MAT122 HW5-6

Problems

1. Find the range of the function $f(x) = 3x^2 + 6x + 8$.

2. Given the function
$$f(x) = \frac{x^2 - 9}{x^2 - 2x - 8},$$
Find all intercepts and vertical asymptotes.

3. If \$10,000 is invested at an annual rate of 5% and is compounded continuously, the amount of money at time t is given by
$$A(t) = 10,000e^{.05t}.$$
How long does it take for your investment to double?

4. The amount of bacteria at time t can be described by the equation
$$B(t) = B_0 e^{rt}$$
where B_0 is the initial amount of bacteria and r is the rate of growth. If there are initially 8000 bacteria and after 7 hours, there are 24,000 bacteria, what is the rate of growth r?

5. The population in Springfield, IL, starting from 2010, is given by the following table:

Years from 2010	0	1	2	3	4	5
Population	46,080	47,080	48,040	49,020	50,050	51,000

Find a linear model which approximates the population growth.

Answer Key

1. $[5, \infty)$
2. The y-intercept is $(0, -\frac{9}{8})$. The x-intercepts are $(3, 0), (-3, 0))$. The vertical asymptotes occur at $x = -2$ and $x = 4$.
3. $t = \dfrac{\ln(2)}{.05} \approx 13.9$ years
4. $r = \dfrac{\ln(3)}{7} \approx .157$
5. $y = 104t + 46,080$

Solutions

1. The x-coordinate of the vertex of the parabola is given by

$$x = -\frac{6}{2(3)} = -1.$$

It follows that the y-coordinate of the vertex is given by

$$f(-1) = 3(-1)^2 + 6(-1) + 8 = 3 - 6 + 8 = 5.$$

Since the coefficient in front of the x^2 term is positive, the vertex is a minimum. It follows that the range of $f(x)$ is $[5, \infty)$.

2. The y-intercept is given by

$$f(0) = \frac{0^2 - 9}{0^2 - 2(0) - 8} = -\frac{9}{8}.$$

The vertical asymptotes occur when the denominator is equal to 0. This occurs when

$$0 = x^2 - 2x - 8 = (x - 4)(x + 2).$$

So the vertical asymptotes occur at $x = -2$ and $x = 4$.

The x-intercepts occur when the numerator is equal to 0 and the denominator is not equal to 0. This occurs when

$$0 = x^2 - 9 = (x - 3)(x + 3)$$

It follows that the x-intercepts occur at $x = -3$ and $x = 3$.

3. If the investment doubles, we have the equation

$$20,000 = 10,000e^{.05t}.$$

This simplifies to $2 = e^{.05t}$. Solving for t gives

$$t = \frac{\ln(2)}{.05} \approx 13.9 \text{ years}.$$

4. Since there are initially 8000 bacteria, $B_0 = 8000$. Since there are 24,000 bacteria after 7 hours, we have the equation

$$24,000 = 8000e^{7r}.$$

This simplifies to $3 = e^{7r}$. Solving for r gives

$$r = \frac{\ln(3)}{7} \approx .157.$$

5. To find the slope of the linear model, we take the average of the differences divided by the total change in time. This is given by

$$m = \frac{100 + 60 + 80 + 130 + 150}{5} = 104$$

Since the y-intercept is given when $t = 0$, the equation for the linear model is

$$y = 104t + 46,080$$