MAT 303: Calculus IV with Applications

Spring 2008

Department of Mathematics
SUNY at Stony Brook

Final Exam info:
Time: Thursday May15, 8am-10:30am
Location: our ordinary classroom, Light Engineering Lab 152

Syllabus

Homework Assignments

Chapter 1 Review and Extra Credit Problems   Instructions are located here.

Chapter 3 Review and Extra Credit Problems (with corrections)   Instructions are located here.

Chapter 5 Reviewand Extra Credit Problems (Instructions within)

Brian's Exam week office hours:
Tuesday, 2-4p
Wednesday, 2-5p

Joseph's review session:
Monday, 12:50 in the math tower, P-131

Brian's review session:
Tuesday, 10:00 in the math tower, P-131

Instructor: Brian Weber
Office: Math Tower 3-121
Office Hours: Tues 2-3p, Thurs 3-4p, or send an email to set up another time
Email: brweber@math.sunysb.edu

Recitation Instructor: Joseph Malkoun
Office: Math Tower 2-112
Office Hours: Tues 11:30a-12:30p, and Wed 3-5p in the MLC
Email: malkoun@math.sunysb.edu

Americans with Disabilities Act

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact

Disability Support Services, ECC (Educational Communications Center) Building, room 128, (631) 632-6748. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students requiring emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information, go to the following web site:

http://www.www.ehs.stonybrook.edu/fire/disabilities.asp

Syllabus for Math 303, Calculus IV with Applications

Spring 2008

Lecturer

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TA

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Course Text

Differential Equations and Boundary Value Problems: Computing and Modelling (Fourth Edition) by Edwards and Penney

Prerequisites

Official prerequisites are a C or higher in one of the following: MAT 127, MAT 132, MAT 142, or AMS 161, or else a level 9 or higher on the mathematics placement exam. Unofficially, you need a firm knowledge of derivatives, integrals, and basic algebra.

Exams

Any necessary special formulas will be provided on the exam, and the problems will be designed so that calculators won't be necessary. Thus all you'll need is your brain and a pencil. No cheatsheets or calculators will be allowed.

Midterm 1: March 4, In-Class (20% of grade)
Midterm 2: April 15, In-Class (20% of grade)
Final: Thursday May 15 (Room TBA) (40% of grade)

Homework (20% of grade)

One problem set will be due each week, at the beginning of recitation. The problem sets consist of roughly 10-20 problems of varying difficulty. Exam questions will be modelled on homework questions, so doing and understanding the homework is the best way to prepare for the tests.

This is a 4 credit course, and as a fair warning, you will have to work hard to be successful. If you fall seriously behind on the homework, you will not be able to keep up in class and will not be prepared for the exams. You are encouraged to work in groups, but you must write up your own solutions.

You must always show your work. No credit will be given for correct answers without correct work, on either exams or homeworks. No exceptions.

Makeup policy

All of your responsibilities for this class have been announced well ahead of time, namely in the first week of classes. Thus almost no requests for makeup homeworks or exams will be granted. The only exceptions, assuming evidence is provided, will be for serious illness, family emergency, or an unforeseeable catastrophe (flood, car wreck, etc).

Grading policy

The grading will be curved. This means your letter grade will be influenced by your performance relative to the rest of the class. An approximate curve will be made after each exam. The final curve, by which your course grade will be determined, will be set using the same process used for the individual exams, so the individual exam curves should be a good measure of how well you are doing.

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Spring 2008

Homework Assignments

Material Presented Week of	Chapter	Problems	Problems Due Week of
1/28	1.1 1.2	1, 6, 11 1, 3, 6, 11, 17, 42, 43	2/4
2/4	1.3 1.4 1.5	11, 14, 20, 22, 29 1, 3, 17, 19, 23, 24, 45, 47 1, 3, 5, 19, 20, 33, 35	2/11
2/11	1.6 2.1 2.2	1, 3, 8, 21, 31, 32, 33, 43, 44, 45, 67 1, 9, 33 2, 4, 21	2/18
2/18	2.3 3.1 3.2	2, 4, 24 3, 11, 16, 17, 19, 33, 35, 37, 39 1, 2, 13, 17, 20, 21, 23, 30	2/25
2/25	3.3	1, 5, 9, 19, 21, 22	3/10
3/3 *Exam I on 3/4*	3.4	15, 17, 19, 12(a)(b)(c)	3/10
3/10	3.5	2, 4, 6, 21, 33, 38, 47, 53, 55	3/24
3/17		*Spring Break*	
3/24	3.6 3.7	1, 4, 7, 11, 13, 15, 18 8, 13, 17	3/31
3/31	4.1 5.1	1, 4, 5, 11, 17 1, 2, 4, 6, 7, 9, 11, 14, 19, 21, 28, 30	4/7
4/7	5.2	1, 6, 11, 17, 27, 35, 38, 39, 41	4/21
4/14 *Exam II on 4/15*	5.3	-, -, -	4/21
4/21	5.4	1, 5, 6, 8, 11, 12, 20, 33	4/28
4/28	5.5 5.6 6.1 6.2	1, 3, 7, 9, 17, 23, 25 1, 5, 29	5/5
5/5		*Review for the Final*	

Chapter 1 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.

Problem I

Consider the multivariale function

$$f(x,y) \quad = \quad \begin{cases} \frac{-2xy}{x^2+y^2} & \text{, if } (x,y) \neq (0,0) \\ 0 & \text{, if } (x,y) = (0,0), \end{cases}$$

and the associated differential equation

$$\frac{dy}{dx} \quad = \quad \frac{-2xy}{x^2 + y^2}.$$

A) Find the points where $f(x,y)$ is discontinuous (remember to justify your answer). Compute $\frac{\partial f}{\partial y}$, and find the points where $\frac{\partial f}{\partial y}$ is discontinuous.

B) For what values of a and b is the IVP $y(a) = b$ guaranteed to have a unique solution? For what values a, b is there no guarantee that the IVP $y(a) = b$ has a solution?

C) Plot a slope field for this differential equation.

D) Considering your answer for (B) and the slope field you drew for (C), do you think any solutions with initial condition $y(0) = 0$ exist, and if so, do you think there is a unique solution?

E) Find the differential equation's general solution (there's an easy way and a hard way!).

Problem II

Consider the multivariale function

$$f(x,y) \quad = \quad \begin{cases} \frac{2xy}{x^2+y^2} & \text{, if } (x,y) \neq (0,0) \\ 0 & \text{, if } (x,y) = (0,0), \end{cases}$$

and the associated differential equation

$$\frac{dy}{dx} \quad = \quad \frac{2xy}{x^2 + y^2}.$$

A) Find the points where $f(x,y)$ is discontinuous (remember to justify your answer). Compute $\frac{\partial f}{\partial y}$, and find the points where $\frac{\partial f}{\partial y}$ is discontinuous.

B) For what values of a and b is the IVP $y(a) = b$ guaranteed to have a unique solution? For what values a, b is there no guarantee that the IVP $y(a) = b$ has a solution?

C) Plot a slope field for this differential equation.

D) Considering your answer for (B) and the slope field you drew for (C), do you think any solutions with initial condition $y(0) = 0$ exist, and if so, do you think there is a unique solution?

E) Find the differential equation's general solution.

Find the general solution. Leave the solution in implicit form when appropriate.

1) $\frac{dy}{dx} = \cos(x)$ *Ans:* $y = \sin(x) + C$

2) $\frac{dy}{dx} = \frac{1}{x^2+3x+2}$ *Ans:* $y = C + \ln\left|\frac{x+1}{x+2}\right|$

3) $\frac{dy}{dx} = \frac{\cos(x) - x\sin(x)}{x\cos(x)}$

4) $\frac{dy}{dx} = \frac{x}{x^2+5x+4}$

5) $x\,\frac{dy}{dx} = 1 - y$ *Ans:* $y = 1 + \frac{C}{x}$

6) $\frac{dy}{dx} = \frac{x^2}{1-y^2}$ *Ans:* $y^3 - 3y + x^3 = C$

7) $x^{-1}\,\frac{dy}{dx} = 1 - y$

8) $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$

9) $\frac{dy}{dx} + \cos(x)\,y = 4\cos(x)$ *Ans:* $y = 4 + Ce^{-\sin(x)}$

10) $\frac{dy}{dx} = \frac{x-2xy}{x^2+1}$ *Ans:* $y = \frac{\frac{1}{2}x^2+C}{x^2+1}$

11) $x\frac{dy}{dx} = -17y + x^3$

12) $x^2\frac{dy}{dx} = -17y + x^3$

13) $\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$ *Ans:* $y = \frac{Cx^2}{1-Cx}$

14) $\frac{dy}{dx} = \frac{x+y}{x}$ *Ans:* $y = x\ln|x| + Cx$

15) $x\,\frac{dy}{dx} = \frac{-xy+y^2}{x}$

16) $\frac{dy}{dx} = -\frac{4x+3y}{2x+y}$

Evaluate $dF(x, y)$ when $F(x, y)$ is the given 2-variable function. Then solve the stated differential equation. If you know how to do these, there is no work whatsoever.

17) $F(x, y) = x^2 + xy + y^3$ *Ans:* $dF = (2x + y)dx + (x + 3y^2)dy$

$\frac{dy}{dx} = -\frac{2x+y}{x+3y^2}$ *Ans:* $y^3 + xy + x^2 = C$

18) $F(x, y) = x^2 \cos(y)$ *Ans:* $dF = 2x\cos(y)dx - x^2\sin(y)dy$

$\frac{dy}{dx} = \frac{2x\cos(y)}{x^2\sin(y)}$ *Ans:* $y = \cos^{-1}(Cx^{-2})$

19) $F(x, y) = xy$

$\frac{dy}{dx} = -\frac{y}{x}$

20) $F(x, y) = xy^2 + x + x^2 - y^5$

$\frac{dy}{dx} = \frac{5y^4-2xy}{y^2+1+2x}$

Find the general solution.

21) $\frac{dy}{dx} = \frac{3x^2-y}{2y+x}$ *Ans:* $y^2 + xy - x^3 = C$

22) $\frac{dy}{dx} = -\frac{y}{3y^2+x}$ *Ans:* $y^3 + xy = C$

23) $\frac{dy}{dx} = \frac{2x - y^{16}}{17y^{16}+16xy^{15}}$

24) $\frac{dy}{dx} = \frac{4y-x}{9x^2+y-1}$

25) $\frac{dy}{dx} = y + y^2$ *Ans:* $y = \frac{1}{-1+Ce^{-x}}$

26) $x\frac{dy}{dx} = y + xy^{-1}$ *Ans:* $y^2 = \frac{2}{3}x + Cx^{-2}$

27) $x\frac{dy}{dx} = y + xy^2$

28) $\frac{dy}{dx} = y + y^{-1}$

Instructions for getting extra credit

- Do either problem I or problem II (or both!)
- Do at least 9 additional problems.
- Turn in your work.

If you don't turn in at least 10 problems or if you don't do either problem I or II, you won't get any extra credit.

You will get 2 points for each correctly solved problem. Only your 10 best problems will be graded, no matter how many are turned in.

This will count as one-half of a homework assignment (which is 12.5% of the total homework grade before Exam I).

Chapter 3 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.

State whether the functions are linearly dependent and/or algebraically dependent (1 point each).

1) $$\begin{aligned} y_1 &= \cos(x) \\ y_2 &= \sin(x) \\ y_3 &= \cos(x) + \sin(x) \end{aligned}$$

Ans: Linearly dependent, Algebraically dependent

2) $$\begin{aligned} x_1 &= \sin(t) \\ x_2 &= e^t \\ x_3 &= e^t \sin(t) - 7e^t \end{aligned}$$

Ans: Algebraically dependent

3) $$\begin{aligned} x_1 &= \cos(t) + \cos(2t) \\ x_2 &= \cos(t) - \cos(2t) \\ x_3 &= e^{3t} + e^{-t} \\ x_4 &= e^{3t} - e^{-t} \\ x_5 &= \cos(t) + e^{-t} \end{aligned}$$

4) $$\begin{aligned} x_1 &= \cos(t) + \cos(2t) \\ x_2 &= \cos(t) - \cos(t) \\ x_3 &= \cos(t) \end{aligned}$$

5) $$\begin{aligned} x_1 &= \cot(t) \\ x_2 &= e^{2t} \\ x_3 &= e^{2t+1} - \pi \cot(t) \end{aligned}$$

6) $$\begin{aligned} y_1 &= \sin(t) \\ y_2 &= \cos(t) \\ y_3 &= 1 \end{aligned}$$

Find the general solution (2 points each)

7) $y'' + 7y' + 12y = 0$

Ans: $y(t) = c_1 e^{-4t} + c_2 e^{-3t}$

8) $y''' + 2y'' - y' - 2y = 0$

9) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0$

10) $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0$

Find the solution (2 points each)

11) $y'' + 7y' + 12y = 0, \quad y(0) = 0,\ y'(0) = 1$

Ans: $y(t) = -e^{-4t} + e^{-3t}$

12) $y''' + 2y'' - y' - 2y = 0, \quad y(0) = 1,\ y'(0) = -2,\ y''(0) = 4$

13) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0, \quad x(0) = 0,\ x'(0) = 3$

14) $x^{(4)} + 4x^{(3)} + 6x'' + 4x' + x = 0, \quad x(0) = 0,\ x'(0) = 0,\ x''(0) = 0,\ x^{(3)}(0) = 0$

Find the solution (4 points each)

15) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos(t), \qquad x(0) = 11/5,\ x'(0) = -29/10$

Ans: $x(t) = \left(2e^{-t} + \frac{1}{5}\right)\cos(2t) + \left(-e^{-t} + \frac{1}{10}\right)\sin(2t)$

16) $\frac{d^2x}{dt^2} + 4x = \cos(2t), \qquad x(0) = 1,\ x'(0) = -1$

17) $y'' - 2y' + y = e^t, \qquad y(0) = 1,\ y(0) = 2$

Find the amplitude $C(\omega)$ of the system's steady-state reponse.
Plot the function $C(\omega)$, and indicate the frequency of practical resonance (if any exists).
(8 points per problem)

18) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos(\omega t)$

Practical resonance at $\omega = \sqrt{3}$

19) $\frac{d^2x}{dt^2} + 5x = \cos(\omega t)$ (same system, but without damping)

20) $x'' + 3x' + x = \cos(\omega t)$

Instructions for getting extra credit

Do as many as you can. Your best 5 problems will count. Since the problems have different point counts, you will want to do more high-counting problems.

Turn in to Brian before Exam II.

Chapter 5 Review/Extra Credit Problems

Remember, no credit will be given for answers without justification.
Instructions: Do #1, either #2 or #3, and either #4 or #5.

1) Let

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

a) (5 pts) Compute $e^{\mathbf{A}}$.

b) (5 pts) Compute $e^{t\mathbf{A}}$.

c) (5 pts) Solve the initial value problem

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}' = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

In the following two problems, you will consider a system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$. Answer the following questions:

a) (10 pts) Find the complimentary solution

b) (5 pts) What is your 'guess' for the particular solution, $\mathbf{x_p}$?
Special instructions: Be *extremely* specific. Do not write, for example, $\mathbf{x_p} = t\mathbf{a} + \mathbf{b}$, but instead write $\mathbf{x_p} = t\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, or maybe $\mathbf{x_p} = \begin{pmatrix} ta_1 + b_1 \\ ta_2 + b_2 \end{pmatrix}$.

c) (10 pts) Plug in your 'guess' for $\mathbf{x_p}$. You will get a system of 4 equation and 4 unknowns. Write this system in augmented matrix form.

d) (5 pts) Find $\mathbf{x_p}$.

2) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 2 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

3) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} -1 & 2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} t \\ 1 + t \end{pmatrix}$

In the following two problems, you will consider a system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Answer the following questions:

a) (10 pts) Find all the generalized eigenvectors of A.

b) (5 pts) Find the general solution.

c) (10 pts) Find $\mathbf{\Phi}(t)$ and compute $\mathbf{\Phi}(0)^{-1}$.

d) (5 pts) Find the solution, given $\mathbf{x}(0) = (-1\ 1\ 0\ 1)^T$.

4)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

5)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}' = \begin{pmatrix} -1 & -1 & -1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$