Homework

1. Compute the anti-derivative of $f(x) = 5x^3 + 9x^2 - 6x + 2$.
2. Compute the anti-derivative of $f(x) = 9x^2 + \frac{2}{x^2}$.
3. Compute the anti-derivative of $f(x) = 5 + 3x + 10x^{99}$.
4. Compute the anti-derivative of $f(x) = x^{\pi e} + 8x^{\pi} + x^{-5}$.
5. If $f'(x) = x^5 + 8x^3 - 5$ and $f(0) = 7$, what is $f(x)$?
6. For $0 \le t \le 5$, the acceleration of a car is given by $a(t) = 10 - 3t$. If $v(t)$ is the velocity of the car and $v(0) = 10$, what is $v(3), v(4), v(5)$? If $s(t)$ is the position of the car and $s(0) = 0$, what is $s(3), s(4), s(5)$?

Solutions

1. Compute the anti-derivative of $f(x) = 5x^3 + 9x^2 - 6x + 2$.

   Solution:

$$\begin{aligned}\int f(x)dx = \int 5x^3 + 9x^2 - 6x + 2dx &= 5\frac{x^4}{4} + 9\frac{x^3}{3} - 6\frac{x^2}{2} + 2x + C \\ &= \frac{5}{4}x^4 + 3x^3 - 3x^2 + 2x + C.\end{aligned}$$

2. Compute the anti-derivative of $f(x) = 9x^2 + \frac{2}{x^2}$.

   Solution:

$$\begin{aligned}\int f(x)dx = \int 9x^2 + \frac{2}{x^2}dx = \int 9x^2 + 2x^{-2}dx& \\ &= 9\frac{x^3}{3} + 2\frac{x^{-1}}{-1} + C \\ &= 3x^3 - 2x^{-1} + C.\end{aligned}$$

3. Compute the anti-derivative of $f(x) = 5 + 3x + 10x^{99}$.

   Solution:

$$\begin{aligned}\int f(x)dx &= \int 5 + 3x + 10x^{99}dx \\ &= 5x + 3\frac{x^2}{2} + 10\frac{x^{100}}{100} + C \\ &= 5x + \frac{3}{2}x^2 + \frac{1}{10}x^{100} + C.\end{aligned}$$

4. Compute the anti-derivative of $f(x) = x^{\pi e} + 8x^{\pi} + x^{-5}$.

   Solution:

$$\begin{aligned}\int f(x)dx &= \int x^{\pi e} + 8x^{\pi} + x^{-5}dx \\ &= \frac{x^{\pi e+1}}{\pi e + 1} + 8\frac{x^{\pi+1}}{\pi + 1} + C \\ &= \frac{x^{\pi e+1}}{\pi e + 1} + \frac{8}{\pi + 1}x^{\pi+1} + C\end{aligned}$$

5. If $f'(x) = x^5 + 8x^3 - 5$ and $f(0) = 7$, what is $f(x)$?

Solution:

$$\begin{aligned}\int f'(x)dx &= \int x^5 + 8x^3 - 5dx \\ &= \frac{x^6}{6} + 8\frac{x^4}{4} - 5x + C \\ &= \frac{x^6}{6} + 2x^4 - 5x + C\end{aligned}$$

Plugging in the point $(0, 7)$ we get $7 = f(0) = \frac{0}{6} + 2 \cdot 0 - 5 \cdot 0 + C = C$. Hence, $C = 7$. And so, $f(x) = \frac{x^6}{6} + 2x^4 - 5x + 7$.

6. For $0 \leq t \leq 5$, the acceleration of a car is given by $a(t) = 10 - 3t$. If $v(t)$ is the velocity of the car and $v(0) = 10$, what is $v(3), v(4), v(5)$? If $s(t)$ is the position of the car and $s(0) = 0$, what is $s(3), s(4), s(5)$?

   Solution: To get velocity from acceleration, we must perform integration.

   $$v(t) = \int a(t)dt = \int 10 - 3tdt = 10t - \frac{3}{2}t^2 + C.$$

   Since $v(0) = 10$ we can solve for $C$ and we get $10 = v(0) = 10 \cdot 0 - \frac{3}{2} \cdot 0 + C = C$. Hence, $C = 10$. And so $v(t) = 10t - \frac{3}{2}t^2 + 7$. Plugging in 3, 4, and 5, we get

   $$\begin{aligned}v(3) &= 10 \cdot 3 - \frac{3}{2}3^2 + 7 = 23.5 \\ v(4) &= 10 \cdot 4 - \frac{3}{2}4^2 + 7 = 23 \\ v(5) &= 10 \cdot 5 - \frac{3}{2}5^2 + 7 = 19.5\end{aligned}$$

   Now that we have velocity, we can get position from velocity (since we also know $s(0) = 0$.)

   $$s(t) = \int v(t)dt = \int 10t - \frac{3}{2}t^2 + 7dt = 5t^2 - \frac{t^3}{2} + 7t + C_1.$$

   Since $s(0) = 0$, we can solve for $C_1$. $0 = s(0) = 5 \cdot 0 - 0/2 + 7 \cdot 0 + C_1 = C_1$. So $C_1 = 0$. Therefore, $s(t) = 5t^2 - \frac{t^3}{2} + 7t$. We can now answer the question: what is $s(3), s(4), s(5)$?

   $$\begin{aligned}s(3) &= 5 \cdot 3^2 - \frac{3^3}{2} + 7 \cdot 3 = 52.5 \\ s(4) &= 5 \cdot 4^2 - \frac{4^3}{2} + 7 \cdot 4 = 76 \\ s(5) &= 5 \cdot 5^2 - \frac{5^3}{2} + 7 \cdot 5 = 97.5\end{aligned}$$

Answer Key

1. Compute the anti-derivative of $f(x) = 5x^3 + 9x^2 - 6x + 2$.

   $\int f(x)dx = \frac{5}{4}x^4 + 3x^3 - 3x^2 + 2x + C.$

2. Compute the anti-derivative of $f(x) = 9x^2 + \frac{2}{x^2}$.

   $\int f(x)dx = 3x^3 - 2x^{-1} + C.$

3. Compute the anti-derivative of $f(x) = 5 + 3x + 10x^{99}$.

   $\int f(x)dx = 5x + \frac{3}{2}x^2 + \frac{1}{10}x^{100} + C.$

4. Compute the anti-derivative of $f(x) = x^{\pi e} + 8x^{\pi} + x^{-5}$.

   $\int f(x)dx = \frac{x^{\pi e+1}}{\pi e+1} + \frac{8}{\pi+1}x^{\pi+1} + C.$

5. If $f'(x) = x^5 + 8x^3 - 5$ and $f(0) = 7$, what is $f(x)$?

   $f(x) = \frac{x^6}{6} + 2x^4 - 5x + 7$

6. For $0 \leq t \leq 5$, the acceleration of a car is given by $a(t) = 10 - 3t$. If $v(t)$ is the velocity of the car and $v(0) = 10$, what is $v(3), v(4), v(5)$? If $s(t)$ is the position of the car and $s(0) = 0$, what is $s(3), s(4), s(5)$?

   $v(3) = 23.5,\ v(4) = 23,\ v(5) = 19.5,\ s(3) = 52.5,\ s(4) = 76,\ s(5) = 97.5$