MAT 544: COMMUTATIVE AND HOMOLOGICAL ALGEBRA (FALL 2020)

CHRISTIAN SCHNELL

Basic Information

We meet Tuesday and Thursday, 9:45am to 11:05am, in room 4–130 of the Mathematics Building. Until the Thanksgiving Break, the course will meet in person twice a week; the remaining two lectures in December will be held online. Please note that everyone is required to wear a mask during class (unless approved for an exception for medical reasons by the unversity). If you are unable to attend class in person, please talk to me as soon as possible about possible accomodations.

Office Hours

My office is 3-117; office hours will be held, via `Zoom`, on Fridays from 2:00pm to 4:00pm (or by appointment). The meeting ID is `296 508 2928`, and the passcode is `liegroup`; here is a direct link:

`https://stonybrook.zoom.us/j/2965082928?pwd=TmJHa2gxUDlGSURBOUdJUVowSEwzdz09`

You can contact me by email at `christian.schnell@stonybrook.edu`.

Course Summary

This course is an introduction to basic techniques of commutative and homological algebra, useful in algebra, algebraic geometry, number theory, and related fields. Specifically, we are going to cover the following topics:

- Review of rings and modules, tensor products and localization.
- Spectrum of prime ideals, connections with algebraic geometry.
- Noetherian and Artinian rings and modules.
- Completion.
- Dimension theory.
- Local rings, Nakayama's lemma.
- Discrete valuation rings and Dedekind domains.
- Integral dependence.
- Chain complexes, projective and injective resolutions.
- Examples of derived functors (such as Ext and Tor)
- Basic category theory (such as adjoint functors, natural transformations, limits and colimits)
- Abelian categories.

Time permitting, we may talk about some other important topics such as flatness, regular local rings, Koszul complexes and regular sequences.

Recommended Books

The classic *Introduction to Commutative Algebra*, by Michael Atiyah and Ian Macdonald, is a concise treatment of basic commutative algebra, with lots of exercises.

A much longer textbook is *Commutative Algebra with a View Towards Algebraic Geometry* by David Eisenbud. It provides lots of motivation and discussion for definitions and theorems, and it shows you how to think about commutative algebra from the point of view of algebraic geometry.

The book *Commutative Algebra* by Hideyuki Matsumura is another standard reference. It contains a wealth of advanced results, but much less motivation.

For homological algebra, the best general textbook is *An Introduction to Homological Algebra* by Charles Weibel. It covers all the things we need, plus a lot more.

Grades

There will be weekly homework assignments, consisting of problems from the above textbooks. My plan is to use the online portion of the course for a group project, with students researching and presenting a more advanced topic. More details on this will be provided during the semester. Your grade will be determined by homework, class participation, and the effort made on the group project.

University Policies

Student Accessibility Support Services. If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Student Accessibility Support Center, ECC (Educational Communications Center) Building, Room 128, `(631)632-6748`. They will determine with you what accommodations, if any, are necessary and appropriate. All information and documentation is confidential. Please see

`https://www.stonybrook.edu/commcms/studentaffairs/sasc/facstaff/syllabus.php`

for more information.

Academic Integrity Statement. Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty is required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center

(School of Health Technology & Management, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty please refer to the academic judiciary website at

`http://www.stonybrook.edu/commcms/academic_integrity/index.html`.

Critical Incident Management Statement. Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of University Community Standards any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Faculty in the HSC Schools and the School of Medicine are required to follow their school-specific procedures. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.