MAT 303: Calculus IV with Applications

Fall 2009

Department of Mathematics
SUNY at Stony Brook

Final Exam Location: Library W0512
Final Exam Date and Time: Wednesday Dec 16, 5:15 - 7:45

Syllabus

Homework Assignments

Here is a link to an Exam I that I gave a year and a half ago, when I last taught 303.
I also made up some Extra Credit/Review questions. I will go over the instructions for these in class...

Old Exam II
Extra Credit/Review questions for Chapter 3

Old Final
Extra Credit/Review Problems for Chapter 5

Review Sheet (no longer eligible for extra credit)

Instructor: Brian Weber
Office: Math Tower 3-121
Office Hours: Monday 12:50-1:50p, Wed 3-4p, or send an email to set up another time
Email: brweber@math.sunysb.edu

Brian's Office Hours for Exam Week

Recitation Instructor: Somnath Basu
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Email: basu@math.sunysb.edu

Final Exam info:
Time: Wednesday Dec 16, 5:15-7:45p

Americans with Disabilities Act

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Disability Support Services, ECC (Educational Communications Center) Building, room 128, (631) 632-6748. They will

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http://www.www.ehs.stonybrook.edu/fire/disabilities.asp

Syllabus for Math 303, Calculus IV with Applications

Fall 2009

Lecturer

Brian Weber
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Course Text *Differential Equations and Boundary Value Problems: Computing and Modelling* (Fourth Edition) by Edwards and Penney

Prerequisites

Official prerequisites are a C or higher in one of the following: MAT 127, MAT 132, MAT 142, or AMS 161, or else a level 9 or higher on the mathematics placement exam. Unofficially, you need a firm enough knowledge of derivatives and integrals to be able to simplify expressions like $\frac{d}{dx}\int_1^x \sqrt[3]{t^2+1}\,dt$ and $\int xe^x\,dx$.

Exams

Any necessary special formulas will be provided on the exam, and the problems will be designed so that calculators won't be necessary. Thus all you'll need is your brain and a pencil. No books, 'cheatsheets,' or calculators will be allowed.

Midterm 1: Oct 2, In-Class (20% of grade)
Midterm 2: Nov 2, In-Class (20% of grade)
Final: Dec 16, 5:15-7:45p (Room TBA) (40% of grade)

Homework (20% of grade)

One problem set will be due each week, at the beginning of recitation. The problem sets consist of roughly 10-20 problems of varying difficulty. Exam questions will be modelled on homework questions, so doing and understanding the homework is the best way to prepare for the tests.

This is a 4 credit course, and as a fair warning, you will have to work hard to be successful. If you fall seriously behind on the homework, you will not be able to keep up in class and will not be prepared for the exams. You are encouraged to work in groups, but you must write up your own solutions.

You must always show your work. No credit will be given for correct answers without correct work, on either exams or homeworks. No exceptions.

Makeup policy

All of your responsibilities for this class have been announced well ahead of time, namely in the first week of classes. Thus almost no requests for makeup homeworks or exams will be granted. The only exceptions, assuming evidence is provided, will be for serious illness, family emergency, or an unforeseeable catastrophe (flood, car wreck, etc).

Grading policy

The grading will be curved. This means your letter grade will be influenced by your performance relative to the rest of the class. An approximate curve will be made after each exam. The final curve, by which your course grade will be determined, will be set using the same process used for the individual exams, so the individual exam grades should be a good measure of how well you are doing.

Chapter 1 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.

Problem I

Consider the multivariable function

$$f(x,y) = \begin{cases} \frac{-2xy}{x^2+y^2} & \text{, if } (x,y) \neq (0,0) \\ 0 & \text{, if } (x,y) = (0,0), \end{cases}$$

and the associated differential equation

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + y^2}.$$

A) Find the points where $f(x,y)$ is discontinuous (remember to justify your answer). Compute $\frac{\partial f}{\partial y}$, and find the points where $\frac{\partial f}{\partial y}$ is discontinuous.

B) For what values of a and b is the IVP $y(a) = b$ guaranteed to have a unique solution? For what values a, b is there no guarantee that the IVP $y(a) = b$ has a solution?

C) Plot a slope field for this differential equation.

D) Considering your answer for (B) and the slope field you drew for (C), do you think any solutions with initial condition $y(0) = 0$ exist, and if so, do you think there is a unique solution?

E) Find the differential equation's general solution (there's an easy way and a hard way!).

Problem II

Consider the multivariale function

$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{, if } (x,y) \neq (0,0) \\ 0 & \text{, if } (x,y) = (0,0), \end{cases}$$

and the associated differential equation

$$\frac{dy}{dx} = \frac{2xy}{x^2 + y^2}.$$

A) Find the points where $f(x,y)$ is discontinuous (remember to justify your answer). Compute $\frac{\partial f}{\partial y}$, and find the points where $\frac{\partial f}{\partial y}$ is discontinuous.

B) For what values of a and b is the IVP $y(a) = b$ guaranteed to have a unique solution? For what values a, b is there no guarantee that the IVP $y(a) = b$ has a solution?

C) Plot a slope field for this differential equation.

D) Considering your answer for (B) and the slope field you drew for (C), do you think any solutions with initial condition $y(0) = 0$ exist, and if so, do you think there is a unique solution?

E) Find the differential equation's general solution.

Find the general solution. Leave the solution in implicit form when appropriate.

1) $\frac{dy}{dx} = \cos(x)$ *Ans:* $y = \sin(x) + C$

2) $\frac{dy}{dx} = \frac{1}{x^2+3x+2}$ *Ans:* $y = C + \ln\left|\frac{x+1}{x+2}\right|$

3) $\frac{dy}{dx} = \frac{\cos(x) - x\sin(x)}{x\cos(x)}$

4) $\frac{dy}{dx} = \frac{x}{x^2+5x+4}$

5) $x\frac{dy}{dx} = 1 - y$ *Ans:* $y = 1 + \frac{C}{x}$

6) $\frac{dy}{dx} = \frac{x^2}{1-y^2}$ *Ans:* $y^3 - 3y + x^3 = C$

7) $x^{-1}\frac{dy}{dx} = 1 - y$

8) $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$

9) $\frac{dy}{dx} + \cos(x)\,y = 4\cos(x)$ *Ans:* $y = 4 + Ce^{-\sin(x)}$

10) $\frac{dy}{dx} = \frac{x-2xy}{x^2+1}$ *Ans:* $y = \frac{\frac{1}{2}x^2+C}{x^2+1}$

11) $x\frac{dy}{dx} = -17y + x^3$

12) $x^2\frac{dy}{dx} = -17y + x^3$

13) $\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$ *Ans:* $y = \frac{Cx^2}{1-Cx}$

14) $\frac{dy}{dx} = \frac{x+y}{x}$ *Ans:* $y = x\ln|x| + Cx$

15) $x\frac{dy}{dx} = \frac{-xy+y^2}{x}$

16) $\frac{dy}{dx} = -\frac{4x+3y}{2x+y}$

Evaluate $dF(x,y)$ when $F(x,y)$ is the given 2-variable function. Then solve the stated differential equation. If you know how to do these, there is no work whatsoever.

17) $F(x,y) = x^2 + xy + y^3$ *Ans:* $dF = (2x+y)dx + (x+3y^2)dy$

$\frac{dy}{dx} = -\frac{2x+y}{x+3y^2}$ *Ans:* $y^3 + xy + x^2 = C$

18) $F(x,y) = x^2 \cos(y)$ *Ans:* $dF = 2x\cos(y)dx - x^2\sin(y)dy$

$\frac{dy}{dx} = \frac{2x\cos(y)}{x^2\sin(y)}$ *Ans:* $y = \cos^{-1}(Cx^{-2})$

19) $F(x,y) = xy$

$\frac{dy}{dx} = -\frac{y}{x}$

20) $F(x,y) = xy^2 + x + x^2 - y^5$

$\frac{dy}{dx} = \frac{5y^4-2xy}{y^2+1+2x}$

Find the general solution.

21) $\frac{dy}{dx} = \frac{3x^2-y}{2y+x}$ *Ans:* $y^2 + xy - x^3 = C$

22) $\frac{dy}{dx} = -\frac{y}{3y^2+x}$ *Ans:* $y^3 + xy = C$

23) $\frac{dy}{dx} = \frac{2x-y^{16}}{17y^{16}+16xy^{15}}$

24) $\frac{dy}{dx} = \frac{4y-x}{9x^2+y-1}$

25) $\frac{dy}{dx} = y + y^2$ *Ans:* $y = \frac{1}{-1+Ce^{-x}}$

26) $x\frac{dy}{dx} = y + xy^{-1}$ *Ans:* $y^2 = \frac{2}{3}x + Cx^{-2}$

27) $x\frac{dy}{dx} = y + xy^2$

28) $\frac{dy}{dx} = y + y^{-1}$

Instructions for getting extra credit

- Do either problem I or problem II (or both!)
- Do at least 9 additional problems.
- Turn in your work.

If you don't turn in at least 10 problems or if you don't do either problem I or II, you won't get any extra credit.

You will get 2 points for each correctly solved problem. Only your 10 best problems will be graded, no matter how many are turned in.

This will count as one-half of a homework assignment (which is 12.5% of the total homework grade before Exam I)

Test II

Name

April 15, 2008

Math 303 - Differential Equations with Applications

- No credit will be given for answers without mathematical or logical justification.
- Simplify answers as much as possible.
- No calculators, notes, or books

Part I

1) Determine whether the following set of column vectors is linearly dependent or linearly independent. (7 pts)

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 17 \\ -20 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 2 \\ -1 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \\ -2 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 3 \end{pmatrix}$$

2) (4 pts each) Perform the indicated operation

a) $\begin{pmatrix} 1 & 2 \\ 2 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

b) $\begin{pmatrix} -101 & 17 & 2 \\ -1 & 29 & 5 \end{pmatrix}^T$

3) a) (5 pts) Express the following system as a matrix equation

$$
\begin{aligned}
x_1' &= 2x_1 + x_2 - x_3 + \cos(2t) \\
x_2' &= x_3 + \sin(3t) \\
x_3' &= -x_1 + x_2 + e^t
\end{aligned}
$$

b) (5 pts) Convert the following system to a single second order equation

$$
\begin{aligned}
x' &= x - y \\
y' &= 2x + y
\end{aligned}
$$

Part II

4) (10 pts) Find the general solution

$$x'' + 9x' + 20x = 0$$

5) (10 pts) Find the solution

$$x'' + 4x' + 13x = 0, \qquad x(0) = 0, \quad x'(0) = -6.$$

6) Consider the differential equation $x'' + 4x' + 13x = 40\cos(t)$ (similar to problem 5).
a) (10 pts) Find the general solution

b) (5 pts) Find the solution, given initial conditions $x(0) = 3$, $x'(0) = -5$.

Part III

7) Consider the equation

$$x^{(4)} + 4x'' + 4x = \cos(2t).$$

a) (5 pts) Find the complimentary solution.

b) (5 pts) If you used the method of undetermined coefficients, what would your 'guess' for the particular solution be? DO NOT actually try to determine the coefficients.

c) (5 pts) Do you think this differential equation could serve as a reasonable model for some simple physical system? Why or why not? (Be brief, but clear.)

8) Consider the mass-spring-dashpot system with the specifications

Mass $m = 1$	Damping constant $c = 2$
Spring constant $k = 2$	External forcing $f(t) = \cos(\omega t)$.

Recall the spring equation $m\,x'' + cx' + kx = f(t)$.

a) (13 pts) Find the amplitude C of the steady periodic response as a function of ω.

b) (12 pts) Find the frequency of practical resonance (if any).

Chapter 3 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.

State whether the functions are linearly dependent and/or algebraically dependent (1 point each).

1) $$\begin{aligned} y_1 &= \cos(x) \\ y_2 &= \sin(x) \\ y_3 &= \cos(x) + \sin(x) \end{aligned}$$

Ans: Linearly dependent, Algebraically dependent

2) $$\begin{aligned} x_1 &= \sin(t) \\ x_2 &= e^t \\ x_3 &= e^t \sin(t) - 7e^t \end{aligned}$$

Ans: Algebraically dependent

3) $$\begin{aligned} x_1 &= \cos(t) + \cos(2t) \\ x_2 &= \cos(t) - \cos(2t) \\ x_3 &= e^{3t} + e^{-t} \\ x_4 &= e^{3t} - e^{-t} \\ x_5 &= \cos(t) + e^{-t} \end{aligned}$$

4) $$\begin{aligned} x_1 &= \cos(t) + \cos(2t) \\ x_2 &= \cos(t) - \cos(t) \\ x_3 &= \cos(t) \end{aligned}$$

5) $$\begin{aligned} x_1 &= \cot(t) \\ x_2 &= e^{2t} \\ x_3 &= e^{2t+1} - \pi \cot(t) \end{aligned}$$

6) $$\begin{aligned} y_1 &= \sin(t) \\ y_2 &= \cos(t) \\ y_3 &= 1 \end{aligned}$$

Find the general solution (2 points each)

7) $y'' + 7y' + 12y = 0$

Ans: $y(t) = c_1 e^{-4t} + c_2 e^{-3t}$

8) $y''' + 2y'' - y' - 2y = 0$

9) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0$

10) $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0$

Find the solution (2 points each)

11) $y'' + 7y' + 12y = 0, \quad y(0) = 0,\ y'(0) = 1$

Ans: $y(t) = -e^{-4t} + e^{-3t}$

12) $y''' + 2y'' - y' - 2y = 0, \quad y(0) = 1,\ y'(0) = -2,\ y''(0) = 4$

13) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0, \quad x(0) = 0,\ x'(0) = 3$

14) $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0, \quad x(0) = 0,\ x'(0) = 0,\ x''(0) = 0,\ x^{(3)}(0) = 0$

Find the solution (4 points each)

15) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos(t), \qquad x(0) = 11/5,\ x'(0) = -29/10$

Ans: $x(t) = \left(2e^{-t} + \frac{1}{5}\right)\cos(2t) + \left(-e^{-t} + \frac{1}{10}\right)\sin(2t)$

16) $\frac{d^2x}{dt^2} + 4x = \cos(2t), \qquad x(0) = 1,\ x'(0) = -1$

17) $y'' - 2y' + y = e^t, \qquad y(0) = 1,\ y(0) = 2$

Find the amplitude $C(\omega)$ of the system's steady-state reponse.
Plot the function $C(\omega)$, and indicate the frequency of practical resonance (if any exists).
(8 points per problem)

18) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos(\omega t)$

Practical resonance at $\omega = \sqrt{3}$

19) $\frac{d^2x}{dt^2} + 5x = \cos(\omega t)$ (same system, but without damping)

20) $x'' + 3x' + x = \cos(\omega t)$

Final Exam

Name

Math 303 - Differential Equations with Applications

May 15, 2008

- No credit will be given for answers without mathematical or logical justification.
- You may leave answers in implicit form, when appropriate.
- Simplify answers as much as possible.
- No calculators, notes, or books.

Part I

1) (7 pts) Solve for x: $\frac{dx}{dt} = tx - \frac{t}{x}, \quad x(0) = 2.$

2) (8 pts) Find the general solution: $\frac{dy}{dx} = \frac{2x - y}{x + 6y}.$

3) (15 pts) Solution with a concentration of 0.1 lbs of salt per gallon pours into a tank at a rate of $\frac{2}{t+1}$ gallons per minute. Also, well-mixed solution leaves the tank at the same rate. How much salt is in the tank after 1 minute, if initially the tank contains 1 gallon of water mixed with 0.9 lb of salt?

4) The equation for a particle caught in the gravitational field of a body of mass M is

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2},$$

where $G \approx 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$ and r is the distance to the center of mass of the body. A particle of dust is caught in the gravitational field of a small, spherically shaped asteroid of mass $\frac{16}{6.67} \times 10^{11} kg$ and radius $100m$.

a) (10 pts) Use the methods of differential equations to find $\frac{dr}{dt}$.

b) (10 pts) Initially the dust particle is motionless relative to the asteroid and $300m$ from its surface. With what speed will it strike the asteroid's surface?

Part II

5) Consider the differential equation

$$x'''' + 2x''' + 2x'' = 4 - 12t$$

a) (10 pts) Find the complimentary solution

b) (15 pts) Find the general solution.

c) (5 pts) Find the solution, given $x(0) = 0$, $x'(0) = 0$, $x''(0) = 8$, $x'''(0) = -6$.
(There's an easy way and a hard way.)

Part III

6) (5 pts) Compute e^{tA}, where $A = \begin{pmatrix} 0 & -1 & 6 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}$.

7) Consider the equation

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ t \end{pmatrix}.$$

a) (5 pts) If you use the method of undetermined coefficients, what would your 'guess' for $\mathbf{x_p}$ be?

b) (15 pts) Find $\mathbf{x_p}$.

Part IV

8) Consider the system $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

a) (10 pts) Find the eigenvalues of the matrix.

b) (20 pts) Find the system's general solution.

(continued from previous page)

c) (10 pts) Write down the fundamental matrix $\Phi(t)$, and compute $\Phi(0)^{-1}$
HINT: This particular $\Phi(0)$ should have some special properties. Before trying to compute $\Phi(0)^{-1}$, see what you get when you multiply $\Phi(0) \cdot \Phi(0)^T$.

d) (5 pts) Given $x(0) = (\,-1\ 1\ 0\ 2\,)^T$, find $x(t)$.

Chapter 5 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.
Instructions: Do #1, either #2 or #3, and either #4 or #5.

1) Let

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & -1 \\ -1 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

a) (5 pts) Show that A can be written as $A = B + C$, where B is the diagonal matrix with 2's along the diagonal, and C is a nilpotent matrix.

b) (5 pts) Use part (a) to compute $e^{\mathbf{A}}$.

c) (5 pts) Compute $e^{t\mathbf{A}}$.

d) (5 pts) Solve the initial value problem

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}' = \begin{pmatrix} 3 & 0 & -1 \\ -1 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

In the following two problems, you will consider a system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$. Perform the following:

a) (10 pts) Find the complimentary solution

b) (5 pts) What is your 'guess' for the particular solution, $\mathbf{x_p}$?
Special instructions: Be *extremely* specific. Do not write $\mathbf{x_p} = t\mathbf{a} + \mathbf{b}$ for example, but instead write $\mathbf{x_p} = t\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, or maybe even $\mathbf{x_p} = \begin{pmatrix} ta_1 + b_1 \\ ta_2 + b_2 \end{pmatrix}$.

c) (10 pts) Plug in your 'guess' for $\mathbf{x_p}$. You will get a system of 4 equation and 4 unknowns. Write this system in augmented matrix form.

d) (5 pts) Find $\mathbf{x_p}$.

2) $$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 2 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

3) $$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} -1 & 2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} t \\ 1 + t \end{pmatrix}$$

In the following two problems, you will consider a system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Answer the following questions:

a) (10 pts) Find all the generalized eigenvectors of A.

b) (5 pts) Find the general solution.

c) (10 pts) Find $\mathbf{\Phi}(t)$ and compute $\mathbf{\Phi}(0)^{-1}$.

d) (5 pts) Find the solution, given $\mathbf{x}(0) = (-1\ 1\ 0\ 1)^T$.

4)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

5)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & -1 & -1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Review Problem Sheet

MAT 303 - Fall '09 Name

1) (15 pts) Which method would you use to solve the following? Justify your answer, but do not actually solve the equation.

- $xy' = y^{-2}x^{-1} + x$
- $y' = xy + x + y + 1$
- $y' = -\frac{2xy+y}{x^2+x}$
- $y'' + x(y')^2 = 0$
- $y' = x^2y + x$
- $xy' = y + y^2$
- $\frac{y''}{y'} - 2y = 0$

2) (25 pts) Find the general solution for any three of the DE's from problem 1.

3) Consider the system $y'' - 2y' + y = 0$.
a) (5 pts) Find the general solution.
b) (10 pts) Convert this equation to a system, and find the general solution.

4) Consider the system $y''' + 2y'' - y' - 2y = 0$.
a) (5 pts) Find the general solution.
b) (20 pts) Convert this equation to a system, and find the general solution.

5) (10 pts) Use Taylor series methods to solve $y'' - 2y' + y = 0$.

6) (10 pts) Use Taylor series methods to find the first three terms of the solution to the *unforced pendulum equation*

$$y'' + \sin(y) = 0, \quad y(0) = 0,\ y'(0) = 1.$$