MAT 638 - Fall 2013 - Topics in Real Analysis

Brownian Motion and Harmonic Measure

[**Course Syllabus**]

Prerequisites: measure theory (at level of MAT 544), elementary theory of harmonic functions in the complex plane (at the level of MAT 542).

Fall 2013 Office Hours	
M	By Appointment
T	12:30 - 3:30
W	By Appointment
H	By Appoitnemtn
F	By Appointment

Office hours are held in Math Tower 4-117.

Credit: Joshua M. Tokle
(Click to Restart)

Description

The name "harmonic measure" was coined by R. Nevanlinna in the 1930s and refers to a fascinating object that arises from studying the classical Dirichlet problem. It has the following nice interpretation: the harmonic measure of a subset E of the boundary of a domain is the probability that a randomly drawn curve inside the domain (Brownian motion) first touches the boundary in E.

There are deep connections between the geometry of a domain and the properties of its harmonic measure. The goal of this course is to illustrate some of these connections. Along the way we will introduce participants to techniques from several areas of analysis, including geometric function theory, geometric measure theory, harmonic analysis, probability theory and ergodic theory.

Syllabus

PART I. Brownian Motion and Harmonic Measure from the Probabilisitic Viewpoint

The course will begin with an introduction to Brownian motion. After studying its basic properties, we will use Brownian motion to define harmonic measure and make a rudimentary investigation into sets of harmonic measure zero.

References for Part I

- Peter Mörters and Yuval Peres, Brownian Motion, Cambridge Unviersity Press, 2010.

PART II. Harmonic Measure in the Plane versus Harmonic Measure in Space

Including the Theorems of F. and M. Riesz, Dahlberg, Makarov, Bourgain, and Wolff

In the second part of the course, we will look at some finer properties of harmonic measure, from classic results to current research. The topics that we select will highlight differences in the theory of harmonic measure in the plane (where the Riemann mapping theorem is an essential tool) and the theory of harmonic measure in space (where we lack non-trivial conformal maps).

References for Part II

- John Garnett and Donald Marshall, Harmonic Measure, Cambridge Unviersity Press, 2005.
- The Literature

Last updated: August 26, 2013

MAT 638, Topics in Real Analysis: Brownian motion and harmonic measure — Syllabus

Physics P122 — Tuesday, Thursday 10:00 – 11:20

Instructor: Matthew Badger (badger@math.sunysb.edu)
Office: Math Tower 4-117
Office Hours: Tuesdays 12:30 - 3:30 or by Appointment

Course Description

The name "harmonic measure", coined by R. Nevanlinna in the 1930s, refers to a fascinating object that arises from studying the classical Dirichlet problem. It has a nice interpretation: the harmonic measure of a subset E of the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{R}^n$ is the probability that a randomly drawn curve inside the domain (Brownian motion) first touches the boundary in the set E. There are deep connections between the geometry of a domain and the properties of its harmonic measure. The goal of this course is to illustrate some of these connections. Along the way we will introduce participants to techniques from several areas of analysis, including geometric measure theory, harmonic analysis, probability theory and ergodic theory.

Part I. Brownian Motion and Harmonic Measure from the Probabilistic Viewpoint. The course will begin with an introduction to Brownian motion. After studying some of its basic properties, we will use Brownian motion to define harmonic measure and make a rudimentary investigation into sets of harmonic measure zero.

Part II. Harmonic Measure in the Plane versus Harmonic Measure in Space. In the second part of the course, we will look at some finer properties of harmonic measure, from classic results to current research. The topics that we select will highlight differences in the theory of harmonic measure in the plane (where the Riemann mapping theorem is an essential tool) and the theory of harmonic measure in space (where we lack non-trivial conformal maps). As time permits, we will examine the theorems on harmonic measure by F. and M. Riesz, Dahlberg, Makarov, Bourgain and Wolff.

References

- C. J. Bishop, *Some questions concerning harmonic measure*, Partial differential equations with minimal smoothness and applications (B. Dahlberg, E. Fabes, R. Fefferman, D. Jerison, C. Kenig, and J. Pipher, eds.), The IMA Volumes in Mathematics and Its Applications, vol. 42, Springer-Verlag, New York, 1992, pp. 89–97.
- J. Bourgain, *On the Hausdorff dimension of harmonic measure in higher dimensions*, Invent. Math. **87** (1987), 477–483.
- B. Dahlberg, *Estimates of harmonic measure*, Arch. Rational Mech. Anal. **65** (1977), no. 3, 275–288.
- J.B. Garnett and D.E. Marshall, *Harmonic Measure*, Cambridge University Press, 2005.
- P. Mörters and Y. Peres, *Brownian Motion*, Cambridge University Press, 2010.
- T. Wolff, *Counterexamples with harmonic gradients in* $\mathbb{R}^3$, in: Essays on Fourier analysis in honor of Elias M. Stein, Princeton Mathematical Series, vol. 42, Princeton University Press, Princeton, NY, 1995, pp. 321–384.

Coursework

There is no graded homework. There are no midterm exams. There is no final exam.

Grades

Grades will be determined by attendance and participation in class.

Disability Support Services

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services (631) 632-6748 or

`studentaffairs.stonybrook.edu/dss/`

They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website:

`www.sunysb.edu/facilities/ehs/fire/disabilities`

Academic Integrity

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at

`www.stonybrook.edu/uaa/academicjudiciary/`

Critical Incident Management

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

Syllabus Revision

The standards and requirements set forth in this syllabus may be modified at any time by the course instructor. Notice of such changes will be by announcement in class and changes to this syllabus will be posted on the course website.

This syllabus was last revised on August 26, 2013.