MAT 203: Calculus III with applications

Instructor: Fabrizio Donzelli



| LEC 1 | M-F 12:50pm-2:10pm | Hvy Engr Lab 201 | Donzelli |
|-------|--------------------|------------------|----------|
| R01 | W 10:40am-11:35am | Chemistry 128 | Aleyasin |
| R02 | Tu 11:20am-12:15pm | Chemistry 128 | Elson |

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Text

Larson, Edwards, Multivariable Calculus, 9th edition.

Course Content

MAT 203 is course on calculus in dimensions two and three. We will extend the notion of differentiation and integration, learned in the previous calculus courses, to functions of two and three variables and vector valued functions. The course covers most of the contents of the book, time permitting.

Office Hours

Monday, 3-5 pm, in my office (3-102).

Homework

The homework will be assigned during the class, and they will be collected by the teaching assistants during the recitation session. For the homework assignment and the sections covered during the class, please click <u>here</u>. You will have at least a week to complete the homework assignment. Late homework will not be accepted.

Exams

There will be a midterm exam and a final exam. The final exam is scheduled on Wednesday, December 16, 2:15pm-4:45pm; the midterm's date will be announced during the semester, two weeks in advance. The opportunity for making up a missed exam is limited to only emergency cases. If such a case should occur the student must provide legitimate documentation of some form as proof that missing the exam was absolutely necessary. Calculators will NOT be allowed for use on exams.

Practice for Final

Please click here.

Grading

The grading will be weighted as follows: homework 30%, midterm 30%, final 40%. The grades are available on blackboard.

Stony Brook University Syllabus Statement

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services at (631) 632-6748 or <u>http://studentaffairs.stonybrook.edu/dss/</u>. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website: <u>http://www.sunysb.edu/ehs/fire/disabilities.shtml</u>.

Syllabus-Homework

| Lectures | Homework | Due date |
|--------------------------|--|---|
| 11.1 | 11.1 : 28,36,52,56,70 | R01: Sept 9 |
| 11.2,11.3 | 11.2 : 32,36,48,74,82,86 11.3 : 8,16,22,24 | R02: Sept 8 R01: Sept 16 R02: Sept 15 |
| 11.3, 11.4, 11.5 | 11.3 : 36,46,48 11.4 : 8,10,16,28,40,42,49,54,56,58* 11.5 : 4,7,20,22,32,39,43,48,56,62 | R01: Sept 23 R02: Sept 22 |
| 11.5,11.6,11.7,12.1,12.2 | 11.5 : 70,74,94,100,102,108,110,130,134 11.6 : 4,44,48,58,64 11.7 : 10,17,38,107 12.1 : 14,17,20,22,36,67,84 | R01: Sept 30 R02: Sept 29 |
| 12.1,12.2,12.4,12.5 | 12.2 : 6,18,20,29,30,62,68 12.4 : 8,21,22,34,40,54,62,69 12.5 : 6,14,22,23,26,30,36,44,96,101,102,103,104 | Oct 9 (during lecture) |
| 13.1-13.5 | 13.1 : 26,30,56 13.2 : 2,29 13.3 : 64,68,78,88,106 13.5 : 6,84,24,26 | R01: Oct 28 R02: Oct 27 |
| 13.6-13.8 | 13.6 : 8,18,40 13.7 : 8,40,43 13.8 :12,26,54 | R01: Nov 4 R02: Nov 3 |
| 14.1,14.2 | 14.1 :46,65,60,66 14.2 :12,16,20,38,40,46,54 | R01: Nov 11 R02: Nov 10 |
| 14.5,14.6, 14.8 | 14.5 :2,8,12,18,38 14.6 :4,14,18,24,32,44,45 14.8 : 24,28,30,31 | R01: Nov 18 R02: Nov 17 |
| 14.3,14.7,15.1,15.2,15,3 | 14.3 (optional): 32,35,40,60 14.7 (optional): 34 15.1 : 34,40,44,59,62,84,90 15.2 : 10,18,30,38,46 15.3 : 7,8,12,20,26,30 | R01: Nov 23 R02: Nov 24 |
| | | R01: Nov |

| | | R02: Dec 01 |
|---------------------|--|-------------|
| 15.5,15.6,15.7,15.8 | 15.5 :24,28 15.6 : 14,18,24,28 15.7 :2,10,14,19,25,26 15.8 :9,13,18,20 | R01: Dec 09 |
| | | R02: Dec 08 |

FINAL EXAM: PRACTICE TEST

Please justify your answers - full marks are only awarded to answers which show the working-out. Partial credit is available where the correct ideas have been indicated but the actual calculation lets you down. Leave answers in exact form - do not compute any decimals, for example. But try to simplify your answers as much as possible.

No calculators allowed. Good luck!

PRACTICE

(1) Graph the level curves f(x, y) = c at the given value of c:

a.
$$f(x,y) = x^2 + y^2$$
; $c = 0, 1, 2$

b.
$$f(x,y) = xy$$
; $c = -1, 0, 1$.

- (2) Find the equation of the tangent plane to the surface given by the equation $z x^2 + y^2 = 0$ at the point P(1, 0, 1).
- (3) Find the directional derivative of $f(x, y) = \sin(xy)$ at $(1, \pi)$ in the direction of the vector < 1, 2 >.
- (4) Find the equations of the normal line to the surface given by the equation $x^2 + y^2 z^2 = 0$ at the point P(1, 0, 1).
- (5) Find and describe the relative extrema of the function $f(x, y) = x^3 + y^3 3xy.$
- (6) Find absolute maxima and minima of the function $f(x,y) = x^2 + y^2$ on the rectangle $-2 \le x \le 3, -2 \le x \le 5$

(7) Evaluate the following:

(a)

$$\iint_D e^{-x^2 - y^2} dA$$

for D=disc of center (0,0) and radius a.

(b)

$$\iint_D x e^y dA$$

for D=region bounded by the parabolas $y = x^2$ and $y = -x^2 + 2$.

(c)

$$\iint_D x dA$$

for $D = \{0 \le x \le \pi, 0 \le y \le \sin x\}$

(d) Area of the ellipse $4x^2 + 9y^2 = 1$

(e) Volume of the region bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane z = 0.

- (8) Evaluate the integral of the vector field $\mathbf{F} = \langle e^x, y^2 + z^2, \frac{-1}{y^2 + z^2} \rangle$ along the curve $\mathbf{r}(t) = \langle t^2, \cos t, \sin t \rangle$, for $0 \leq t \leq 1$
- (9) (a) Find a potential f for the vector field $\mathbf{F} = \langle ze^{xz}, 1, xe^{xz} \rangle$.

(b) Evaluate the integral of **F** along the curve $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$, for $0 \le t \le \pi$.

PRACTICE

(c) Evaluate the integral of **F** along the segment joining (0,1,0) to $(\pi,-1,0)$.

(10) Show that the following vector fields are not conservative:

(a)
$$\mathbf{F} = \langle y^2, x^2, z + x \rangle$$

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(b)
$$\mathbf{F} = < \frac{y}{x^2 + y^2}, \frac{-x}{x^2 + y^2} >$$

(11) Prove the following identities (for f=scalar function, **F**=vector field):

(a) $div(curl\mathbf{F}) = 0$

(b) $curl(\nabla f) = 0$

(12) Let $\mathbf{F} = \nabla f$. Prove that, for any closed curve C,

$$\int_C \mathbf{F} d\mathbf{r} = 0$$

(13) Let $\mathbf{F} = curl(\mathbf{A})$. Prove that, for any closed oriented surface S,

$$\iint_{S} \mathbf{F} \cdot \mathbf{N} dS = 0$$

PRACTICE

(14) Evaluate the line integral of the vector fields:

(a) $\mathbf{F} = \langle xy, x^2 + y^2 \rangle$ along the boundary of the square with vertices (0,0), (0,2), (2,0), (2,2).

(b)
$$\mathbf{F} = \langle y^2, x^{\frac{4}{3}} \rangle$$
 on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.

- (15) Compute the flux of the vector field $\mathbf{F} = \langle x, y, z \rangle$ across the boundary of the solid region defined by the equations: $x^2 + y^2 + z^2 \leq 4, -1 \leq z \leq 1$
- (16) Let $\mathbf{F} = \frac{-\mathbf{r}}{||\mathbf{r}||^3}$ where $\mathbf{r} = \langle x, y, z \rangle$. Evaluate

$$\int \int_S {\bf F} \cdot {\bf N} dS$$

where S is the sphere centred at the origin of radius 1. I claim that you will get the same answer if you compute

$$\int \int_{S} \mathbf{F} \cdot \mathbf{N} dS$$

where S is the cube of sidelength 29 with centre point at the origin. Why?

(17) Assuming Stoke's Theorem, prove Green's Theorem.

(18) Compute $\int_C \mathbf{F} d\mathbf{r}$, where $\mathbf{F} = \langle 4xz, y, 4xy \rangle$ and C is the boundary (oriented counterclockwise of the surface) $z = 9 - x^2 - y^2$, $z \ge 0$.

(19) Find the surface area of the cone given by

$$z = 10 - \sqrt{x^2 + y^2}, \ 0 \le z \le 10.$$

(20) Verify the divergence theorem by evaluating both sides of the equation

$$\iint_{S} \mathbf{F} \cdot \mathbf{N} dS = \iint_{V} \int_{V} \operatorname{div}(\mathbf{F}) \mathrm{dV}$$

where V is the solid region bounded by the coordinate planes and the plane x + 2y + z = 4, and S is the boundary of V, and

$$\mathbf{F} = x\mathbf{i} + yz\mathbf{j} + z\mathbf{k}.$$

(21) I have a a vector field $\mathbf{F} = \langle F_1, F_2, F_3 \rangle$. I know that:

- (1) $\operatorname{div}(\mathbf{F}) = 0;$
- (2) $F_1 = xy^2 z;$
- (3) $F_2 = z^2;$

.

What is one possibility for the third component of \mathbf{F} ?