1)$

$y - 23 = \tan\frac{\pi}{5}(x - 3)$ ← (*) this is an acceptable answer on an exam since you cannot use calculators on the exam.

$y - 23 = .727(x - 3)$

③ If $\sin x = \frac{2}{3}$ and $\frac{\pi}{2} < x < \pi$, find $\cos x$

this helps us label our sides of the Δ.



to find the missing side we use Pythagorean Thm.

↳ this tells us that the angle x is in quadrant II.

⊛ this type of question will be on the exam!

$2^2 + b^2 = 3^2$

$4 + b^2 = 9$

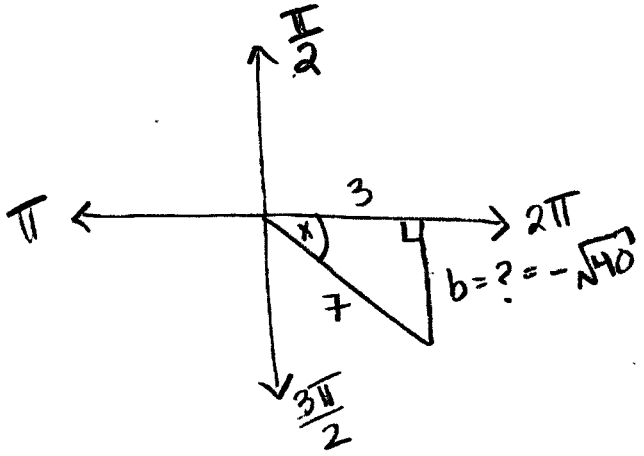
$b^2 = 5$

$b = \pm\sqrt{5}$

Answer

$$\cos x = -\frac{\sqrt{5}}{3}$$

④ If $\cos x = \frac{3}{7}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\sin x$



$3^2 + b^2 = 7^2$

$9 + b^2 = 49$

$b^2 = 40$

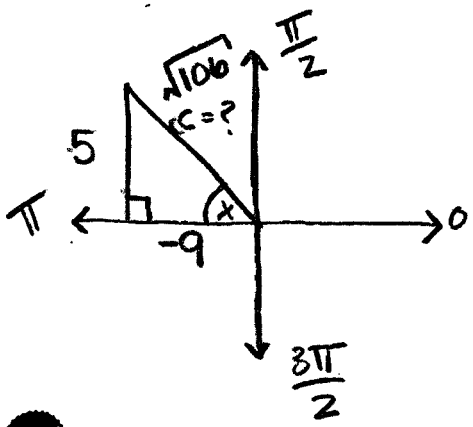
$b = \pm\sqrt{40}$

Answer

$$\sin x = -\frac{\sqrt{40}}{7}$$

⑤ If $\tan x = \frac{-5}{9}$ and $\frac{\pi}{2} < x < \pi$, find $\cos x$

⊛ note: the hypotenuse is always positive!



$5^2 + (-9)^2 = c^2$

$25 + 81 = c^2$

$106 = c^2$

$c = \pm\sqrt{106}$

$$\cos x = \frac{-9}{\sqrt{106}}$$ Answer