Lecture 16

Applications of Linear Equations

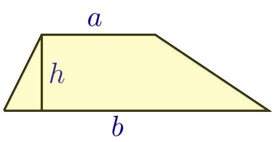
Linear equations in mathematics, physics, and beyond

In this lecture, we will show how

- to solve linear equations originated in **mathematics** and **physics**
- how to use linear equations for solving **word problems**.

Area of trapezoid

Problem 1. The area A of a trapezoid with bases a, b and the height h is given by the formula



$$A = \frac{a+b}{2}h.$$

Using this formula, express b in terms of A, a, and h.

Solution. We have to solve out b from the equation $A = \dfrac{a+b}{2}h$.

Multiply the equation by 2: $2A = (a+b)h$,

divide both sides by h: $\dfrac{2A}{h} = a+b$,

and move a to LHS: $\dfrac{2A}{h} - a = b$.

Answer: $b = \dfrac{2A}{h} - a$.

Motion with constant acceleration

Problem. A car moving at a constant speed of v_0 starts to accelerate with a constant acceleration of a.
How long will it take for the car to increase the speed up to v,
if the initial speed v_0, the terminal speed v, the acceleration a, and the time t are related by the formula $v = v_0 + at$?

Solution. We have to solve out t from the equation $v = v_0 + at$.

For this, we subtract v_0 from both sides: $v - v_0 = at$,

and divide both sides by a: $\dfrac{v - v_0}{a} = t$.

Answer: $t = \dfrac{v - v_0}{a}$.

Newton's law

Example. According to Newton's law of universal gravitation,

$$F = G\,\frac{m_1 m_2}{R^2},$$

where F is the gravitational force between masses m_1 and m_2, G is the gravitational constant, and R is the distance between the centers of the masses.
Use this equation to find m_1 in terms of F, G, m_2, and R.

Solution. To solve out m_1 from the equation $F = G\,\dfrac{m_1 m_2}{R^2}$,

multiply both sides by R^2: $FR^2 = G\,m_1 m_2$,

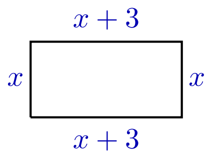
and divide by Gm_2: $\dfrac{FR^2}{Gm_2} = m_1$.

Answer: $m_1 = \dfrac{FR^2}{Gm_2}$

Perimeter of a rectangle

Problem. In a rectangle, one side is 3 feet longer than the other side. Find the lengths of the sides, if the perimeter of the rectangle is 34 feet.

Solution.



Let x be the length of the short side.

Then the length of the long side is $x+3$.

The **perimeter** is the sum of the lengths of all sides: $x+(x+3)+x+(x+3)$.

Simplify this expression: $x+(x+3)+x+(x+3)=4x+6$.

Since the perimeter is 34 feet, $4x+6=34$.

Solve this equation: $4x+6=34 \iff 4x=28 \iff \boxed{x=7}$ feet.

The short side is 7 feet, the long side is $7+3=10$ feet.

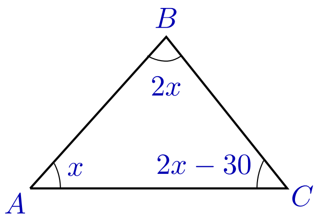
Answer. The lengths of the sides are 7 and 10 feet.

6 / 9

Angles in a triangle

Problem. In a triangle ABC, the angle B is twice as large as the angle A, and the angle C is $30°$ less than the angle B. Find the angles.

Solution.



Let x be the measure of A.

Then the measure of B is $2x$,

and the measure of C is $2x-30$.

The **sum** of the angles in a triangle is $180°$. In our case,

$$x+2x+(2x-30)=180.$$

This is a linear equation to solve:

$x+2x+(2x-30)=180 \iff 5x-30=180 \iff 5x=210 \iff x=42.$

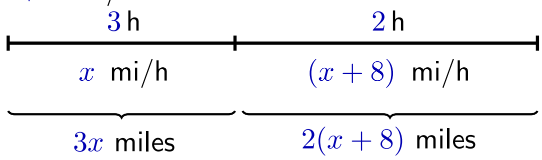
The measure of A is $\boxed{42°}$, the measure of B is $2\cdot 42=\boxed{84°}$, the measure of C is $84-30=\boxed{54°}$.

7 / 9

Uniform motion

Problem. A car traveled for 3 hours at a constant speed. Then it increased the speed by 8 mi/h and traveled for another 2 hours. During this trip, the car traveled for 271 miles. Find the speed of the car on both intervals of driving.

Solution. Let x mi/h be the speed of the car on the first interval of driving. Then the speed on the second interval of driving is $x+8$ mi/h.



The **total** distance is $3x+2(x+8)$ miles, which is equal to 271 miles.
Therefore, $3x+2(x+8)=271$. Let us solve this equation to find x.
$3x+2(x+8)=271 \iff 3x+2x+16=271 \iff 5x=255 \iff x=51$

So the speed on the first interval is 51 mi/h, and the speed on the second interval is $51+8=59$ mi/h.

Answer. 51 mi/h and 59 mi/h.

Summary

In this lecture, we have learned

- ☑ how to solve linear equations "with letters" arising from **mathematics and physics**
- ☑ how to solve **word problems** leading to linear equations