MAT 539

Algebraic Topology

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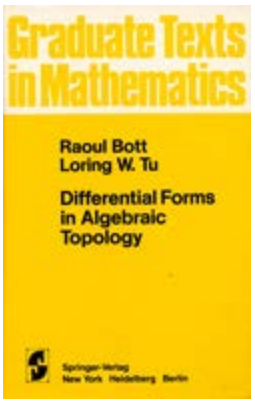
Prerequisites

A basic introduction to geometry/topology, such as **MAT 530** and **MAT 531**. Thus prior exposure to basic point set topology, homotopy, fundamental group, covering spaces is assumed, as well as some acquaintance with differentiable manifolds and maps, differential forms, the Poincaré Lemma, integration and volume on manifolds, Stokes' Theorem. We will briefly review some of this material in the first week of classes.

Textbook

Differential forms in algebraic topology, by Raoul Bott and Loring W. Tu, GTM **82**, Springer Verlag 1982.



The guiding principle of the book is to use differential forms and in fact the de Rham theory of differential forms as a prototype of all cohomology thus enabling an easier access to the machineries of algebraic topology in the realm of smooth manifolds. The material is structured around four core sections: de Rham theory, the Cech-de Rham complex, spectral sequences, and characteristic classes, and includes also some applications to homotopy theory.

Other recommended texts:

- *Algebraic Topology: A first Course*, W. Fulton, GTM **153**, Springer Verlag 1995
- *Topology from the Differentiable Viewpoint*, J. Milnor, U. of Virginia Press 1965
- *Algebraic Topology*, A. Hatcher (on-line), Cambridge University Press, to appear
- *Characteristic classes*, J. Milnor and J. Stasheff, Princeton University Press 1974

Course description

The book contains more material than can be resonably covered in a one-semester course. We will hopefully cover the following sections:

- **de Rham theory:** the de Rham complex, orientation and integration, Poincaré lemmas, the Mayer-Vietoris argument, Poincaré duality on an orientable manifold, Thom class and the Thom isomorphism (orientable vector bundle case)
- **the Cech-de Rham complex:** the generalized Mayer-Vietoris argument, sheaves and Cech cohomology, the de Rham theorem, sphere bundles, Euler class, the Hopf index theorem, the Thom isomorphism in general, monodromy
- **characteristic classes:** Chern classes of complex vector bundles, the splitting principle, Whitney's

product formula explicit computations of Chern classes, Pontrjagin classes of real vector bundles, the universal bundle, infinite grassmannians
- **spectral sequences**: spectral sequence of a double complex, products, applications and explicit computations (all these only if time permits)

Homework & Exams

I will assign problems in each lecture, ranging in difficulty from routine to more challenging. There will be also a take-home midterm and a final exam. Course grades will be based on these problems (and any other participation); solving at least half of them will be considered a perfect score.

Software

Here are some pointers to software that may be used to visualize topological objects:

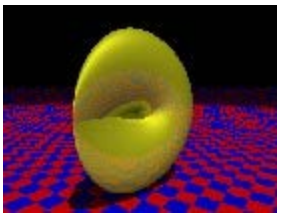
- KnotPlot. Download binaries from the following site.
- Java View: a 3d geometry viewer written in Java. Among the demos you may find a Klein Bottle
- LiveGraphics3D: a Java applet to display and rotate three-dimensional graphics. For instance two visualizations of the Borromean Rings
- Geomview another interactive 3D viewing program.

Links & 3D-models

History of topology:

- "Topology enters mathematics": a brief overview of the early developments (MacTutor History of Mathematics archive).
- A Brief History of Topology by E.C. Zeeman
- Stable algebraic topology 1945-1966, by J.P. May

Topological zoo:



- "The Topological Zoo" at the Geometry Center: a visual dictionary of surfaces and other mathematical objects.
- Images of "classical" topological "objects" from the Geometry Center.
- "A Knot Zoo". Here is another one. Or Hyperbolic knots. All these sites are part of an exciting collection of knots and links available at "The KnotPlot Site". Very instructive are also the VRML knot models.
- Raytraced images: Sphere, Torus, projective plane: a Crosscap, a Steiner surface, a Boy surface, and a genus 3 orientable surface.
- VRML models: a Möbius band, a Klein bottle and a Trefoil Knot. Download here a vrml viewer for Linux.
- David Eppstein's "Geometry Junkyard": a collection of pointers, clippings, research blurbs, and other stuffs related to discrete, computational geometry, and topology.
- Paul Bourke's collection of raytraced surfaces. Here is for instance the animation of a transition from a Steiner surface into a Boy surface.
- A picture of the Hopf fibration created by Ken Shoemake. Click here for a better quality TIFF version of the picture. The picture visualizes well the remarkable geometric fact that any two fibres (=circles) of the Hopf fibration are *linked*. Here is another page and an mpeg animation of the

Hopf fibration (created with Knotplot).

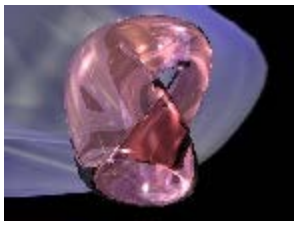
Art & Topology:

- "Symbolic Sculpture and Mathematics"
- "Mathematics & Knots Exhibition"
- Benno Artmann's Topological Models
- The Scherk-Collins Sculpture Generator: a program to generate Scherk-Collins towers and toroids (by Carlo H. Séquin)
- Helaman Ferguson's sculptures. For instance here is "Klein's modular quartic" which is on the patio of MSRI Berkeley. Or Alexander's horned sphere...
- More art links on Carlo H. Séquin's web site.
- Knots from the Alhambra de Grenada

Archives:

- The "Topology Atlas"
- The Hopf Topology Archive
- Rob Kirby's Problems in Low-Dimensional Topology (380 pages)
- An Algebraic Topology Discussion List

Fun:



- Torus and Klein Bottle Games: a collection of Java applets/games played on the surface of a torus or a Klein bottle (chess, tic-tac-toe, crossword puzzles, and more).
- Glass Klein Bottles!

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