Lecture 21

Linear Systems. Part 1

What is a linear system?

We will study systems consisting of two linear equations in two unknowns,

like this: $\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1 \end{cases}$ x, y are called unknowns.

To **solve a system** means to find all values of x and y which satisfy **both** equations.

The brace $\{$ means that **both** equations should be satisfied by the same values of x and y.

The values $x = 1$ and $y = -2$ satisfy $\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1, \end{cases}$

because $\begin{cases} -2 \cdot 1 + 3(-2) = -2 + (-6) = -8 \\ 5 \cdot 1 + 2(-2) = 5 + (-4) = 1. \end{cases}$ Therefore,

$\begin{cases} x = 1 \\ y = -2 \end{cases}$ (or just the pair $(1, -2)$) is a **solution** of $\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1. \end{cases}$

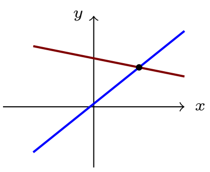
Are there **other solutions?** To solve a system means to find **all** its solutions!

How many solutions may a system have?

The graph of each equation of the system is a **line**.

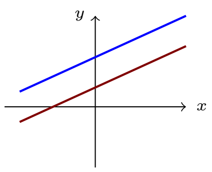
A solution of the system is a point which belongs to **both** lines.

How can two lines on a plane be **positioned** with respect to each other?



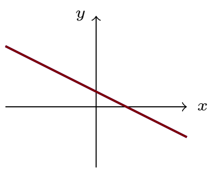
lines intersect at one point

system has **one** solution



lines are parallel

system has **no** solution



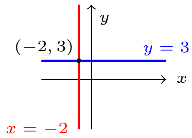
lines coincide

system has **infinitely many** solutions

How to solve a system?

Some systems are easy.

$\begin{cases} x = -2 \\ y = 3 \end{cases}$ is a linear system,
but it looks like a solution,
and it is a **solution** for itself.



To solve a more complicated system, we propose to turn it into an easy one by a sequence of elementary **transformations**.

The transformations must preserve the set of all solutions.

If two systems have the same solutions, we call them equivalent .
and write $\iff$ between the systems,

like this:

$$\begin{cases} x + 3 = 1 \\ 2y = 6 \end{cases} \iff \begin{cases} x = -2 \\ y = 3 \end{cases}$$

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Elementary transformations

There are **three** elementary transformations.

1. Adding equations, that is replacing one equation by its sum with the other equation.

$$\begin{cases} -x + 2y = 3 \\ x - \ \ y = 0 \end{cases} \begin{matrix} \xrightarrow{\text{sum up}} \\ \xrightarrow{\text{keep}} \end{matrix} \iff \begin{cases} -x + 2y + (x - y) = 3 + 0 \\ x - y \ = \ \ 0 \end{cases}$$

$$\iff \begin{cases} -x + 2y + (x - y) = 3 + 0 \\ x - y \ = \ \ 0 \end{cases} \iff \begin{cases} y = 3 \\ x - y = 0 \end{cases}$$

Adding the first equation to the second one completes the solution:

$$\begin{cases} y = 3 \\ x - y = 0 \end{cases} \iff \begin{cases} y = \ \ 3 \\ x - y + y = 0 + 3 \end{cases} \iff \begin{cases} y = 3 \\ x = 3 \end{cases}$$

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Elementary transformations

2. Subtracting equations.

$$\begin{cases} \quad y = -1 \\ x + y = 1 \end{cases} \begin{matrix} \xrightarrow{\text{keep}} \\ \xrightarrow{\text{subtract}} \end{matrix} \iff \begin{cases} y & = & -1 \\ x + y - y & = & 1 - (-1) \end{cases} \iff \begin{cases} y = -1 \\ x = 2 \end{cases}$$

3. Multiplying an equation by a non-zero number.

$$\begin{cases} -\frac{1}{2}x = 1 \\ 3y = -5 \end{cases} \begin{matrix} \xrightarrow{\text{multiply by } (-2)} \\ \xrightarrow{\text{divide by } 3} \end{matrix} \begin{cases} x = -2 \\ y = -\frac{5}{3} \end{cases}$$

Division by 3 is multiplication by $\frac{1}{3}$.

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Summary

In this lecture, we have learned

- ☑ what a **linear system** is
- ☑ what **solutions** of a linear system are
- ☑ what it means to **solve** a system
- ☑ **how many solutions** a linear system may have
- ☑ which systems are called **equivalent**
- ☑ what **elementary transformations** are

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