Lecture 4

Addition and Multiplication

Properties of operations

Addition and multiplication are **basic** arithmetic operations.

They share two useful **properties**.

These properties are

- **commutativity**
- **associativity**

In this lecture, we will study these properties

and learn how to make use of them.

Commutativity of addition

When adding two numbers, the order of the numbers doesn't matter.

For example, $2 + 3 = 3 + 2$.

This property of addition can be written using variables:

$a + b = b + a$ for any a and b

Since a and b can represent **any** numbers, this formula represents infinitely many equalities.
For example, if $a = 8$ and $b = 5$, then $a + b = b + a$ becomes

$8 + 5 = 5 + 8$.

If $a = x$ and $b = 5$, then $a + b = b + a$ becomes

$x + 5 = 5 + x$.

This property of addition is called **commutativity**.

Commutativity of multiplication

Multiplication is also **commutative**.

When multiplying two numbers, the order of the numbers doesn't matter.

For example, $2 \cdot 3 = 3 \cdot 2$.

This property is expressed using variables as follows:

$$a \cdot b = b \cdot a \quad \text{for any } a \text{ and } b$$

Since a and b represent **any** numbers, this formula represents infinitely many equalities.

For example, if $a = 4$ and $b = 7$, then $a \cdot b = b \cdot a$ becomes
$4 \cdot 7 = 7 \cdot 4$,

if $a = 2$ and $b = x$, then $a \cdot b = b \cdot a$ becomes
$2 \cdot x = x \cdot 2$.

4 / 10

Associativity of addition

When we add **three** numbers, the result does **not** depend on the order of operations:

$$(1 + 2) + 3 = 3 + 3 = 6$$
$$1 + (2 + 3) = 1 + 5 = 6 .$$

That is, $(1 + 2) + 3 = 1 + (2 + 3)$.

In general,

$$(a + b) + c = a + (b + c) \quad \text{for any } a,\ b \text{ and } c$$

This property of addition is called **associativity**.

Associativity helps to make calculations easier. Compare:

$428 + 13999 + 1 = (428 + 13999) + 1 = 14427 + 1 = 14428$ and

$428 + 13999 + 1 = 428 + (13999 + 1) = 428 + 14000 = 14428$.

5 / 10

Associativity of multiplication

Multiplication is also **associative**:

$$(ab)c = a(bc) \text{ for any } a, b \text{ and } c$$

Associativity of multiplication is useful:

$53 \cdot 25 \cdot 4 = 53 \cdot (25 \cdot 4) = 53 \cdot 100 = 5300.$

In the next examples, **both** associativity and commutativity are used:

$5 \cdot 97 \cdot 20 = (5 \cdot 97) \cdot 20 = (97 \cdot 5) \cdot 20 = 97 \cdot (5 \cdot 20) = 97 \cdot 100 = 9700,$

$2x \cdot 3y = 2(x \cdot 3)y = 2(3x)y = (2 \cdot 3)xy = 6xy.$

When can we leave out parentheses?

Due to **associativity**,
when we perform either additions only, or multiplications only,
the result does **not** depend on the order of operations:

$$((1+2)+3)+4 = (1+(2+3))+4 = 1+((2+3)+4)$$
$$((2\cdot 3)\cdot 4)\cdot 5 = (2\cdot(3\cdot 4))\cdot 5 = 2\cdot((3\cdot 4)\cdot 5)\,.$$

Therefore, we do **not** use parentheses in a formula
which involves additions **only** or multiplications **only**, like this

$$1+2+3+4\,, \qquad 2\cdot 3\cdot 4\cdot 5$$

Moreover, due to **commutativity**, the order of numbers **doesn't** matter:

$$1+2+3+4 = 2+3+4+1 = 4+2+1+3 = \dots$$

$$2\cdot 3\cdot 4\cdot 5 = 2\cdot 3\cdot 5\cdot 4 = 4\cdot 2\cdot 5\cdot 3 = \dots$$

Recall that if **both** addition and multiplication are present,
then the order **does** matter: $(1+2)\cdot 3 \neq 1+2\cdot 3$

Special numbers: 0 and 1

$$a + 0 = a \quad \text{for any } a$$

Numbers a and $-a$ are called **opposite** to each other.

For example, -2 is opposite to 2, and 2 is opposite to -2.

$$a + (-a) = 0 \quad \text{for any } a$$

The product of any number by 0 equals 0:

$$a \cdot 0 = 0 \quad \text{for any } a$$

The product of any number by 1 equals this number:

$$a \cdot 1 = a \quad \text{for any } a$$

8 / 10

Reciprocals

Numbers a and b are called **reciprocals** if $a \cdot b = 1$.

For example, 2 and $\frac{1}{2}$ are reciprocals, since $2 \cdot \frac{1}{2} = 1$.

Numbers a and $\frac{1}{a}$ are reciprocals for any non-zero a.

$$a \cdot \frac{1}{a} = 1 \quad \text{for any non-zero } a$$

0 has **no** reciprocal, because there is **no** number b such that $0 \cdot b = 1$.

Indeed, $0 \cdot b = 0$ for any b.

9 / 10

Summary

In this lecture, we have learned

- ☑ commutativity of addition: $a+b=b+a$
- ☑ commutativity of multiplication: $ab=ba$
- ☑ associativity of addition: $(a+b)+c=a+(b+c)$
- ☑ associativity of multiplication: $(ab)c=a(bc)$
- ☑ when parentheses are not needed
- ☑ identities involving 0 and 1: $a+0=a,\ a\cdot 1=a,\ a\cdot 0=0$
- ☑ opposite numbers
- ☑ reciprocal numbers