MAT 645
Introduction to Floer theory

Joa Weber, Spring 2000
TU TH 9.50-11.10am, MATH 5-127

Instructor: Joa Weber, 3-109 Math Tower, phone 632 82 55

Office Hours: MO 11.30-12.30 TH 12-1

Prerequisites: basic knowledge of differential geometry, algebraic topology and analysis

Topics covered: Arnold conjecture for symplectic fixed points, Morse homology (the toy model), symplectic action functional, periodic orbits of hamiltonian vector fields, Conley-Zehnder index, moduli spaces of pseudo-holomorphic cylinders, transversality, compactness, Floer homology for (monotone) closed symplectic manifolds, Floer homology for cotangent bundles and the heat flow.

Abstract: A weak version of Arnold's conjecture for non-degenerate fixed points of exact symplectomorphisms claims a lower bound of such by the sum of the Betti numbers of a closed symplectic manifold. Our goal is to prove the conjecture in the monotone case by constructing Floer homology groups for the symplectic manifold.

As a warm-up to understand Floer homology we study its finite dimensional toy model, namely the Morse-Witten complex of a closed Riemannian manifold. Here mainly methods from differential topology and dynamical systems will be applied.

The main part of the course will be devoted to analyze the space of solutions to Floer's nonlinear elliptic PDE with prescribed non-degenerate boundary conditions. We are going to deal with certain aspects of the analysis in depth, such as Fredholm theory and compactness, whereas others will be only sketched, such as transversality and orientation of moduli spaces.

If time permits we explain our approach to Floer cohomology for cotangent bundles, its relation to the heat flow on the underlying closed Riemannian manifold M and its isomorphism to singular homology of the free loop space of M.

Text: The main text we follow are the Park City Lecture notes on Floer homology by Dietmar Salamon. More references will be provided as the lecture course advances.

Grades: An s-grade depends on regular attendance of lectures.

Special Needs: If you have a physical, psychiatric, medical, or learning disability that could adversely affect your ability to carry out assigned course work, I urge you to contact me or the staff in the Disabled Student Services office (DSS), Room 133 Humanities, 632-6748/TDD. DSS will review your situation and determine, with you, what accomodations are necessary and appropriate. All information and documentation of disability is confidential.

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Main text we follow

- [Sa1] Salamon D.A., Lectures on Floer homology (1430kB), Lecture Notes for the IAS/PCMI Graduate Summer School on Symplectic Geometry and Topology, Preprint, December 1997. In Synplectic Geometry and Topology, edited by Y. Eliashberg and L. Traynor, IAS/Park City Mathematics series, Vol 7, 1999, pp. 143--230.

Morse theory

- **Background in differential topology and dynamical systems**
  - [Hi] Hirsch M.W., Differential topology, Corrected reprint of the 1976 original. Graduate Texts in Mathematics, 33, Springer-Verlag, New York, 1994.
  - [PdM] Palis J.Jr., de Melo W., Geometric theory of dynamical systems. An introduction. Translated from the Portuguese by A. K. Manning. Springer-Verlag, New York-Berlin, 1982.
- **Foundations**
  - [Mi] Milnor J., Morse theory. Based on lecture notes by M. Spivak and R. Wells. Annals of Mathematics Studies, No. 51 Princeton University Press, Princeton, N.J. 1963 vi+153 pp.
- **Morse homology**
  - [Sch]Schwarz M., Morse homology. Progress in Mathematics, 111. Birkhäuser Verlag, Basel, 1993.
  - [Wi] Witten E., Supersymmetry and Morse theory. J. Differential Geom. 17 (1982), no. 4, 661--692 (1983).

Symplectic geometry

- [McDS1] McDuff D., Salamon D.A., Introduction to Symplectic Topology, Oxford University Press, April 1995.

  - foundational material of symplectic geometry in part I

- **Some analytical background**
  - [McDS2] McDuff D., Salamon D.A., J-holomorphic Curves and Quantum Cohomology, AMS, University Lecture Series, Vol. 6, Providence, Rhode Island, 1994.

Some history of the weak non-degenerate Arnold conjecture

- **monotone case**
  - [Fl1] Floer, A., Symplectic fixed points and holomorphic spheres, Comm. Math. Phys. 120 (1989), no. 4, 575--611.
- **weakly monotone case**
  - [HS1] Hofer H., Salamon D.A., Floer homology and Novikov rings, The Floer Memorial Volume, edited by H. Hofer, C. Taubes, A. Weinstein, and E. Zehnder, Birkhäuser 1995, pp 483-524. paper
  - [On1] Ono K., On the Arnol'd conjecture for weakly monotone symplectic manifolds, Invent. Math. 119 (1995), no. 3, 519--537.
- **general case**
  - [FO] Fukaya K., Ono K., Arnold conjecture and Gromov-Witten invariant, Topology 38 (1999), no. 5, 933--1048.
  - [LT] Liu G., Tian G., Floer homology and Arnold conjecture, J. Differential Geom, 49 (1998), no. 1, 1--74. paper
  - [Ru] Ruan Y., Virtual neighborhoods and pseudo-holomorphic curves, Proceedings of 6th Gökova Geometry-Topology Conference. Turkish J. Math. 23 (1999), no. 1, 161--231. preprint

Remark: Some links refer to .dvi, .pdf or .ps files. Configuration of netscape to start automatically an appropriate viewer such as xdvi, ghostview or the Adobe acrobat pdf-reader is recommended. Let me know in case you have trouble with that.