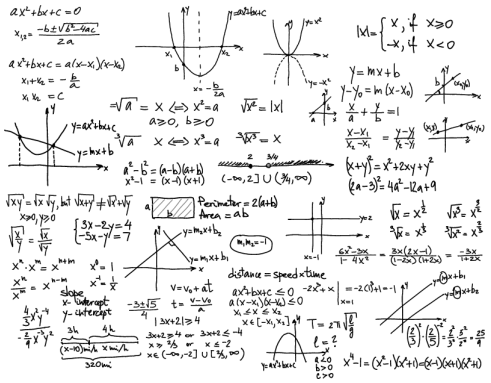
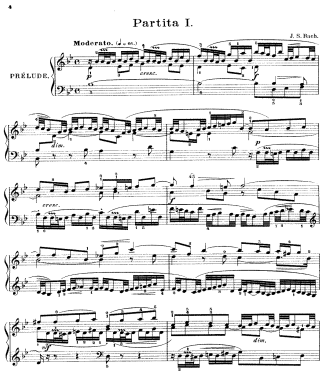
Lecture 1

Numbers and Operations

From chaos to harmony

For many of us, mathematics is a messy jumble of incomprehensible formulas:,
just the way musical scores are for someone who can't read music.





Both formulas and scores hide meaning and harmony.

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What this course is about

This a proficiency course in Algebra.

We will learn the basics of **algebraic literacy**:

- how to read, understand and manipulate algebraic expressions,
- how to solve simple equations and inequalities,
- how to visualize formulas by drawing graphs.

Enjoy the course!

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What Algebra studies

• **Numbers:** $1,\ \frac{5}{7},\ -27.4,\ 0,\ \sqrt{2},\ \pi,\ \ldots$

• **Operations** with numbers:
addition $1+2=3$,
subtraction $3-1=2$,
multiplication $3\cdot 2=6$,
division $6\div 3=2$,
exponentiation $2^3=8$,
taking the radical $\sqrt{49}=7$,
and their combinations $-5+2^3\cdot(3-\sqrt{4})=3$.

Often numbers are denoted by symbols (letters):

$$a,\ b,\ c,\ \ldots,\ x,\ y,\ z,\ A,\ B,\ C,\ \ldots,\ X,\ Y,\ Z,\ \alpha,\ \beta,\ \gamma,\ \ldots$$

Symbols are connected by operations into **formulas**:

$$1+2x,\quad x-3y,\quad x^2-x+1,\quad x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a},\ \ldots$$

Numbers

Positive integers: $1,\ 2,\ 3,\ 4,\ 5,\ldots$

Integers: $\ldots -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots$

Rational numbers are quotients $\frac{p}{q}$, where p and q are integers and $q\neq 0$.

For example, $\frac{1}{2},\ \frac{2}{1},\ \frac{6}{3},\ \frac{-4}{7}$ are rational numbers.

Any integer is a rational number. For example, $3=\frac{3}{1}$.

Irrational numbers are numbers which cannot be represented as a quotient of two integers.

For example, $\sqrt{2},\ \sqrt[3]{5},\ \sqrt{2}+\sqrt{3},\ \pi$.

Decimal presentations

Rational numbers can be represented as **decimals**:

$$\frac{9}{2} = 4.5, \quad \frac{1}{3} = 0.333\ldots = 0.\overline{3}$$

Any rational number is presented as either a **finite** decimal, like $\frac{9}{2} = 4.5$ or $\frac{7}{8} = 0.875$,

or a **repeating** decimal, like $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$ or $\frac{168}{11} = 15.272727\ldots = 15.\overline{27}$.

Irrational numbers also have decimal representations. They are infinite and not repeating.

For example, $\sqrt{2} = 1.41421356\ldots$ and $\pi = 3.14159265\ldots$.

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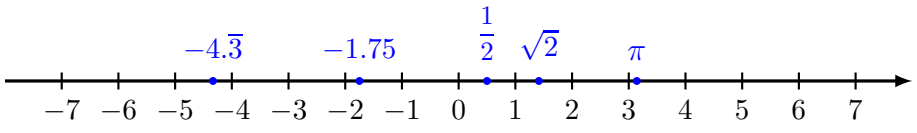
Real numbers and the real line

Both rational and irrational numbers are called **real** numbers.

Real numbers live on the **real line**:



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Summary

In this lecture, we have learned

- ☑ what this course is about
- ☑ what Algebra studies (**numbers, operations, formulas**)
- ☑ what kinds of numbers we are going to deal with (**integers, rational and irrational numbers**)
- ☑ what is a decimal representation of a number
- ☑ what real numbers are
- ☑ what the real line is

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