Stony Brook University
The Graduate School

Doctoral Defense Announcement

Abstract

From Herman Rings to Herman Curves

By

Willie Rush Lim

Given a non-trivial holomorphic map on a Riemann surface, an invariant connected set on which the map is conjugate to rigid rotation is either a disk (Siegel disk), an annulus (Herman ring), or a single Jordan curve (Herman curve). The last one is the least understood.

In this dissertation, we study a family of rational maps admitting a Herman ring with bounded type rotation number. For such a family, we prove *a priori bounds* that are independent of their conformal moduli via careful analysis of near-degenerate surfaces in the spirit of Kahn, Lyubich, and D. Dudko. As a major application, we study the limits of degenerating Herman rings and obtain the first examples of Herman curves which are not equivalent to round circles. The rigidity properties of such Herman curves are also explored.

This dissertation also initiates the study of renormalization theory of critical quasicircle maps, i.e. analytic self homeomorphisms of a quasicircle with a single critical point. We prove a rigidity theorem, which implies dynamical universality and exponential convergence of renormalizations towards a horseshoe attractor. Moreover, we prove the hyperbolicity of renormalization periodic points of critical quasicircle maps by developing an operator called *Corona Renormalization*, a doubly connected version of *Pacman Renormalization* for Siegel disks.

Date: April 29, 2024
Time: 11.15 am
Place: Math Tower, P-131

Program: Mathematics
Dissertation Advisor: Dzmitry Dudko