**Stony Brook University** Mathematics Department Joseph Helfer Logic, Language and Proof MAT 200, Lec 02 Fall 2024

# Syllabus

**Course description:** The goal of the course is to introduce the student to logical reasoning and proofs. This course serves as an introduction to rigorous mathematics used in upper-division mathematics courses. We discuss logical language and operations, and methods of proof in general. Then we focus on sets and maps between them - the foundational objects of mathematics. Finally, we study cardinality. We apply the rigorous language to systematically define and study some notions of number theory, elementary analysis, and Euclidean geometry. There is considerable focus on mathematical writing.

Credits: 3

Course meeting time and location: Tuesday/Thursday, 2 - 3:20 PM in Physics P130

Instructor: Joseph Helfer

email: joseph.helfer@stonybrook.edu Office hours: see https://www.math.stonybrook.edu/cards/helferjoseph.html

Grader: Brennan Williams

email: brennan.williams@stonybrook.edu

**Textbook:** Peter J. Eccles, An Introduction to Mathematical Reasoning, Cambridge University Press.

**Brightspace.** All course information (besides homework) will be posted to Brightspace. Check Announcements and Content regularly!

**Homework:** will be assigned roughly biweekly through **Gradescope**. The emphasis of the course is on writing proofs, so please try to write legibly and explain your reasoning clearly and fully. You are encouraged to discuss the homework problems with others, but your write-up must be your own work. Suspiciously similar papers won't be graded.

Homework should be submitted to Gradescope according to the Gradescope rules. Incorrect submission format will lead to a grade reduction. If you cannot access Gradescope, please contact the instructor.

Late homework won't be accepted. Homework in the form of e-mail won't be accepted.

**Examination system:** Two Midterms and Final exam. Missing any of the exams without any serious and documented reason will result in failure in the course.

Final exam is on Tuesday, December 17, 2024 at 2:15-5:00pm.

Preliminary dates for the (in class) Midterms: Thursday, September 12 (Midterm 1) and Thursday, November 14 (Midterm 2).

There will also be roughly weekly quizzes in class.

MAT 200 and MAT 250. By the results of Midterm 1, some students will be proposed an option to move up to MAT 250 (Introduction to Advanced Mathematics). This is a 4 credit alternative to MAT 200. It covers the same material, but at an advanced level. The goal of MAT 250 is to prepare for advanced mathematical courses and for the challenges of a graduate or professional school at the finest universities.

#### Grading system:

Homework	10%
Quizzes	10%
Midterm	25%
Midterm	25%
Final	30%

**Make-up policy:** Make-up examinations are given only for work missed due to unforeseen circumstances beyond the student's control.

#### **REQUIRED SYLLABUS STATEMENTS**

#### Student Accessibility Support Center Statement:

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact the Student Accessibility Support Center, Stony Brook Union Suite 107, (631) 632-6748, or at sasc@stonybrook.edu. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

#### Academic Integrity Statement

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty is required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center (School of Health Professions, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty please refer to the academic judiciary website at http://www.stonybrook.edu/commcms/academic\_integrity/index. html

#### **Critical Incident Management**

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Student Conduct and Community Standards any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Faculty in the HSC Schools and the School of Medicine are required to follow their school-specific procedures. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.

## Weekly Plan (tentative)

Note: we will read the chapters of the book in the following order: 1, 2.1, 6.1, 7.1-7.2, 7.6, 2.2-2.3, 3-4, 7.3-7.4, 5, 7.5, 6, 8, 7.7, 9, 19-21, 22, 10-11, 14

## Week 1 (Tu 8/27, Th 8/29).

**Topics:** Propositions and predicates. Logical connectives. Truth tables. Compound propositions. Conditional and biconditional sentences. Converse, contrapositive. Examples from arithmetic. Universal statements.

**Supplementary topics:** Tautologies and contradictions. De Morgan's laws, the law of excluded middle and the law of consistency. Inverse of a conditional statement. Denials. Conjunctive and disjunctive normal form.

**Reading:** Chapter 1 and section 2.1.

## Week 2 (Tu 9/3, Th 9/5).

**Topics:** Basic set notation. Universal and existential quantifier. Negating statements with quantifiers. Interchanging quantifiers.

Supplemental topics: Unique existential quantifier.

**Reading:** Sections 6.1, 7.1, 7.2, and 7.6

Homework: Homework 1 due on Gradescope by Thursday at 11:59 PM.

## Week 3 (Tu 9/10, Th 9/12).

Review and Midterm 1.

#### Week 4 (Tu 9/17, Th 9/19).

**Topics:** Structure of a mathematical theory: basic objects, axioms, definitions and theorems. Direct proofs. Constructing proofs backwards.

Reading: Section 2.2-2.3 and Chapter 3

Homework: Homework 2 due on Gradescope by Thursday at 11:59 PM.

## Week 5 (Tu 9/24, Th 9/26).

**Topics:** Proof by contraposition, proof by contradiction, proof by exhaustion. Proving quantified statements.

Reading: Chapters 4 and Sections 7.3 and 7.4.

## Week 6 (Tu 10/1, Th 10/3). Proof techniques (continued).

Homework: Homework 3 due on Gradescope by Thursday at 11:59 PM.

#### Week 7 (Th 10/10, no classes Tu 10/8).

Topics: Principle of mathematical induction. Definition by induction. Strong induction.

**Reading:** Chapter 5 and section 7.5.

## Week 8 (Tu 10/15, Th 10/17)

**Topics:** Basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement, equality of sets. Ways of specifying sets. The power set. Relations between logical and set-theoretical operations. Set-theoretic identities. Products of sets. Functions: definitions and notations. Composition of functions. Domain, codomain, image of a function. Graphs of functions. Examples of functions: functions in one variable, numerical sequences, identity function, constant function, inclusion function, Restrictions of functions.

Reading: Chapters 6 and 8, and section 7.7.

Homework: Homework 4 due on Gradescope by Thursday at 11:59 PM.

## Week 9 (Tu 10/22, Th 10/24).

Topics: Injections, surjections and bijections. Preimage. Inverses. Peano's axioms.

Reading: Chapter 9.

Homework: Homework 5 due on Gradescope by Thursday at 11:59 PM.

Week 10 (Tu 10/29, Th 10/31).

Review and Midterm 2.

Week 11 (Tu 11/5, Th 11/7).

**Topics:** Congruence modulo m. Modular arithmetic. The remainder map. Linear congruence. Congruence classes. Operations on congruence classes.

Reading: Chapters 19, 20, and 21.

Homework: Homework 6 due on Gradescope by Thursday at 11:59 PM.

Week 12 (Tu 11/12, Th 11/14).

Modular arithmetic (continued).

Week 13 (Tu 11/19, Th 11/21). Topics: Relations. Equivalence relations and partitions. Quotient sets. Constructions of integers and rational numbers.

Reading: Chapter 22.

Homework: Homework 7 due on Gradescope by Thursday at 11:59 PM.

## Week 14 (Tu 11/26, Thanksgiving break on Th 11/28)

**Topics:** Cardinality of finite sets. Infinite sets. Cardinality of disjoint unions and products. Inclusion-exclusion principle. Pigeonhole principle.

**Reading:** Chapters 10, and 11.

Week 15 (Tu 12/3, Th 12/5).

**Topics:** Cardinality of infinite sets. Countable and uncountable set. Cantor's theorem. Cantor-Schröder-Bernstein theorem

Reading: Chapter 14.

Homework: Homework 8 due on Gradescope by Thursday at 11:59 PM.

Final exam is on Tuesday, December 17, 2024 at 2:15-5:00pm.