MAT122 HW10-12

Problems

1. Use the definition of the derivative to calculate derivative of
$$f(x) = 4x^2 - 7x$$
at $x = 1$.

2. Use the definition of the derivative to calculate derivative of
$$f(x) = \frac{1}{x-1}$$
at $x = 2$.

3. Use the definition of the derivative to calculate the derivative of
$$f(x) = \sqrt{1-9x}.$$

4. Use the definition of the derivative to calculate the derivative of
$$f(x) = x + \frac{1}{x}.$$

5. Compute the derivatives of the following functions:
   (a) $f(x) = x^{\frac{2}{3}} + x^{\frac{-2}{3}}$.
   (b) $f(x) = 7x^3 + 2\sqrt{x^5} - 8x + \dfrac{7}{x}$
   (c) $f(x) = \dfrac{x^2 + 3x + 27}{x^2}$

Answer Key

1. $f'(1) = 1$
2. $f'(2) = -1$
3. $f'(x) = \dfrac{-9}{2\sqrt{1-9x}}$
4. $f'(x) = \dfrac{x^2-1}{x^2}$
5. (a) $f'(x) = \dfrac{2}{3}x^{\frac{-1}{3}} - \dfrac{2}{3}x^{\frac{-5}{3}}$

   (b) $f'(x) = 21x^2 + \dfrac{5}{2}x^{\frac{3}{2}} - 8 - 7x^{-2}$

   (c) $f'(x) = -3x^{-2} - 54x^{-3}$

Solutions

1. By definition of the derivative,

$$\begin{aligned}
f'(1) &= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \\
&= \lim_{h\to 0} \frac{4(1+h)^2 - 7(1+h) - (-3)}{h} \\
&= \lim_{h\to 0} \frac{4+8h+4h^2-7-7h+3}{h} \\
&= \lim_{h\to 0} \frac{h-4h^2}{h} \\
&= \lim_{h\to 0} \frac{1-4h}{1} \\
&= 1
\end{aligned}$$

2. By definition of the derivative,

$$\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{1+h}-1}{h} \\
&= \lim_{h\to 0} \frac{\frac{-h}{1+h}}{h} \\
&= \lim_{h\to 0} \frac{-1}{1+h} \\
&= -1
\end{aligned}$$

3. By definition of the derivative,

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1-9(x+h)}-\sqrt{1-9x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1-9(x+h)}-\sqrt{1-9x}}{h}\,\frac{\sqrt{1-9(x+h)}+\sqrt{1-9x}}{\sqrt{1-9(x+h)}+\sqrt{1-9x}} \\
&= \lim_{h\to 0} \frac{1-9(x+h)-(1-9x)}{h\sqrt{1-9(x+h)}+\sqrt{1-9x}} \\
&= \lim_{h\to 0} \frac{-9}{\sqrt{1-9(x+h)}+\sqrt{1-9x}} \\
&= \frac{-9}{2\sqrt{1-9x}}
\end{aligned}$$

4. Notice that
$$x + \frac{1}{x} = \frac{x^2+1}{x}.$$
By definition of the derivative,
$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h} \\
&= \lim_{h\to 0} \frac{x(x^2+2xh+h^2+1) - (x+h)(x^2+1)}{hx(x+h)} \\
&= \lim_{h\to 0} \frac{x^3+2x^2h+xh^2+x-(x^3+x+x^2h-h)}{hx(x+h)} \\
&= \lim_{h\to 0} \frac{x^2h+xh^2-h}{hx(x+h)} \\
&= \lim_{h\to 0} \frac{h(x^2+xh-1)}{hx(x+h)} \\
&= \lim_{h\to 0} \frac{x^2+xh-1}{x(x+h)} \\
&= \frac{x^2+x(0)-1}{x(x+0)} \\
&= \frac{x^2-1}{x^2}
\end{aligned}$$

5. (a) By the power rule,
$$f'(x) = \frac{2}{3}x^{\frac{-1}{3}} - \frac{2}{3}x^{\frac{-5}{3}}$$

(b) Notice that
$$f(x) = 7x^3 + x^{\frac{5}{2}} - 8x + 7x^{-1}.$$
It follows from the power rule that
$$f'(x) = 21x^2 + \frac{5}{2}x^{\frac{3}{2}} - 8 - 7x^{-2}.$$

(c) Notice that
$$f(x) = 1 + 3x^{-1} + 27x^{-2}.$$
It follows from the power rule that
$$f'(x) = -3x^{-2} - 54x^{-3}.$$