



MAT 545

Complex Geometry

[HOME](#)[ABOUT THE COURSE](#)[TEXTBOOK](#)[TOPICS](#)[HOMEWORK](#)

Instructor:

Sorin Popescu (office: Math 3-109, tel. 632-8255, e-mail [sorin at math.sunysb.edu](mailto:sorin@math.sunysb.edu))

Schedule:

TuTh 9:50am-11:10am Physics P123

About the course

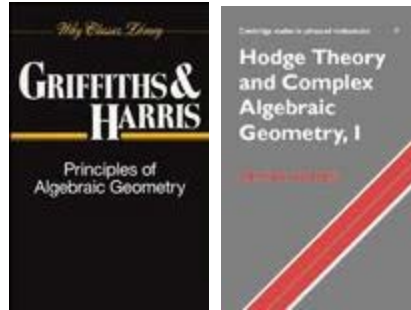
The course aims to give brief introduction to basic notions and techniques in complex differential geometry and complex algebraic geometry. The language will be mainly geometric and/or analytic and we will assume material covered in **MAT 530/531**, **MAT 534/535** as well as **MAT 539** (Algebraic Topology), and **MAT 542/543** (Complex Analysis). It should however be possible to fill in some of the gaps during the semester.

Textbook(s)

No particular text will be used exclusively for the course, but the plan is to cover parts of chapters 0-1-2 in Ph. Griffiths' and J. Harris' textbook *Principles of Algebraic Geometry* and part of Claire Voisin's textbook *Hodge Theory and Complex Algebraic Geometry I*. (These two textbooks will be in the library on reserve.)

Other textbooks that you may find useful are the following:

- *Complex analytic and algebraic geometry*, Jean-Pierre Demailly (a PDF file of the current version).
Download [here](#) a DjVu (searchable) version.
- *Complex Differential Geometry*, Sh. Kobayashi, H.H. Wu
- *Basic Algebraic Geometry 1* & *Basic*



Algebraic Geometry 2, I.R.

Shafarevich, Springer 1995

- *Topologie Algébrique et Théorie des Faisceaux*, R. Godement.
- *Coherent Analytic Sheaves*, H. Grauert and R. Remmert.
- *Complex Algebraic Curves*, F. Kirwan

Topics

The following is a tentative list of what we will try to cover in class:

- Notions of several complex variables/analytic geometry: Cauchy's Integral Formula, Hartog's theorem, Weierstrass preparation theorems, analytic Nullstellensatz, Riemann's extension theorem, etc.
- Basic Sheaf theory: presheaves, sheaves, cohomology of sheaves, long exact sequence of cohomology, basic computations and applications (among them the de Rham and Dolbeault theorems).
- Complex differential geometry: complex vector bundles, hermitian metrics, connections, hermitian vector bundles, the metric connection, sub-bundles and quotient bundles, tensor bundles, curvature, Chern classes (differential geometric approach), positivity.
- Kaehler differential geometry: The Kaehler condition, projective manifolds, basic topological consequences of the Kaehler condition, the Kaehler identities.
- Hodge theory: The Hodge Theorem: statement (no proof), and consequences (harmonic forms=Dolbeault cohomology, finite dimensionality, Poincaré duality, Serre duality), Hodge decomposition for compact Kaehler manifolds. The Kodaira (Akizuki-Nakano) vanishing theorem, the Hard Lefschetz Theorem.

- Divisors and line bundles: Divisors and line bundles, linear systems and maps to projective space Bertini's Theorem, exponential sequence and Chern classes, adjunction formula, Cartan-Serre's Theorems A and B, Lefschetz theorem on $(1,1)$ classes, blowing up, Kodaira embedding theorem, projective bundles, Chern classes revisited.

If time allows any of the following topics may perhaps be also discussed:

- Topology of projective varieties: Weak Lefschetz Theorem, Lefschetz pencils, monodromy, Picard-Lefschetz, relation with the Hard Lefschetz Theorem.
- Curves: Riemann-Roch, compact Riemann surfaces are projective, Riemann-Hurwitz, genus formula, Abel's Theorem, the Jacobian, Jacobi inversion, tori, Albanese variety.

Homework

I will assign problems in each lecture, ranging in difficulty from routine to more challenging. They are due the following week.

DSS advisory. If you have a physical, psychiatric, medical, or learning disability that may affect your ability to carry out the assigned course work, please contact the office of Disabled Student Services (DSS), ECC (Educational Communications Center) Building, room 128, 632-6748/TDD. DSS will review your concerns and determine what accommodations may be necessary and appropriate. All information and documentation of disability is confidential.

Sorin Popescu
2005-1-15



Sorin Popescu

Department of Mathematics
Stony Brook University
Stony Brook, NY 11794-3651

email: sorin@math.sunysb.edu
Office: Math 3-109
Phone: (631)-632-8255
Fax: (631)-632-7631

Research Interests: Algebraic Geometry, Commutative Algebra, Combinatorics and Computational methods

Teaching:

Spring 2006









[MAT 311 Number Theory](#)

[MAT 614 Topics in Algebraic Geometry](#)












Previous years

[Teaching Archive](#)

Algebra, Geometry and Physics seminar: [Spring 2006](#)



Publications & E-Prints: Unless otherwise indicated, the files below are DVI files () , PostScript files () , PDF files () , or tar gzipped DVI and PostScript files (). Files marked as () or () are hyperlinked PDF or Macromedia Flash files formatted for screen viewing. Other formats (source, PS using Type I fonts) can be obtained via the UC Davis Front to the [Mathematics ArXiv](#). Click on () or () for related [Macaulay2](#), or [Macaulay](#) code.

Syzygies:










- [Gale Duality and Free Resolutions of Ideals of Points](#) [], [], [] [] [] , *Invent math* **136** (1999) 2, 419-449
David Eisenbud and Sorin Popescu
- [The Projective Geometry of the Gale Transform](#) [], [], [] [] , *J. Algebra* **230** (2000), no. 1, 127-173
David Eisenbud and Sorin Popescu
(in the D. Buchsbaum anniversary volume of *J. Algebra*)
- [Syzygy Ideals for Determinantal Ideals and the Syzygetic Castelnuovo Lemma](#) [] [] , [[MathSci](#)] ,

Springer 1999

David Eisenbud and Sorin Popescu

- [Extremal Betti Numbers and Applications to Monomial Ideals](#) [] [] [] [], *J. Algebra* **221** (1999), no. 2, 497-512
Dave Bayer, Hara Charalambous and Sorin Popescu
- [Lagrangian Subbundles and Codimension 3 Subcanonical Subschemes](#) [], [] [] [], *Duke Math. J.* **107** (2001), no. 3, 427-467
David Eisenbud, Sorin Popescu and Charles Walter
- [Enriques Surfaces and other Nonpfaffian Codimension 3 Subcanonical Subschemes](#) [] [] [] [], *Comm. Algebra* **28** (2000), 5629-5653
David Eisenbud, Sorin Popescu and Charles Walter
(in the Hartshorne anniversary volume of *Comm. Algebra*)
- [Syzygies of Unimodular Lawrence Ideals](#) [] [] [] [], *J. Reine Angew. Math* **534** (2001), 169-186
Dave Bayer, Sorin Popescu and Bernd Sturmfels
- [Hyperplane Arrangement Cohomology and Monomials in the Exterior Algebra](#) [] [] [] [] [], *Trans. AMS.* **355** (2003), 4365-4383
David Eisenbud, Sorin Popescu and Sergey Yuzvinsky
- [Exterior algebra methods for the Minimal Resolution Conjecture](#) [] [] [] [], *Duke Math. J.* **112** (2002), no. 2, 379-395
David Eisenbud, Frank-Olaf Schreyer, Sorin Popescu and Charles Walter
- [Symmetric resolutions of coherent sheaves](#) [] [] []
David Eisenbud, Sorin Popescu and Charles Walter
- [A note on the Intersection of Veronese Surfaces](#) [] [] [] [] [],
David Eisenbud, Klaus Hulek and Sorin Popescu
- [Restricting linear syzygies: algebra and geometry](#) [] [] [] [] [], *Compositio Math.* **141** (2005), no.6, 1460-1478
David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu
- [Small schemes and varieties of minimal degree](#) [] [] [] [] [], *Amer. J of Math* (2005), to appear
David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu





Abelian varieties, modular varieties and equations:

- [Equations of \(1,d\)-polarized abelian surfaces](#) [] [] [], *Math. Ann.* **310** (1998), no. 2, 333-377
Mark Gross and Sorin Popescu
- [The moduli space of \(1,11\)-polarized abelian surfaces is unirational](#) [] [] [], *Compositio Math.* **126** (2001), no. 1, 1-24
Mark Gross and Sorin Popescu
- [Calabi-Yau threefolds and moduli of abelian surfaces I](#) [] [] [], *Compositio Math.* **127**, no. 2, (2001), 169-228
Mark Gross and Sorin Popescu




























[Calabi-Yau threefolds and moduli of abelian surfaces II](#) [] [] []




Mark Gross and Sorin Popescu

- [Elliptic functions and equations of modular curves](#) [] [] [] [], *Math. Ann.* **321** (2001), no. 3, 553-568
Lev A. Borisov, Paul Gunnells, and Sorin Popescu

Surfaces in P^4 and threefolds in P^5 :

- [The Geometry of Bielliptic Surfaces in \$P^4\$](#) [], [] [], *Internat. J. Math.* **4** (1993), no. 6, 873-902
A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- [On Surfaces in \$P^4\$ and Threefolds in \$P^5\$](#) [] [] [], [**MathSci**], LMSLN **208**, 69--100
W. Decker and S. Popescu
- [Surfaces of degree 10 in \$P^4\$ via linear systems and linkage](#) [] [] [] [] [], *J. Algebraic Geom.* **5** (1996), no. 1, 13-76
S. Popescu and K. Ranestad
- [Syzygies of Abelian and Bielliptic Surfaces in \$P^4\$](#) [] [] [], *Internat. J. Math.* **8** (1997), no. 7, 849-919
A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- [Examples of smooth non general type surfaces in \$P^4\$](#) [] [] [] [] [], *Proc. London Math. Soc.* (3) **76** (1998), no. 2, 257-275
S. Popescu
- [Surfaces of degree \$\geq 11\$ in the Projective Fourspace](#) [] [] [] + [Appendix](#) [] [] []
S. Popescu

PRAGMATIC 1997: A summer school in Catania, Sicily

- [Research Problems for the summer school](#) [], [] [], [**MathSci**], *Matematiche* (Catania) **53** (1998), 1-14
David Eisenbud and Sorin Popescu

Algorithmic Algebra and Geometry: Summer Graduate Program (1998) at MSRI:

- Poster [] [], [lecture slides and streaming video](#) , CD ROM,
Dave Bayer and Sorin Popescu






Linear algebra notes

- [On circulant matrices](#) [], [] [] [] [],
Daryl Geller, Irwin Kra, Sorin Popescu and Santiago Simanca

Upcoming conferences:

- DARPA FunBio Mathematics-Biology Kick-off meeting, Princeton, September 21-23, 2005
- [MAGIC 05: Midwest Algebra, Geometry and their Interactions Conference](#), University of Notre Dame, Notre Dame, October 7-11, 2005
- [AMS Special Session on Resolutions](#), Eugene, OR, November 12-13, 2005
- [Clay Workshop on Algebraic Statistics and Computational Biology](#), Clay Mathematics Institute, November 12-14, 2005
- [CIMPA School on Commutative Algebra](#), December 26, 2005 - January 6, 2006, Hanoi, Vietnam
- [AMS Special Session on Syzygies in Commutative Algebra and Geometry](#), San Antonio, TX, January 12-15, 2006
- [KAIST Workshop on Projective Algebraic Geometry](#), January 23-25, 2006, Korean Advanced Institute of Science and Technology, Daejeon
- [AMS Special Session on the Geometry of Groebner bases](#), San Francisco, CA, April 29-30, 2006
- [Castnuovo-Mumford regularity and related topics](#), Workshop at CIRM, Luminy, France, May 9-13, 2006
- [Commutative Algebra and its Interaction with Algebraic Geometry](#), Workshop at CIRM, Luminy, France, May 22-26, 2006
- [Syzygies and Hilbert Functions](#), Banff International Research Meeting, Canada, October 14-19, 2006

Past conferences:

- A [conference](#) on algebraic geometry to celebrate Robin Hartshorne's 60th birthday, Berkeley, August 28-30, 1998
- [Western Algebraic Geometry Seminar](#), MSRI, Berkeley, December 5-6, 1998
- [Conference on Groebner Bases, Guanajato](#), Mexico, February 8-12, 1999
- [The Pacific Northwest Geometry Seminar](#) 
- [Computational Commutative Algebra and Combinatorics](#), Osaka, July 21-30, 1999. 
- [Kommutative Algebra und Algebraische Geometrie](#), Oberwolfach, August 8-14, 1999. 
- [AMS Western Section Meeting](#) Salt Lake City, UT, September 25-26, 1999.
- [Algebra and Geometry of Points in Projective Space](#), Napoli, February 9-12, 2000.
- [AMS Spring Eastern Sectional Meeting](#) Lowell, MA, April 1-2, 2000.
- [Algèbre commutative et ses interactions avec la géométrie algébrique](#), Centre International de Rencontres Mathématiques, June 5-9, 2000.
- [Topics in Classical Algebraic Geometry](#), Oberwolfach, June 18-24, 2000 
- [AMS Fall Central Section Meeting](#) Toronto, Ontario Canada, September 22-24, 2000
- [AMS Fall Eastern Section Meeting](#), New York, Columbia U. in New York, November 4-5, 2000
- [Exterior algebra methods and other new directions in Algebraic Geometry, Commutative Algebra and Combinatorics](#), 8-15 September 2001, Ettore Majorana Centre, Erice, Sicily, Italy. [Photos](#) from the conference.
- [Classical Algebraic Geometry](#), Oberwolfach, May 26 - June 1, 2002 
- [Current trends in Commutative Algebra](#), Levico, Trento, June 17-21, 2002
- [Birational and Projective Geometry of Algebraic Varieties](#), Ferrara, September 2-8, 2002
- [Commutative Algebra, Singularities and Computer Algebra](#), Sinaia, September 17-22, 2002. [Photos](#) from the conference.
- [James H. Simons Conference on Quantum and Reversible Computation](#), Stony Brook, May 25-31, 2003

- [Conference on Commutative Algebra](#), Lisbon, June 23-27 2003. [Photos](#) from the conference. Also [photos](#) from Belém.
- [Commutative Algebra and Interactions with Algebraic Geometry and Combinatorics](#), ICTP, Trieste, June 6-11
- [III Iberoamerican Congress on Geometry](#), Salamanca, June 7-12
- [Projective Varieties: A Conference in honour of the 150th anniversary of the birth of G. Veronese](#), Siena, June 8-12 , 2004. [Photos](#) from the conference.
- [Algebraic Geometry: conference in honour of Joseph Le Potier & Christian Peskine](#), Paris, June 15-18, 2004
- [Classical Algebraic Geometry](#), Oberwolfach, June 27-July 3, 2004
- [Combinatorial Commutative Algebra](#), Oberwolfach, July 4-10th, 2004

Last updated on 10 Dec 2003

