Lecture 13

Composing Algebraic Expressions

Translating English to Algebra

In this lecture, we will learn how to **compose** algebraic expressions after word descriptions.

Composing algebraic expressions is an important **skill** for solving "**real-life**" problems that you may encounter in the math classroom and beyond.

To **translate** successfully English phrases into algebraic expressions, we need to understand the **meaning** of each phrase and **express** this meaning algebraically.

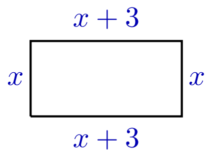
This translation may **require** some basic knowledge from other fields, for example

- geometric formulas for area, volume, and perimeter,
- formula for uniform motion: distance=speed×time ,
- common facts about money system (cents, nickels, dimes, quarters), pricing,
- percentage.

Perimeter of a rectangle

Problem. In a rectangle, one side is x feet long. The other side is 3 feet longer. Compose an algebraic expression (in terms of x) for the **perimeter** of the rectangle. Simplify the expression. Find the value of this expression for $x = 5$ feet.

Solution.



The **perimeter** is the sum of the lengths of all sides.

The perimeter of our rectangle is

$$x + (x + 3) + x + (x + 3)\,.$$

Simplify this expression:

$$x + (x + 3) + x + (x + 3) = \boxed{4x + 6}\,.$$

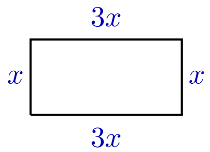
Find the value of the expression at $x = 5$:

$$4x + 6\Big|_{x=5} = 4 \cdot 5 + 6 = 20 + 6 = \boxed{26}\text{ (feet)}.$$

Area of a rectangle

Problem. In a rectangle, the width is x feet. The length is 3 times as long as the width. Compose an algebraic expression (in terms of x) for the **area** of the rectangle. Simplify this expression. Find the value of this expression for $x = 5$ feet.

Solution.



The **area** of a rectangle is
the product of the width by the length.

The area of our rectangle is $x(3x)$.

Simplify this expression: $x(3x) = \boxed{3x^2}$.

Find the value of the expression at $x = 5$:

$$3x^2\Big|_{x=5} = 3 \cdot 5^2 = 3 \cdot 25 = \boxed{75}\ (\text{ft}^2).$$

Counting money

Problem. In a piggy bank, there are dimes and quarters. The number of quarters is 5 less than the number of dimes. Compose an algebraic expression for the total amount of money in the piggy bank, if the number of dimes is x. Find the value of the expression if $x = 20$.

Solution. There are x dimes in the piggy bank. Their total value is $10x$ cents.

The number of quarters is 5 less than the number of dimes (which is x).
So there are $x - 5$ quarters. Their total value is $25(x - 5)$ cents.

The total money value in the piggy bank is the value of dimes plus the value of quarters:

$$10x + 25(x - 5).$$

Let us simplify the expression:

$$10x + 25(x - 5) = 10x + 25x - 125 = \boxed{35x - 125},$$

and evaluate it at $x = 20$:

$35x - 125\Big|_{x=20} = 35 \cdot 20 - 125 = 700 - 125 = \boxed{575}$ cents.

Uniform motion

Problem 1. A car moved for 4 hours at a constant speed of x mi/h.
Compose an algebraic expression (in terms of x) for the distance covered.

Solution. For **uniform motion** (motion with a constant speed),
the distance, speed and time are related by the formula

$$\text{distance}=\text{speed}\times\text{time}.$$

Therefore, the distance that the car covered traveling for 4 hours at a constant speed of x mi/h is $\boxed{4x}$ (miles).

Problem 2. It took x seconds for an athlete to run the distance of 300 meters. Compose an algebraic expression for the **speed** of the athlete.

Solution. Given: time $= x$ seconds, distance $= 300$ meters. Find the speed.

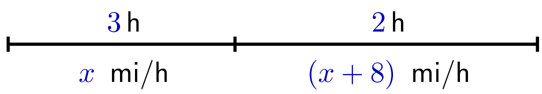
Since $\text{distance}=\text{speed}\times\text{time}$, then $\text{speed} = \dfrac{\text{distance}}{\text{time}}$.

In our case, the **speed** of the athlete is $\dfrac{300}{x}$ (m/s).

Uniform motion

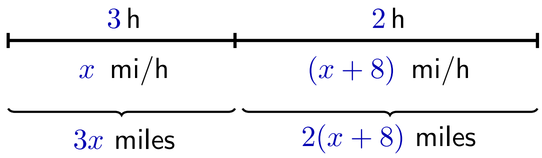
Problem. A car traveled for 3 hours at a constant speed of x mi/h. Then it increased the speed by 8 mi/h and traveled for another 2 hours. Compose an algebraic expression for the **total distance** covered by the car. Simplify the expression. Find the value of the expression for $x = 50$ mi/h.

Solution. Let us show schematically what is given in the problem:



The total distance is the **sum** of two distances.

Uniform motion



The total distance is $3x + 2(x+8)$ miles.

Simplify the expression:

$$3x + 2(x+8) = 3x + 2x + 16 = \boxed{5x+16}.$$

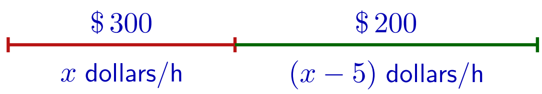
Find the value of the expression for $x = 50$ mi/h:

$$5x + 16\Big|_{x=50} = 5 \cdot 50 + 16 = 250 + 16 = \boxed{266} \text{ miles.}$$

Pay rate

Problem. This week, Rob earned $\$\,300$ while tutoring for x dollars per hour, and $\$\,200$ working at an office, where the pay rate is $\$\,5$ per hour less than for tutoring. Compose an algebraic expression for the **total time** that Rob spent working this week.

Solution. Let us show schematically what is given in the problem:



The **total** time is the time spent on tutoring plus the time spent in office.

Pay rate

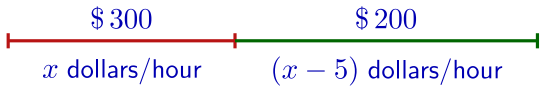
The amount earned, the pay rate, and the time are related by the formula

$$\text{amount earned} = \text{pay rate} \times \text{time}.$$

From which we get

$$\text{time} = \frac{\text{amount earned}}{\text{pay rate}}.$$

Calculate the time spent on each job **separately**:



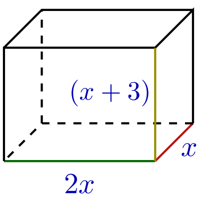
The time spent while tutoring is $\frac{300}{x}$. The time spent in office is $\frac{200}{x-5}$.

The total time is $\frac{300}{x} + \frac{200}{x-5}$ (hours).

Volume

Problem. The width of a rectangular aquarium is x inches, the length is twice as long as the width, and the height is 3 inches more than the width. Compose an algebraic expression for the **volume** of the aquarium. Simplify the expression.

Solution.



The **volume** of a rectangular box is width×length×height.

By this, the volume of the aquarium is $x \cdot (2x) \cdot (x+3)$.

Simplify this expression:

$$x \cdot (2x) \cdot (x+3) = 2x^2(x+3) = \boxed{2x^3 + 6x^2}.$$

Electric bill

Problem. An electric company charges a flat rate of $\$50$ per month plus $\$x$ per kWh. The sales tax is 2.5%. Compose an algebraic expression showing total charges in the electric bill for this month, if 1000 kWh have been consumed. Simplify the expression.

Solution. The charge for consumed 1000 kWh is $1000x$. Adding the flat rate of $\$50$, we get the charge before tax: $50 + 1000x$.
Upon the top, we have to add 2.5% tax, which is 2.5% of before-tax amount. Since $2.5\% = \frac{2.5}{100} = 0.025$, the tax is $0.025(50 + 1000x)$.

The total charge is

$$\underbrace{50}_{\text{flat rate}} + \underbrace{1000x}_{\text{consumed}} + \underbrace{0.025(50 + 1000x)}_{\text{tax}}.$$

Simplify the expression:
$50 + 1000x + 0.025(50 + 1000x) = 50 + 1000x + 1.25 + 25x = \boxed{51.25 + 1025x}$ (dollars).

Summary

In this lecture, we have learned

- ☑ how to translate English phrases into algebraic language
- ☑ which additional information may be required:
  - formulas for area, volume, perimeter of geometric figures
  - formula for uniform motion
  - percentage
- ☑ how to make schematic drawings for problems