Lecture 23

Linear Systems. Part 3

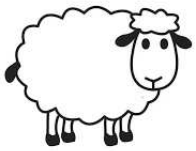
Applications of linear systems

In this lecture, we will learn how to solve **word problems** using systems of linear equations.

2 / 9

On a farm

Problem. On a farm, there are sheep and chicken.



All together,
they have 44 feet and 17 heads.
How many sheep and
how many chicken
are on the farm?

Solution. Let x be the number of sheep, and y be the number of chicken.

How many feet do all sheep have? $4x$

How many feet do all chicken have? $2y$

How many feet do sheep and chicken have all together? $4x + 2y$

How many heads do they have all together? $x + y$

What is given in the problem?

- all together they have 44 feet, so $4x + 2y = 44$.
- all together they have 17 heads, so $x + y = 17$.

3 / 9

Solve a system

How to find x, the number of sheep, and y, the number of chicken?

Solve the system

$$\begin{cases} 4x+2y=44 \\ x+y=17 \end{cases} \iff \begin{cases} 2x+y=22 \\ x+y=17 \end{cases} \iff \begin{cases} x=5 \\ x+y=17 \end{cases} \iff \begin{cases} x=5 \\ y=12 \end{cases}$$

Therefore, the number of sheep is 5, the number of chicken is 12.

Let us check if our answer is correct.

How many feet do 5 sheep and 12 chicken have?

$$4 \cdot 5 + 2 \cdot 12 = 20 + 24 = 44 \ \checkmark$$

How many heads do 5 sheep and 12 chicken have?

$$5 + 12 = 17 \ \checkmark$$

The problem is solved correctly!

Answer. There are 5 sheep and 12 chicken on the farm.

In a movie theater

Problem. A family of two adults and five children pays $\$61$ for tickets in a movie theater.
A family of three adults and two children pays $\$53$.
Find a ticket price for an adult and a ticket price for a child.

Solution. Let $\$x$ be the price for an adult ticket, and $\$y$ be the price for a child ticket.

How much a family of **two** adults and **five** children will pay then? $\$(2x+5y)$

How much a family of **three** adults and **two** children will pay? $\$(3x+2y)$

What is given in the problem?

A family of **two** adults and **five** children pays $\$61$. So $2x+5y=61$.

A family of **three** adults and **two** children pays $\$53$. So $3x+2y=53$.

How to find x and y?

Solve a system

$$\begin{cases} 2x+5y=61 \\ 3x+2y=53 \end{cases} \iff \begin{cases} 6x+15y=183 \\ 6x+4y=106 \end{cases} \iff \begin{cases} 11y=77 \\ 3x+2y=53 \end{cases} \iff$$

$$\begin{cases} y=7 \\ 3x+2\cdot 7=53 \end{cases} \iff \begin{cases} y=7 \\ 3x=53-14 \end{cases} \iff \begin{cases} y=7 \\ 3x=39 \end{cases} \iff \begin{cases} x=13 \\ y=7 \end{cases}$$

Therefore, the price for an adult ticket is $\$13$, and the price for a children ticket is $\$7$.

Let us check if our answer is correct.

How much a family of **two** adults and **five** children will pay, in dollars?

$2\cdot 13+5\cdot 7=26+35=61$ ✓

How much a family of **thee** adults and **two** children will pay, in dollars?

$3\cdot 13+2\cdot 7=39+14=53$ ✓

Answer. The ticket price for an adult is $\$13$, the ticket price for a child is $\$7$.

6 / 9

In a winery

Problem. A winemaker has in his cellar 1620 liters of wine aging in **three** small and **five** large barrels. Find the volumes of the barrels if a large barrel contains 20 liters more than a small one.

Solution. Let x be the volume (in liters) of a small barrel, and y be the volume (in liters) of a large barrel.

What is the **total** volume of **three** small and **five** large barrels?

$3x+5y$ (liters)

What is the difference in volumes between a large and a small barrel? $y-x$

What is given in the problem?

- total volume: $3x+5y=1620$
- the difference in volumes: $y-x=20$

7 / 9

Solve a system

$$\begin{cases} 3x+5y=1620 \\ -x+\ \ y=20 \end{cases} \iff \begin{cases} 3x+5y=1620 \\ y=x+20 \end{cases} \iff \begin{cases} 3x+5(x+20)=1620 \\ y=x+20 \end{cases}$$

$$\begin{cases} 3x+5x+100=1620 \\ y=x+20 \end{cases} \iff \begin{cases} 8x=1520 \\ y=x+20 \end{cases} \iff \begin{cases} x=190 \\ y=190+20 \end{cases} \iff$$

Therefore, a small barrel contains 190 liters,

$$\begin{cases} x=190 \\ y=210 \end{cases}$$

and a large one contains 210 liters.

Let us check if our answer is correct.

What is the total volume of **three** small barrels and **five** large ones?

$3 \cdot 190 + 5 \cdot 210 = 570 + 1050 = 1620$ ✓

How many liters more does a large barrel contain than a small one?

$210 - 190 = 20$ ✓

Answer. 190 and 210 liters.

Summary

In this lecture, we have learned

- ☑ how to solve word problems using linear systems
- ☑ how to check if the answer is correct