MAT131 Homework 7-9

Problems

1. Compute the following limits:
   (a) $\lim_{x\to 1} 8 - 3x^2 + 12x^3$
   (b) $\lim_{t\to -2} \dfrac{4+2t}{2t^2+1}$
2. Compute the following limits:
   (a) $\lim_{x\to 8} \dfrac{2x^2 - 17x + 8}{8-x}$
   (b) $\lim_{h\to 0} \dfrac{(6+h)^2 - 36}{h}$
   (c) $\lim_{x\to 1} \dfrac{\sqrt{z}-1}{z-1}$
   (d) $\lim_{x\to 0} \dfrac{x}{3-\sqrt{x+9}}$
3. Compute the following limits:
   (a) $\lim_{x\to 1} \dfrac{\sqrt{z}-1}{z-1}$
   (b) $\lim_{x\to 0} \dfrac{x}{3-\sqrt{x+9}}$
4. Consider the function

$$f(x) = \begin{cases} 19-3x & x<3 \\ x^2+1 & x\geq 3 \end{cases}$$

   Evaluate the following limits, if they exist:
   (a) $\lim_{x\to 0} f(x)$
   (b) $\lim_{x\to 3} f(x)$
5. Evaluate the following limits, if they exist:
   (a) $\lim_{x\to 5} (10 + |x-5|)$

(b) $\lim_{t \to 1} \frac{t-1}{|t-1|}$

6. Evaluate the following limits:

   (a) $\lim_{x \to \infty} \frac{x^6 - 6x^4 + 8x^2 - 1}{3x^6 + 2x^3 + 9}$

   (b) $\lim_{x \to -\infty} \frac{2x^7 - 5x^2 + 1}{5 - 10x^5}$

   (c) $\lim_{x \to \infty} \frac{x^3 - 6x + 4}{1 - 3x^4}$

Answer Key

1. (a) $\lim_{x\to 1} 8 - 3x^2 + 12x^3 = 17$

   (b) $\lim_{t\to -2} \dfrac{4+2t}{2t^2+1} = 0$

2. (a) $\lim_{x\to 8} \dfrac{2x^2 - 17x + 8}{8-x} = -15$

   (b) $\lim_{h\to 0} \dfrac{(6+h)^2 - 36}{h} = 12$

3. (a) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1} = \dfrac{1}{2}$

   (b) $\lim_{x\to 0} \dfrac{x}{3-\sqrt{x+9}} = -6$

4. (a) $\lim_{x\to 0} f(x) = 19$

   (b) $\lim_{x\to 3} f(x) = 10$

5. Evaluate the following limits, if they exist:

   (a) $\lim_{x\to 5} (10 + |x-5|) = 10.$

   (b) $\lim_{t\to 1} \dfrac{t-1}{|t-1|}$ does not exist.

6. (a) $\lim_{x\to\infty} \dfrac{x^6 - 6x^4 + 8x^2 - 1}{3x^6 + 2x^3 + 9} = \dfrac{1}{3}.$

   (b) $\lim_{x\to -\infty} \dfrac{2x^7 - 5x^2 + 1}{5 - 10x^5} = -\infty.$

   (c) $\lim_{x\to\infty} \dfrac{x^3 - 6x + 4}{1 - 3x^4} = 0.$

Solutions

1. (a) $\lim_{x\to 1} 8 - 3x^2 + 12x^3 = 8 - 3(1)^2 + 12(1)^3 = 17$

   (b) $\lim_{t\to -2} \dfrac{4+2t}{2t^2+1} = \dfrac{4+2(-2)}{2(-2)^2+1)} = 0$

2. Compute the following limits:

   (a) If we plug in $x = 8$, we get

   $$\frac{2(8)^2 - 17(8) + 8}{8-8} = \frac{0}{0}.$$

   However, notice that the numerator can be factored as

   $$2x^2 - 17x + 8 = (x-8)(2x-1).$$

   It follows that

   $$\lim_{x\to 8} \frac{2x^2 - 17x + 8}{8-x} = \lim_{x\to 8} \frac{(x-8)(2x-1)}{8-x} = \lim_{x\to 8} -(2x-1) = -(2(8)-1) = -15.$$

   (b) If we plug in $h = 0$, we get

   $$\frac{(6+0)^2 - 36}{0} = \frac{0}{0}.$$

   However, notice that

   $$(6+h)^2 - 36 = 36 + 12h + h^2 - 36 = 12h + h^2.$$

   It follows that

   $$\lim_{h\to 0} \frac{(6+h)^2 - 36}{h} = \lim_{h\to 0} \frac{12h + h^2}{h} = \lim_{h\to 0} 12 + h = 12$$

3. Compute the following limits:

   (a) If we plug in $x = 1$, we get

   $$\frac{\sqrt{1} - 1}{1-1} = \frac{0}{0}.$$

   However, notice that the denominator can be factored as

   $$x - 1 = \left(\sqrt{x} - 1\right)\left(\sqrt{x} + 1\right)$$

   It follows that

   $$\lim_{x\to 1} \frac{\sqrt{x} - 1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x} - 1}{(\sqrt{x} - 1)(\sqrt{x} + 1)} = \lim_{x\to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{1+1} = \frac{1}{2}.$$

(b) If we plug in $x = 0$, we get

$$\frac{0}{3 - \sqrt{0+9}} = \frac{0}{0}.$$

However, notice that

$$\frac{x}{3 - \sqrt{x+9}} \left(\frac{3 + \sqrt{x+9}}{3 + \sqrt{x+9}} \right) = \frac{x\left(3 + \sqrt{x+9}\right)}{9 - (x+9)} = -(3 + \sqrt{x+9}).$$

It follows that

$$\lim_{x \to 0} \frac{x}{3 - \sqrt{x+9}} = \lim_{x \to 0} -(3 + \sqrt{x+9}) = -(3 + \sqrt{0+9}) = -6.$$

4. Consider the function

$$f(x) = \begin{cases} 19 - 3x & x < 3 \\ x^2 + 1 & x \geq 3 \end{cases}$$

Evaluate the following limits, if they exist:

(a) Near $x = 0$, $f(x) = 19 - 3x$. It follows that

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} 19 - 3x = 19 - 3(0) = 19.$$

(b) The limit from the left is

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} 19 - 3x = 19 - 3(3) = 10.$$

The limit from the right is

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2 + 1 = (3)^2 + 1 = 10.$$

5. (a) The limit from the left is

$$\lim_{x \to 5^-} (10 + (5 - x)) = 10 + 5 - 5 = 10.$$

The limit from the right is

$$\lim_{x \to 5^+} (10 + (x - 5)) = 10 + 5 - 5 = 10.$$

Since the limits from both sides are equal, The limit from the left is

$$\lim_{x \to 5} (10 + (5 - x)) = 10.$$

(b) The limit from the left is

$$\lim_{t \to 1^-} \frac{t-1}{1-t} = -1.$$

The limit from the right is

$$\lim_{t \to 1^+} \frac{t-1}{t-1} = 1.$$

Since the limit from the left does not equal the limit from the right, the limit does not exist.

6. Evaluate the following limits:

   (a) Since the highest power in the numerator is equal to the highest power in the denominator, the limit is the ratio of the coefficients in front of the highest powers. It follows that

   $$\lim_{x \to \infty} \frac{x^6 - 6x^4 + 8x^2 - 1}{3x^6 + 2x^3 + 9} = \frac{1}{3}$$

   (b) Since the highest power in the numerator is larger than the highest power in the denominator and the highest power in the numerator is odd,

   $$\lim_{x \to -\infty} \frac{2x^7 - 5x^2 + 1}{5 - 10x^5} = -\infty$$

   (c) Since the highest power in the numerator is smaller than the highest power in the denominator,

   $$\lim_{x \to \infty} \frac{x^3 - 6x + 4}{1 - 3x^4} = 0$$