MAT122 Homework 1-4

Problems

1. Let $f(x) = 3 + 5x - x^2$. Perform the following function evaluations:
   (a) $f(-1)$
   (b) $f(0)$
   (c) $f(1)$
   (d) $f(x+h)$

2. If $f(x) = 5 + 2x$ and $g(x) = 8 - 2x$,
   (a) $(f \circ g)(x)$,
   (b) $(g \circ f)(x)$

3. Find the domain and range of the following functions:
   (a) $f(x) = 3 + \sqrt{x^2 + 4}$
   (b) $f(x) = 5 - |8 - x|$

4. Find the domain of the following functions:
   (a) $f(x) = \dfrac{5}{x^3 + 10x^2 + 9x}$
   (b) $f(x) = \dfrac{4}{x-2} - \sqrt{x^2 - 1}$

5. Find the equation of the line with slope 2 passing through the ordered pair $(x, y) = (1, 4)$.

6. Find the equation of the line passing through the ordered pairs $(x_1, y_1) = (-1, 3)$, $(x_2, y_2) = (1, 6)$.

7. A shirt has been marked down 30% and is now on sale for \$15. How much was the original price of the shirt?

Answer Key

1. (a) $f(-1) = -3$
   (b) $f(0) = 3$
   (c) $f(1) = 7$
   (d) $f(x+h) = 3 + 5(x+h) - (x+h)^2$
2. (a) $(f \circ g)(x) = 21 - 4x$.
   (b) $(g \circ f)(x) = -2 - 4x$.
3. (a) Domain: $(-\infty, \infty)$, Range: $[5, \infty)$
   (b) Domain: $(-\infty, \infty)$, Range: $(-\infty, 5]$
4. (a) $(-\infty, -9) \cup (-9, -1) \cup (-1, 0) \cup (0, \infty)$
   (b) $(-\infty, -1) \cup (1, 2) \cup (2, \infty)$
5. $y = 2x + 2$
6. $y = \dfrac{3}{2}x + \dfrac{9}{2}$
7. The original price is \$20

Solutions

1. (a) $f(-1) = 3 + 5(-1) - (-1)^2 = 3 - 5 - 1 = -3$
   (b) $f(0) = 3 + 5(0) - (0)^2 = 3$
   (c) $f(1) = 3 + 5(1) - (1)^2 = 3 + 5 - 1 = 7$
   (d) $f(x + h) = 3 + 5(x + h) - (x + h)^2$

2. (a) $(f \circ g)(x) = f(8 - 2x) = 5 + 2(8 - 2x) = 5 + 16 - 4x = 21 - 4x.$
   (b) $(g \circ f)(x) = g(5 + 2x) = 8 - 2(5 + 2x) = 8 - 10 - 4x = -2 - 4x.$

3. (a) To find the domain of f, recall that the domain of $\sqrt{x}$ is $[0, \infty)$. It follows that we must have $x^2 + 1 > 0$. Since this is true for all real numbers, the domain of $f(x)$ is $(-\infty, \infty)$.
   Since the range of x^2 is $[0, \infty)$, the range of $x^2 + 4$ is $[4, \infty)$. It follows that the range of $\sqrt{x^2 + 4}$ is $[2, \infty)$. We conclude that the range of $f(x)$ is $[5, \infty)$.
   (b) Since the domain of $|x|$ is $(-\infty, \infty)$, the domain of $|8 - x|$ is $-\infty, \infty)$. It follows that the domain of $f(x)$ is $(-\infty, \infty)$.
   Since range of $-|8 - x|$ is $(-\infty, 0]$, the range of $f(x)$ is $(-\infty, 5]$.

4. (a) Notice that the denominator can be factored as
$$x^3 + 10x^2 + 9x = x(x^2 + 10x + 9) = x(x + 9)(x + 1)$$
   Since the denominator of the fraction cannot be 0, the domain of $f(x)$ is all real numbers except $x = 0, -1, -9$. In interval notation, the domain of $f(x)$ is written as
$$(-\infty, -9) \cup (-9, -1) \cup (-1, 0) \cup (0, \infty).$$
   (b) The fraction $\dfrac{4}{x - 2}$ is undefined when $x = 2$. The function $\sqrt{x^2 - 1}$ is undefined when $x^2 - 1 < 0$. This occurs for all x in the interval $(-1, 1)$. It follows that the domain for f is
$$(-\infty, -1) \cup (1, 2) \cup (2, \infty).$$

5. Using the point-slope formula,
$$y - 4 = 2(x - 1) = 2x - 2.$$
   It follows that $y = 2x + 2$.

6. Using the formula for the slope.
$$m = \frac{6 - 3}{1 - (-1)} = \frac{3}{2}.$$

Using the point-slope formula,

$$y - 3 = \frac{3}{2}(x - (-1)) = \frac{3}{2}x + \frac{3}{2}.$$

It follows that $y = \dfrac{3}{2}x + \dfrac{9}{2}$

7. If p is the original price of the shirt, then

$$p - .3p = 15.$$

It follows that $.7p = 15$. Solving for p gives $p = \$20$.