Stony Brook University
Mathematics Department
Julia Viro

Introduction to Advanced Mathematics
MAT 250
Fall 2021

Syllabus

Course description: An introduction to the Advanced Track mathematics program. Provides the core of basic logic, elementary set theory and language of maps. The rigorous language will be applied to define and study some notions of number theory, combinatorics, elementary analysis, Euclidean geometry, topology, etc. No preliminary knowledge of advanced mathematics is required. MAT 250 serves as an alternative to MAT 200 for students in the Advanced Track.

Credits: 4.

STEM+: A grade of C or better in this course fulfills the Science, Technology, Engineering, and Mathematics (STEM+) objective in the Stony Brook Curriculum.

Instructor: Julia Viro
e-mail: Julia.Viro@stonybrook.edu
Online MLC hours: Tuesday at 3pm-4pm.
Office hours: Tuesday and Thursday at 5pm-6pm.
Zoom personal meeting room:
https://stonybrook.zoom.us/j/9792031214

Grader: Dylan Galt
e-mail: Dylan.Galt@stonybrook.edu
Online MLC hours: Wednesday at 1:30pm-2:30pm.
Online office hours: Tuesday at 3pm-5pm.

Textbook: Peter J. Eccles, *An Introduction to Mathematical Reasoning*, Cambridge University Press.

Meetings: TuTh 1:15pm-2:35pm in Chemistry 126.

Homework will be assigned weekly through Blackboard (Assignments). Your solutions should be submitted to Blackboard. Each submission should contain a single pdf-file. You may use any app which consolidate your pictures in a single pdf-file (for example, CamScan). Submission in a wrong format (multiple files, jpg-format, links to the cloud, etc.) will be accepted but with reduced score. Late submission will be accepted but with reduced score.

The emphasis of the course is on writing proofs, so please try to write legibly and explain your reasoning clearly and fully. You are encouraged to discuss the homework problems with others, but your write-up must be your own work. Suspiciously similar submissions won't be graded. Submitting somebody's else work is a serious violation of university integrity policy and will be treated respectively. See Academic integrity statement below.

Quizzes will be given weekly in class.

Exams: two midterms and final exam. Missing any of the exams without any serious and documented reason will result to failure in the course.

Grading system: your grade for the course will be based on: homework 5%, quizzes 10%, class active participation 10%, two midterms 25% each, final exam 25%.

☞ All your work should be done by you and nobody else. Submitting somebody's else work is a serious violation of university integrity policy and will be treated respectively. See Academic integrity statement below.

Make-up policy: Make-up examinations are given only for work missed due to unforeseen circumstances beyond the student's control.

Student Accessibility Support Center (SASC) statement: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact SASC (631) 632-6748 or http://studentaffairs.stonybrook.edu/dss/. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and SASC. For procedures and information go to the following website:
http://www.stonybrook.edu/ehs/fire/disabilities/asp.

Academic integrity statement: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary

Critical incident management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Student Conduct and Community Standards any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Until/unless the latest COVID guidance is explicitly amended by SBU, during Fall 2021"disruptive behavior" will include refusal to wear a mask during classes.

For the latest COVID guidance, please refer to:

https://www.stonybrook.edu/commcms/strongertogether/latest.php

Weekly Plan (tentative)

Week 1 (Tu 8/24, Th 8/26).

Learning objectives: Introduction to logic. Propositions and predicates. Logical connectives. Truth tables. Compound propositions. Conjunctive and disjunctive normal forms. Conditional and biconditional sentences. Denials. Logical identities.

Reading: 1, 2.

Learning outcomes. A student should be able to
1. outline the scope of the course and list the main topics to be studied
2. identify whether a phrase is a proposition
3. distinguish a proposition and a predicate
4. manipulate correctly with five logical connectives (negation, conjunction, disjunction, implication, and equivalence).
5. understand the nature of truth tables
6. identify logical connectives given with emotional attributions (logical conjunction vs. colloquial *and, but, though, nevertheless*, etc.).
7. compose propositional forms and identify their truth values
8. determine equivalent propositional forms
9. identify conditional and biconditional sentences
10. use the whole range of linguistic expressions associated with conditionals and biconditionals ("sufficient", "necessary", "sufficient and necessary", "whenever", "if and only if", etc.)
11. understand the difference between implication in mathematics and causation in language/everyday life
12. list and prove at least 10 logical identities
13. define what a tautology and contradiction mean
14. formulate and prove de Morgan's laws, the law of excluded middle and the law of consistency
15. construct useful denials of propositional forms
16. construct the contrapositive, the converse, and the inverse of a conditional statement.
17. explain what a normal form of a proposition is and how to construct disjunctive and conjunctive normal forms

Week 2 (Tu 8/31, Th 9/2).

Learning objectives: Quantifiers and quantified sentences. Analyzing and constructing propositions involving several quantifiers.

Reading: 7.

Learning outcomes. A student should be able to
1. recognize three quantifiers (universal, existential, and unique existential) in both written and colloquial environment
2. translating propositions formulated in a colloquial English into symbolic forms and the other way around
3. analyze and construct propositions involving several quantifiers

4. identify free and dummy variables in logical structures
5. list the situations when quantifiers commute and when they don't
6. construct useful denials of propositional forms and quantified sentences

Week 3 (Tu 9/7, Th 9/9).

Learning objectives: Logical structure of definitions and theorems. How to read and understand mathematical texts. Structure of a mathematical theory: basic objects, axioms, definitions and theorems. The role of proofs. Examples and counterexamples.

Reading: 3; Lecture notes.

Learning outcomes. A student should be able to
1. comprehend the logical structure of a definition
2. treat mathematical definitions as biconditional sentences with a single free variable
3. be aware about the agreement about conditional colloquial expressions in definitions
4. present three signs/criteria for identification of a definition
5. comprehend the logical structure of a theorem
6. explain the impossibility of free variables in formulations of theorems
7. distinguish a definition from a theorem and example using the logical criteria
8. identify definitions, theorems, and examples in an unknown mathematical text
9. put known definitions and theorems in an appropriate logical structure, both in words and symbols.
10. comprehend the structure of a mathematical theory: identify the basic objects, axioms and theorems
11. explain the role of proofs in mathematics
12. distinguishing the formulation (statement) of a theorem and its proof and see the difference between motivation and proof
13. understand the nature of examples and counterexamples
14. explain when and why examples can't replace a proof
15. understand the structure of a mathematical text
16. list several techniques how to read and understand a mathematical text

Week 4 (Th 9/14, Th 9/16).

Learning objectives: Proof techniques: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion. Strategies for constructing proofs.

Reading: 3, 4.

Learning outcomes. A student should be able to
1. describe four standard proof techniques: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion
2. implement standard proving schemes for simplest proofs
3. evaluate pros and contra of different proof techniques
4. make comparative analysis of various proofs of the same fact
5. master symbolic writing within appropriate logical framework
6. identify three most common logical mistakes in a making a proof

Week 5 (Tu 9/21, Th 9/23). Review and Midterm 1.

Week 6 (Tu 9/28, Th 9/30).

Learning objectives: Principle of mathematical induction in various forms: induction, strong induction, well-ordering principle.

Reading: 5.

Learning outcomes. A student should be able to
1. describe the principle of mathematical induction in various forms (induction, strong induction, well- ordering principle)
2. identify the situations when a proof by induction is suitable and the situations when it is not
3. conduct proofs by induction of various statements from combinatorics, algebra, geometry and analysis.

Week 7 (Tu 10/5, Th 10/7).

Learning objectives. Basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement. Families of sets. Relations between logical and set-theoretical operations. Set-theoretic identities. Maps: definitions and notations. Basic terminology associated with maps: domain, codomain, image and preimage. Examples of maps: functions in one variable, numerical sequences, identity map, constant map.

Reading: 6, 8

Learning outcomes. A student should be able to
1. operate freely with basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement
2. use Venn diagrams to illustrate set-theoretical events
3. explain why Venn diagram can't serve as a proof
4. explain why Venn diagram can serve as a counterexample
5. establish relations between logical and set-theoretical operations, like negation and complement, conjunction and intersection etc.
6. explain what is a set-theoretical identity and how to prove it
7. understand the concept of families of sets and give several examples of families of sets
8. give definition of the power set and list several properties of the power set
9. define a map from one set to another, provide synonyms for the word *map*
10. use freely basic terminology related to maps: the domain, codomain, and range of a map; the image and preimage of a set
11. use correct symbols related to maps
12. provide examples of maps from different parts of mathematics
13. operate with special maps: identity map and constant map.

Week 8 (Tu 10/12: no classes: Fall break, Th 10/14).

Learning objectives. Composition of maps: definition and properties. Inclusion map. Restriction of a map to a subset. Submap. Characteristic function of a set. Power set and the set of all maps to a two-element set. The set of all maps $X \to Y$.

Injections, surjections and bijections. Definition and properties of inverse map. Equivalence between invertibility and bijectivity.

Reading: 9.

Learning outcomes. A student should be able to
1. define a composition of maps and list its properties
2. define inclusion map, submap and restrictions of a map
3. define and list properties of the characteristic function of a set
4. work with the set of all maps from one set to another
5. establish relation between the power set and the set of all maps to a two-element set
6. provide definitions of Injections, surjections and bijections. List synonyms for these words
7. provide definitions of inverse map, left inverse,and right inverse
8. list basic examples of functions and their inverse: exponential and logarithmic, tangent and arctangent, etc.
9. state and prove equivalence of invertibility and bijectivity

Week 9 (Tu 10/19, Th 10/21).

Learning objectives. Cartesian product of sets. Coordinate projections and fibers. Graph of a map. Relations. Functions of several variables as functions on a Cartesian product. Metric on a set. Equivalence relations and partitions. Quotient sets. Canonical factorization of a map into composition of surjection, bijection and injection. Constructions of integers and rational numbers. Construction of complex numbers.

Reading: 13; Lecture notes.

Learning outcomes. A student should be able to
1. give definition of the Cartesian product of sets and list several properties of the Cartesian product
2. describe coordinate projections and fibers
3. define graph os a map
4. define a metric on a set
5. provide examples of different metrics on the same sets
6. define and give example of a relation from one set to another and a relation on a set
7. list several properties of relations
8. discuss properties associated with a binary relation on a set: reflexivity, irreflexivity, symmetry, antisymmetry, transitivity
8. define strict partial order, non-strict partial order, and linear order
10. provide basic examples of sets with strict partial order, non-strict partial order, and linear order
11. define equivalence relation on a set
12. provide five examples of equivalence relations
13. define partition of a set and establish connection between equivalence relations and partitions of a set
14. describe equivalence classes, the quotient set, and the quotient map

Week 10 (Tu 10/26, Th 10/28).

Learning objectives. Congruence classes. Modular arithmetic.

Reading: 19, 20, 21.

Learning outcomes. A student should be able to
1. define congruence modulo m and prove that this is an equivalence relation
2. define congruence classes
3. define operations of addition and multiplication on congruence classes
3. give definition of a ring
4. prove that $\mathbb{Z}_m$ is a ring
5. use modular arithmetic for solving various divisibility problems and control of calculations
6. define ring homomorphism and prove that the canonical projection $\mathbb{Z} \to \mathbb{Z}_m$ is a ring homomorphism

Week 11 (Tu 11/2, Th 11/4). Review and Midterm 2.

Week 12 (Tu 11/9, Th 11/11).

Learning objectives. Number systems. Peano's axioms. Integers, rational, real and complex numbers as quotient sets. Definitions of equipotent sets and cardinality of a set. Finite and infinite sets. Finite arithmetic. Pigeonhole principle.

Reading: 10, 11, 12.

Learning outcomes. A student should be able to
1. define which sets are called equipotent
2. define the cardinality of a set
3. explain what are the cardinal numbers of the empty set and a singleton
4. explain why natural numbers and integers have the same cardinality
5. define which sets are called finite and infinite
6. formulate and prove the pigeonhole principle
7. state and prove five corollaries for the pigeonhole principle
8. solve problems using the pigeonhole principle
9. establish one-to-one correspondence between natural numbers and cardinalities of finite sets.

Week 13 (Tu 11/16, Th 11/18).

Learning objectives. Denumerable, countable and uncountable sets. Examples of infinite sets of the same and different cardinalities. Hilbert's Grand Hotel. Cantor theorem about non-equipotency of a set and its power set.

Reading: 14.

Learning outcomes. A student should be able to
1. explain which sets are called denumerable, countable and uncountable
2. explain counting principles for finite sets (addition, multiplication, inclusion-exclusion)
3. count the number of permutations of a finite set
4. state and prove absorption theorem for denumerable sets

Week 14 (Tu 11/23, Th 11/24 no classes: Thanksgiving break)

Learning objectives. Denumerable arithmetic. Countable and uncountable sets.

Reading: 14.

Learning outcomes. A student should be able to
1. explain how to define the product of infinite cardinal numbers
2. prove that the set of rational numbers is denumerable
3. state and prove Cantor's theorem about uncountability of $\mathbb{R}$
4. prove that an open interval in uncountable

Week 15 (Tu 11/30, Th 12/2)

Learning objectives. Cantor-Shröder-Bernstein theorem. Ordering of cardinal numbers. Cantor's theorem about uncountability of $\mathbb{R}$. Continuum hypotheses.

Reading: 14.

Learning outcomes. A student should be able to
1. define inequalities for cardinal numbers
2. state the Continuum hypothesis
3. state and prove Cantor's theorem about cardinalities of a set and its power set
4. state Cantor-Shröder-Bernstein theorem and use it for problem solving
5. prove that the power set of natural numbers is uncountable

Final exam is on Tuesday 12/14 at 2:15pm-5:00pm.