Lecture 2

Numerical Expressions

Numerical expressions

A **numerical expression** consists of

numbers, symbols of operations and parentheses,

and describes an algorithm (a set of instructions) for calculation.

For example, $3 - 8 \div 4 \cdot (1 + 2)$.

The result of the calculation is called the **value** of the numerical expression.
The process of calculation is called **evaluation**.

In this lecture we will learn **how to evaluate** a numerical expression.
For example, here is the evaluation of the numerical expression given above:
$$3 - 8 \div 4 \cdot (1 + 2) =$$
$$3 - 8 \div 4 \cdot 3 =$$
$$3 - 2 \cdot 3 =$$
$$3 - 6 =$$
$$-3.$$

In particular, we will learn in which **order** to perform the arithmetic operations.

Without parentheses

Multiplication and division have to be done **before** addition and subtraction,

if the formula does not contain parentheses.

By this rule, $1 + 2 \cdot 3 = 1 + 6 = 7$.

If the formula contains several multiplications and divisions(and still no parentheses),
the multiplications and divisions are performed in order **from left to right.**

For example, $6 \div 3 \cdot 5 = \boxed{\boxed{6 \div 3} \cdot 5} = 2 \cdot 5 = 10$,

$6 \div 3 + 4 \cdot 5 = \boxed{6 \div 3} + \boxed{4 \cdot 5} = 2 + \boxed{4 \cdot 5} = 2 + 20 = 22$.

Additions and subtractions are done after all multiplications and divisions, also **from left to right**:

$5 - 4 \div 2 + 3 \cdot 2 \div 6 = 5 - \boxed{4 \div 2} + \boxed{\boxed{3 \cdot 2} \div 6} = 5 - 2 + 1 = \boxed{5 - 2} + 1 = 3 + 1 = 4$.

Two kinds of parentheses

In expressions, parentheses play **two** different roles.

• First, they help describe the **order of operations**:
$$(1 + 2) \cdot 3 = 3 \cdot 3 = 9 \,.$$
Notice that the expression above **without** parentheses has a different value:
$$1 + 2 \cdot 3 = 1 + 6 = 7 \,.$$

• Second, parentheses have to surround a **negative** number,

when the number comes after the sign of an arithmetic operation, as in
$$2 + (-3) \,, \quad \text{or} \quad 2 \cdot (-3) \,.$$

Parentheses around a negative number

Parentheses around a negative number **do not matter** for the order of operations.

If all parentheses in a formula are of that kind,

then calculations should be performed as if there were no parentheses:

first, all multiplications and divisions from left to right,

then all additions and subtractions from left to right:

$$(-4) \div 2 + 3 \cdot (-5) = \boxed{(-4) \div 2} + \boxed{3 \cdot (-5)} = -2 + (-15) = -17 \,.$$

Parentheses rule

If a formula contains parentheses which surround more than one number, then

1. find the innermost parentheses of this kind,

2. evaluate the formula within the parentheses,

3. and continue if needed.

For example,
$$(3 - 1) \cdot (1 + 4 \div (3 - 5)) =$$
$$2 \cdot (1 + 4 \div (3 - 5)) =$$
$$2 \cdot (1 + 4 \div (-2)) =$$
$$2 \cdot (1 + (-2)) =$$
$$2 \cdot (-1) =$$
$$-2 \,.$$

Summary

In this lecture, we have learned

- ☑ what a **numerical expression** is
- ☑ what the **value** and **evaluation** of a numerical expression are
- ☑ how parentheses are used in a numerical expression
- ☑ in which order arithmetic operations are performed