Lecture 3

Variables and Algebraic Expressions

Variables

A **variable** is a letter representing a number.

Why do we need letters?

- Some numbers are **special**,
  but don't have any convenient representation, like π.
- Some numbers are given by a formula, which is too **bulky** to deal with,
  like the golden ratio $\varphi = \dfrac{1+\sqrt{5}}{2}$.
- Sometimes we don't know the number, but want to **find** it.
  For example, when we are solving the equation $2x + 1 = 7$.
- Sometimes we want to express a **relationship** between quantities,
  like $d = v \cdot t$, where d is distance, v is speed and t is time.

Variables for numbers are like **names** (or nicknames) for people.

Algebraic expressions

We already know (from Lecture 2) that a **numerical expression**
consists of numbers, symbols for operations and parentheses,
and describes an algorithm for calculation.

For example, $1 \cdot 2 - 3 \cdot (1+2) \div 4$ is a numerical expression.

An **algebraic expression** (or simply "an expression")
consists of numbers, **variables**, symbols for operations and parentheses,
and becomes a numerical expression when we substitute (plug in)
a numerical value for each variable.

Example 1. $3 \cdot x - 4 \cdot (x+1)$ is an algebraic expression. It involves the numbers $3, 4, 1$, the variable x, and the operations multiplication, addition and subtraction. How many operations are there in this expression? Four.

Example 2. $x \cdot y - \dfrac{5 \cdot (x+y)}{4}$ is an algebraic expression. It involves the numbers $5, 4$, the variables x, y, and the operations multiplication, division, addition and subtraction. How many operations are there in this expression? Five.

When can the multiplication dot be omitted?

It is customary **not** to write the multiplication dot in front of a variable or parenthesis:

$a \cdot b$ is written as ab,

$2 \cdot x$ is written as $2x$,

but the dot has to be present in $x \cdot 2$ and $2 \cdot 2$,

$a \cdot (b + c)$ is written as $a(b + c)$,

$(a + b) \cdot (c + d)$ is written as $(a + b)(c + d)$.

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Evaluation of expressions

An algebraic expression becomes a numerical expression
if we **substitute** (plug in) a numerical value for each variable.

For example, if we plug $x = 2$ into the expression $3x - 4(x + 1)$, we get

$$3x - 4(x + 1)\Big|_{x=2} = 3 \cdot 2 - 4(2 + 1),$$

which is a numerical expression. Its value is

$$3 \cdot 2 - 4(2 + 1) = 6 - 4 \cdot 3 = 6 - 12 = -6.$$

This process is called **evaluation** at $x = 2$.

A numerical expression is a **special kind** of algebraic expression.

A **numerical** expression is an **algebraic** expression which contains **no** variables.

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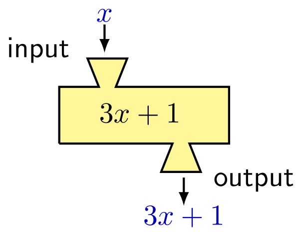
An expression as a program

An expression may be understood as a **program** (or algorithm, or set of instructions) describing a calculation.

For example, the expression $3x+1$ represents the following procedure:

$$x \xrightarrow{\text{multiply by } 3} 3x \xrightarrow{\text{add } 1} 3x+1.$$

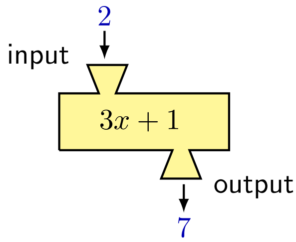
For each value of the variable x (for each input), this program delivers an output, which is called the **value** of the expression $3x+1$.



Evaluating an expression is running the program

When we **evaluate** an expression at a number, we run the corresponding program.

For example, to evaluate the expression $3x+1$ at the number 2,
we need to plug $x=2$ into $3x+1$:



We denote this evaluation as follows:

$$3x+1\Big|_{x=2} = 3\cdot 2+1 = 7\,.$$

Examples of evaluations

Example 1. Evaluate the expression $\dfrac{2x-1}{x+1}$ at $x=-3$.

Solution.

$$\left.\frac{2x-1}{x+1}\right|_{x=-3} = \frac{2(-3)-1}{(-3)+1} = \frac{-6-1}{-2} = \frac{-7}{-2} = \frac{7}{2}.$$

Example 2. Find the value of the expression $3(x-1)+2y$ at $x=1$, $y=-2$.

Solution.

$$\left.3(x-1)+2y\right|_{x=1,\, y=-2} = 3(1-1)+2(-2) = 3\cdot 0-4 = 0-4 = -4.$$

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Why algebraic expressions are important

Algebraic expressions and operations with them are fundamentally **involved** in all parts of Algebra.

So far, we have met only the **simplest** of them.

Later in the course we will study **more complex** expressions and operations.

Fluency in operating with algebraic expressions is crucial for your success in the course.

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Summary

In this lecture, we have learned

- ☑ what a variable is
- ☑ what an algebraic expression is
- ☑ how to evaluate an expression at a number
- ☑ how to understand an expression as a program
- ☑ why algebraic expressions are important