Homework

1. Compute the derivative of $f(x) = 9^x - 8^x$.
2. Compute the derivative of $f(x) = x^\pi \pi^x$.
3. Compute the derivative of $f(x) = 4^{\sin(3x)}$.
4. Compute the derivative of $f(x) = \ln(x^2 - \frac{2}{x^2})$.
5. Compute the tangent line to the graph of f at $x = \pi/4$ for $f(x) = \log_{10}(\sec x)$.
6. Compute the tangent line to the graph of f at $x = 1$ for $f(x) = e^{2x^3} + \ln(x^2)$.

Solutions

1. Compute the derivative of $f(x) = 9^x - 8^x$.

   Solution: We use the derivative rule that $\frac{d}{dx}[a^x] = a^x \ln a$. Applying this rule gives us

   $$f'(x) = 9^x \ln 9 - 8^x \ln 8.$$

2. Compute the derivative of $f(x) = x^\pi \pi^x$.

   Solution: We use the derivative rule that $\frac{d}{dx}[a^x] = a^x \ln a$, the power rule, and the product rule. Applying these rules gives us

   $$f'(x) = \frac{d}{dx}[x^\pi]\pi^x + x^\pi \frac{d}{dx}[\pi^x] = \pi x^{\pi-1}\pi^x + x^\pi \pi^x \ln \pi.$$

3. Compute the derivative of $f(x) = 4^{\sin(3x)}$.

   Solution: We use the derivative rule that $\frac{d}{dx}[a^x] = a^x \ln a$ and the chain rule. Applying these rules gives us

   $$f'(x) = 4^{\sin(3x)} \frac{d}{dx}[\sin(3x)] = 4^{\sin(3x)} \cos(3x) \frac{d}{dx}[3x] = 4^{\sin(3x)} \cos(3x)3.$$

4. Compute the derivative of $f(x) = \ln(x^2 - \frac{2}{x^2})$.

   Solution: We use the derivative rule that $\frac{d}{dx}[\ln x] = 1/x$ and the chain rule. Applying these rules gives us

   $$f'(x) = \frac{1}{x^2 - \frac{2}{x^2}} \frac{d}{dx}[x^2 - \frac{2}{x^2}] = \frac{1}{x^2 - \frac{2}{x^2}}(2x - \frac{-4}{x^3}).$$

   Recall $\frac{1}{x^2} = x^{-2}$ and so $\frac{d}{dx}[\frac{1}{x^2}] = \frac{d}{dx}[x^{-2}] = -2x^{-3} = \frac{-2}{x^3}$.

5. Compute the tangent line to the graph of f at $x = \pi/4$ for $f(x) = \log_{10}(\sec x)$.

   Solution: The slope of the tangent line at $x = \pi/4$ is given by $f'(\pi/4)$ so we start by computing f'.

   $$f'(x) = \frac{1}{\sec x \ln 10} \frac{d}{dx}[\sec x] = \frac{1}{\sec x \ln 10} \sec x \tan x = \frac{\tan x}{\ln 10}.$$

   Hence, $f'(\pi/4) = \dfrac{\tan(\pi/4)}{\ln 10} = 1/\ln 10$. In order to find our tangent line we also need the y value corresponding to $x = \pi/4$.

   $$f(\pi/4) = \log_{10}(\sec(\pi/4)) = \log_{10}(1/\cos(\pi/4)) = \log_{10}(1/(\sqrt{2}/2)) = \log_{10}(\sqrt{2}).$$

   The formula for the tangent line at $(x_1, f(x_1))$ is $y - f(x_1) = f'(x_1)(x - x_1)$. Plugging in our values gives us $y - \log_{10}(\sqrt{2}) = \dfrac{1}{\ln 10}(x - \pi/4)$.

6. Compute the tangent line to the graph of f at $x = 1$ for $f(x) = e^{2x^3} + \ln(x^2)$.

   Solution: The slope of the tangent line at $x = 1$ is given by $f'(1)$ so we start by computing f'.

   $$f'(x) = e^{2x^3}\frac{d}{dx}[2x^3] + \frac{1}{x^2}\frac{d}{dx}[x^2] = e^{2x^3}6x^2 + \frac{1}{x^2}2x = e^{2x^3}6x^2 + \frac{2}{x}.$$

   Hence, $f'(1) = 6e^2 + 2$. In order to find our tangent line we also need the y value corresponding to $x = 1$. $f(1) = e^2 + \ln 1 = e^2$. The formula for the tangent line at $(1, f(1))$ is $y - f(1) = f'(1)(x - 1)$. Plugging in our values gives us $y - e^2 = (6e^2 + 2)(x - 1)$.

Answer Key

1. Compute the derivative of $f(x) = 9^x - 8^x$.

   $f'(x) = 9^x \ln 9 - 8^x \ln 8$

2. Compute the derivative of $f(x) = x^\pi \pi^x$.

   $f'(x) = \pi x^{\pi-1}\pi^x + x^\pi \pi^x \ln \pi$

3. Compute the derivative of $f(x) = 4^{\sin(3x)}$.

   $f'(x) = 4^{\sin(3x)} \cos(3x)3$

4. Compute the derivative of $f(x) = \ln(x^2 - \frac{2}{x^2})$.

   $$f'(x) = \frac{2x - \frac{-4}{x^3}}{x^2 - \frac{2}{x^2}}$$

5. Compute the tangent line to the graph of f at $x = \pi/4$ for $f(x) = \log_{10}(\sec x)$.

   $$y - \log_{10}(\sqrt{2}) = \frac{1}{\ln 10}(x - \pi/4)$$

6. Compute the tangent line to the graph of f at $x = 1$ for $f(x) = e^{2x^3} + \ln(x^2)$.

   $y - e^2 = (6e^2 + 2)(x - 1)$