Exercise 1. Compute the definite integral $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$.

Solution. Let $u = \sqrt{x}$ so $du = \frac{1}{2\sqrt{x}}dx$. Then the integral becomes $\int_{\sqrt{1}}^{\sqrt{9}} 2e^u du$, which is $2\int_1^3 e^u du = 2e^u|_1^3$. This is $2(e^3 - e)$.

Exercise 2. Compute the definite integral $\int_1^2 x\sqrt{x-1}dx$.

Solution. Let $u = x - 1$. Then $du = dx$ and the integral becomes $\int_0^1 (u+1)\sqrt{u}du$. Multiplying in $\sqrt{u}$ into the parentheses gives us $\int_0^1 u^{\frac{3}{2}} + u^{\frac{1}{2}}$. By power rule we get this integral is $\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}|_0^1$. This evaluates to $\frac{2}{5} + \frac{2}{3}$ which is $\frac{16}{15}$.

Exercise 3. Compute the definite integral $\int_0^2 \frac{e^x+1}{e^x+x}dx$.

Solution. Let $u = e^x + x$. Then $du = (e^x + 1)dx$. So the integral becomes $\int_1^{e^2+2} \frac{1}{u}du$ which is $ln(u)|_1^{e^2+2}$. This computes to $ln(e^2 + 2) - ln(1) = ln(e^2 + 2)$.

Exercise 4. Evaluate the indefinite integral $\int xsin(x^2)dx$.

Solution. Let $u = x^2$ so $du = 2xdx$. Then the integral is $\int \frac{1}{2}sin(u)du$. So this is $-\frac{1}{2}cos(u) + C$. Substituting back in x gives us $-\frac{1}{2}cos(x^2) + C$.

Exercise 5. Evaluate the indefinite integral $\int cos^3xsinxdx$.

Solution. Let $u = cosx$. Then $du = sinxdx$ and the integral becomes $\int u^3du$ which is $\frac{u^4}{4} + C$. Substituting back in x gives us $\frac{cos^4x}{4} + C$.