Homework

1. With $n = 4$, (the number of rectangles), compute both the left endpoint and right endpoint Riemann sums for $y = \sin(\pi x)$ on $[0, 1]$.

2. With $n = 4$, compute both the left endpoint and right endpoint Riemann sums for $y = 3x^2 + 1$ on $[-1, 3]$.

3. Recall that in the context of integrals and Riemann sums area can be negative. Approximate the area under the curve $y = x^3 - 4x^2 + 1$ with $n = 5$ on $[0, 5]$ using a left endpoint and right endpoint Riemann sum.

4. Compute the left endpoint Riemann sum for $y = x^2 - 2x$ on $[0, 4]$ for

   (a) $n = 2$

   (b) $n = 4$

   (c) $n = 8$

5. Compute the right endpoint Riemann sum for $y = \ln x$ on $[1, 4]$ with $n = 6$.

Solutions Throughout this solution set, let L_n, R_n denote the left hand and right hand Riemann sums, respectively, with n rectangles.

1. With $n = 4$, (the number of rectangles), compute both the left endpoint and right endpoint Riemann sums for $y = \sin(\pi x)$ on $[0, 1]$.

   Solution: The length of each rectangle is the length of the interval divided by the total number of rectangles which in this case is $1/4$. Thus,

$$\begin{aligned} L_4 &= 0.25y(0) + 0.25y(0.25) + 0.25y(0.5) + 0.25y(0.75) \\ &= 0.25(y(0) + y(0.25) + y(0.5) + y(0.75)) = 0.25(\sin(0) + \sin(\pi/4) + \sin(\pi/2) + \sin(3\pi/4)) \\ &= 0.25(0 + \sqrt{2}/2 + 1 + \sqrt{2}/2) = 0.25(1 + \sqrt{2}) \simeq 0.6036 \end{aligned}$$

   And,

$$\begin{aligned} R_4 &= 0.25y(0.25) + 0.25y(0.5) + 0.25y(0.75) + 0.25y(1) \\ &= 0.25(y(0.25) + y(0.5) + y(0.75) + y(1)) = 0.25(\sin(\pi/4) + \sin(\pi/2) + \sin(3\pi/4) + \sin(\pi)) \\ &= 0.25(\sqrt{2}/2 + 1 + \sqrt{2}/2 + 0) = 0.25(1 + \sqrt{2}) \simeq 0.6036 \end{aligned}$$

2. With $n = 4$, compute both the left endpoint and right endpoint Riemann sums for $y = 3x^2 + 1$ on $[-1, 3]$.

   Solution: The length of each rectangle is $(2 - (-2))/4 = 4/4 = 1$. Therefore,

$$\begin{aligned} L_4 &= 1y(-1) + 1y(0) + 1y(1) + 1y(2) = y(-1) + y(0) + y(1) + y(2) \\ &= (3(-1)^2 + 1) + (3(0)^2 + 1) + (3(1)^2 + 1) + (3(2)^2 + 1) = 22 \end{aligned}$$

   And,

$$\begin{aligned} R_4 &= 1y(-1) + 1y(0) + 1y(1) + 1y(2) = y(-1) + y(0) + y(1) + y(2) \\ &= (3(0)^2 + 1) + (3(1)^2 + 1) + (3(2)^2 + 1) + (3(3)^2 + 1) = 46 \end{aligned}$$

3. Recall that in the context of integrals and Riemann sums area can be negative. Approximate the area under the curve $y = x^3 - 4x^2 + 1$ with $n = 5$ on $[0, 5]$ using a left endpoint and right endpoint Riemann sum.

   Solution: The length of each rectangle is $(5 - 0)/5 = 1$. As such the area of each rectangle is given by 1 times the height of the rectangle.

$$\begin{aligned} L_5 &= 1y(0) + 1y(1) + 1y(2) + 1y(3) + 1y(4) = y(0) + y(1) + y(2) + y(3) + y(4) \\ &= ((0)^3 - 4(0)^2 + 1) + ((1)^3 - 4(1)^2 + 1) + ((2)^3 - 4(2)^2 + 1) + ((3)^3 - 4(3)^2 + 1) + ((4)^3 - 4(4)^2 + 1 \end{aligned}$$

   And,

$$\begin{aligned} R_5 &= 1y(1) + 1y(2) + 1y(3) + 1y(4) + 1y(5) = y(1) + y(2) + y(3) + y(4) + y(5) \\ &= ((1)^3 - 4(1)^2 + 1) + ((2)^3 - 4(2)^2 + 1) + ((3)^3 - 4(3)^2 + 1) + ((4)^3 - 4(4)^2 + 1) + ((5)^3 - 4(5)^2 + 1 \end{aligned}$$

4. Compute the left endpoint Riemann sum for $y = x^2 - 2x$ on $[0, 4]$ for

   (a) $n = 2$

   For $n = 2$ the length of each rectangle is $(4 - 0)/2 = 2$. Therefore, the left endpoints for the rectangles are at $x = 0, 2$. Summing the area of the rectangles gives us

$$\begin{aligned} L_2 &= 2y(0) + 2y(2) = 2(y(0) + y(2)) \\ &= 2(((0)^2 - 2(0)) + ((2)^2 - 2(2)) = 0 \end{aligned}$$

   (b) $n = 4$

   For $n = 4$ the length of each rectangle is $(4 - 0)/4 = 1$. Therefore, the left endpoints for the rectangles are at $x = 0, 1, 2, 3$. Summing the area of the rectangles gives us

$$\begin{aligned} L_4 &= 1y(0) + 1y(1) + 1y(2) + 1y(3) = 1(y(0) + y(1) + y(2) + y(3)) \\ &= ((0)^2 - 2(0)) + ((1)^2 - 2(1)) + ((2)^2 - 2(2)) + ((3)^2 - 2(3)) = 2 \end{aligned}$$

   (c) $n = 8$

   For $n = 4$ the length of each rectangle is $(4 - 0)/8 = 0.5$. Therefore, the left endpoints for the rectangles are at $x = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5$. Summing the area of the rectangles gives us

$$\begin{aligned} L_8 &= 0.5y(0) + 0.5y(0.5) + 0.5y(1) + 0.5y(1.5) + 0.5y(2) + 0.5y(2.5) + 0.5y(3) + 0.5y(3.5) \\ &= 0.5(y(0) + y(0.5) + y(1) + y(1.5) + y(2) + y(2.5) + y(3) + y(3.5)) \\ &= 0.5(((0)^2 - 2(0)) + ((0.5)^2 - 2(0.5)) + ((1)^2 - 2(1)) + ((1.5)^2 - 2(1.5)) \\ &\qquad + ((2)^2 - 2(2)) + ((2.5)^2 - 2(2.5)) + ((3)^2 - 2(3)) + ((3.5)^2 - 2(3.5))) \\ &= 3.5 \end{aligned}$$

5. Compute the right endpoint Riemann sum for $y = \ln x$ on $[1, 4]$ with $n = 6$.

   Solution: The length of each rectangle is $(4 - 1)/6 = 0.5$. As such the area of each rectangle is given by 0.5 times the height of the rectangle. Summing the area of the rectangles gives us

$$\begin{aligned} R_6 &= 0.5y(1.5) + 0.5y(2) + 0.5y(2.5) + 0.5y(3) + 0.5y(3.5) + 0.5y(4) \\ &= 0.5(y(1.5) + y(2) + y(2.5) + y(3) + y(3.5) + y(4)) \\ &= 0.5(\ln(1.5) + \ln(2) + \ln(2.5) + \ln(3) + \ln(3.5) + \ln(4)) \simeq 2.876 \end{aligned}$$

Answer Key

1. With $n = 4$, (the number of rectangles), compute both the left endpoint and right endpoint Riemann sums for $y = \sin(\pi x)$ on $[0, 1]$.

   $L_4 \simeq .6036$, $R_4 \simeq .6036$

2. With $n = 4$, compute both the left endpoint and right endpoint Riemann sums for $y = 3x^2 + 1$ on $[-1, 3]$.

   $L_4 = 22$, $R_4 = 46$

3. Recall that in the context of integrals and Riemann sums area can be negative. Approximate the area under the curve $y = x^3 - 4x^2 + 1$ with $n = 5$ on $[0, 5]$ using a left endpoint and right endpoint Riemann sum.

   $L_5 = -15$, $R_5 = 10$

4. Compute the left endpoint Riemann sum for $y = x^2 - 2x$ on $[0, 4]$ for

   (a) $n = 2$

   $L_2 = 0$

   (b) $n = 4$

   $L_4 = 2$

   (c) $n = 8$

   $L_8 = 3.5$

5. Compute the right endpoint Riemann sum for $y = \ln x$ on $[1, 4]$ with $n = 6$.

   $R_6 \simeq 2.876$