MAT122 Homework for Lectures 39-42

July 5, 2021

1 Problems

1. Integrate using u substitution:

$$\int \frac{7x+1}{14x^2+4x-3}\,dx$$

2. Integrate using u substitution:

$$\int_e^{e^2} \frac{dx}{x\ln(x^3)}.$$

3. Find the limit of

$$\lim_{x\to 4} \frac{-13}{x-4}.$$

4. Use the **definition** of a derivative to find the derivative of $f(x) = 2x - x^2$.

5. Differentiate $g(x) = (x^3-4x)^3(-3x^4-2x-3)^2$. It is fine to leave it unexpanded.

6. Differentiate $y = \frac{(x^3-2x+2)^3}{(x^2-1)^2}$. It is fine to leave it unexpanded.

7. Let $f(x) = -x^3 + x + 7$. Find the max/min, written in (x, y) form. Be sure to say which point is the max and which is the min. Find the intervals on which the function is increasing and decreasing.

2 Answer Key

1. $\frac{1}{4}\ln|14x^2+4x-3|+C$.
2. $\frac{\ln(2)}{3}$.
3. The limit does not exist.
4. $f'(x) = 2-2x$.
5. $$g'(x) = 3(x^3-4x)^2(3x^2-4)(-3x^4-2x-3)^2 + 2(x^3-4x)^3(-3x^4-2x-3)(-12x^3-2).$$
6. $$\frac{dy}{dx} = \frac{3(x^2-1)^2(x^3-2x+2)(3x^2-2)-2(x^3-2x+2)^3(x^2-1)(2x)}{(x^2-1)^4}.$$
7. The minimum is $(-\frac{1}{\sqrt{3}}, 7-\frac{2}{3\sqrt{3}})$. The maximum is $(\frac{1}{\sqrt{3}}, 7+\frac{2}{3\sqrt{3}})$. The function is increasing on the interval $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and decreasing on $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$.

3 Solutions

1. Let $u = 14x^2 + 4x - 3$. $du = (28x + 4)dx = 4(7x + 1)dx$. So the integral becomes $\frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |14x^2 + 4x - 3| + C$.

2. Note that $\ln(x^3) = 3\ln(x)$. So the indefinite integral equals $\frac{1}{3} \int \frac{dx}{x \ln(x)}$. Let $u = \ln(x), du = \frac{dx}{x}$. So now the definite integral is $\frac{1}{3} \int_1^2 \frac{du}{u} = \frac{1}{3} \ln |u||_1^2 = \frac{\ln(2)}{3}$.

3. $\lim_{x \to 4^+} \frac{-13}{x-4} = -\infty$ but $\lim_{x \to 4^-} \frac{-13}{x-4} = +\infty$. Therefore, the limit does not exist.

4. Using the definition:

$$\lim_{h \to 0} \frac{1}{h}(2(x + h) - (x + h)^2 - (2x - x^2)).$$

   The numerator becomes $2x+2h-x^2-2xh-h^2-2x+x^2 = 2h-2xh-h^2 = h(2-2x-h)$ so the limit becomes $\lim_{h \to 0} 2 - 2x - h = 2 - 2x$.

5. Use the product and chain rules.

   $g'(x) = 3(x^3 - 4x)^2(3x^2 - 4)(-3x^4 - 2x - 3)^2 + 2(x^3 - 4x)^3(-3x^4 - 2x - 3)(-12x^3 - 2).$

6. Use the quotient and chain rules.

$$\frac{dy}{dx} = \frac{3(x^2 - 1)^2(x^3 - 2x + 2)(3x^2 - 2) - 2(x^3 - 2x + 2)^3(x^2 - 1)(2x)}{(x^2 - 1)^4}.$$

7. $f'(x) = -3x^2 + 1$ so $x = \pm\frac{1}{\sqrt{3}}$. The critical points are indeed max and min; this can be checked by the 2nd derivative $f''(x) = -6x$. The minimum is $(-\frac{1}{\sqrt{3}}, 7 - \frac{2}{3\sqrt{3}})$. The maximum is $(\frac{1}{\sqrt{3}}, 7 + \frac{2}{3\sqrt{3}})$. The function is increasing on the interval $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and decreasing on $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$.