Department of Mathematics
Mat 322: Analysis in Several Dimensions
Spring 2006

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Course Grader: Young Nam (ynam@math.sunysb.edu). Office: Math Tower 2-117. Office hours: Tu. 2:00-3:00PM and Th. 4:00-5:00PM or by appointment.

Classroom: Harriman Hall 115. Time: TTh 2:20-3:40.

Text: *Vector Calculus, Linear Algebra, and Differential Forms*, 2nd edition, by J. H. Hubbard & B. B. Hubbard, Prentice Hall.

Description: This is a rigorous course in Calculus of several variables. We will analyze techniques to study functions whose domains are subsets of R^n, essential tools in the pursuit of advanced level mathematics, and many sciences. We shall begin with the geometric description of R^n, and discuss matrices as linear transformations. We will develop criteria for differentiability of functions, solve basic linear equations, relate the dimensions of kernel and images of linear transformations, and discuss the inverse and implicit function theorems. Then, we will proceed with the definition and study of higher order derivatives of functions, quadratic forms, integration, and submanifolds of R^n, the latter subject emphasizing the case of curves and surfaces. The course will end with a description of forms and the exterior differentiation operator, the statement of Stokes' Theorem and some of its applications.

Homework: There will be homework assignments throughout the semester, approximately weekly. *No late homework will be accepted.*

Examinations: There will be a midterm and a final examination, as indicated below. The time of these exams is given now so that you can resolve any possible conflict before the course starts. There will be no makeup exams due to conflicts.

Test	Date	Time
Midterm	March 21	02:20-03:40

Final	May 16	2:00-4:30PM

FINAL EXAM REVIEW: Exam Review
This review sheet is good for the midterm as well. Go over everything up to and including Newton's method. Stop before the contraction lemma.

Grading: The homework assignments, the midterm, and the final examination will count for 20%, 30%, and 50% of your final grade, respectively.

SECTIONS TO BE COVERED (approximate)

Feb. 14: Sections 1.4, 1.5
Feb. 21: Sections 1.6, 1.7
Feb. 28: Section 1.8
March 7: Section 1.9
March 21: Sections 2.7, 2.8
March 28- April 18: Section 2.9
April 20: Section 3.3
April 24: Sections 3.4-3.6
May 1: Sections 3.7-3.8

HOMEWORK

Due: Jan 26: Read Chapter 0, except 0.6 and prepare a list of questions on its content that you might ask in class.
Due Feb. 7: Problems 1.1.4, 1.1.8, 1.2.2a,c,d, 1.2.4, 1.2.8, 1.2.10, 1.2.12, 1.2.13
Due Feb. 14: Problems 1.3..4, 1.3.9, 1.3.10, 1.3.12, 1.3.14, 1.3.19, 1.3.20, 1.3.21, 1.4.3, 1.4.4, 1.4.10
Due Feb. 21: Problems 1.4.12, 1.4.16, 1.4.23, 1.4.24, 1.5.1a,b,d,f, 1.5.3, 1.5.4, 1.5.6
Due March 2: Problems1.5.8, 1.5.10, 1.5.13, 1.5.20, 1.5.21a,b, 1.6.3, 1.6.5, 1.6.10, 1.6.11
Due March 14: Problems 1.7.2, 1.7.6, 1.7.10a, 1.7.12, 1.7.19, 1.7.22
Due March 30: Problems 1.8.2, 1.8.6, 1.8.10, 1.9.2, 2.7.1, 2.7.3, 2.7.6, 2.7.9, 2.7.13 a,b
Due April 7: Problems 2.8.2, 2.8.3, 2.8.4, 2.8.5, 2.8.8, the last problem is difficult
Due April 18: Problems 2.9.2, 2.9.3, 2.9.5, 2.9.6, 2.9.8, 2.9.11, 2.9.12, 2.9.15, 2.9.7
Due April 27: http://www.math.sunysb.edu/~daryl/322prob.pdf
Due May 4: http://www.math.sunysb.edu/~daryl/322prob2.pdf

If you have a physical, psychological, medical or learning disability that my impact on your ability to carry out assigned course work, please contact the staff in the Disabled Student Service Office, Room 133, Humanities, 632-6784/TDD. DSS will review your concerns and determine with you what accommodations are necessary and appropriate. All information and documentation of disability is confidential.

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Here is a list of review topics for the final exam:

Linear Transformations; e. g. rotations
The matrix of a transformation. Composition of transformations and multiplication of matrices
Invertible transformations
Dot Product, Schwartz Inequality, Cross Product in $\mathbf{R}^3$
Norms of vectors and of linear transformations, operator norms
Open and closed sets and neighbourhoods
Limits of sequences and of functions, continuity
Composition of functions and continuity
Partial and Directional Derivatives, Differentiable functions
Differentiability implies continuity
Every linear transformation on $\mathbf{R}^n$ is continuous
Geometric series of matrices, $|A| < 1$ or $||A|| < 1$ implies

$$(I - A)^{-1} = \sum_{n=0}^{\infty} A^n$$

The set of invertible linear transformations is open in the set of all transformations
A closed bounded subset of $\mathbf{R}^n$ is compact
A function on a compact set attains maximum and minimum values
A sequence in a compact set has a convergent subsequence
Partial derivatives of a function give the matrix of the linear transformation which is its derivative
The map which takes A into A^{-1} is differentiable. Compute its derivative
Rules for computing derivatives: sums, products, compositions, bi-linear functions
Mean value theorem for $f : \mathbf{R}^n \to \mathbf{R}$
A function with continuous partial derivatives is differentiable. Prove it.
A function whose derivative is bounded is Lipschitz.
Newton's method of solving equations and the inverse function theorem which comes from it.
The contraction lemma, and using it to prove an inverse function theorem without Newton's method.
Comparason of Newton's method with the iteration which uses the contraction lemma.
O(3), SO(3), O(n), SO(n), Square roots of some matrices
Parametrization of O(n) given by the implicit function theorem.
Proof that $||A|| = ||A^t||$
Quadratic forms on $\mathbf{R}^n$.
Taylor polynomials and Taylor's Theorem for functions of n variables
Classification of critical points of functions on $\mathbf{R}^n$.