Stony Brook University
Mathematics Department
Maxim Jeffs

Logic, Language and Proof
MAT 200, Lec 04
Fall 2023

Syllabus

Notice: the most up-to-date course information can be found on Brightspace.

Course description: A basic course in the logic of mathematics, the construction of proofs, and the writing of proofs, aimed to prepare students for proof-oriented courses such MAT 310 and MAT 319. The mathematical content is primarily logic and proofs, set theory, combinatorics, functions and relations. There will be considerable focus on writing logically correct proofs in colloquial English.
Prerequisites: Level 4 on the mathematics placement examination or equivalent course or permission of the instructor.
Credits: 3
Meetings: Tuesdays and Thursdays 10:00-11:20 am, Physics P113

Instructor: Maxim Jeffs
Email: maxim.jeffs@stonybrook.edu
MLC hour: 1-2 pm Tuesdays, see here for the most up-to-date information.
Office hours: 2-3 pm Tuesdays and 12-1 pm Thursdays in SCGP 303, see here for the most up-to-date information.

Grader: Bryan Wong
Email: bryan.wong@stonybrook.edu
MLC hours: W 10:00am-11:00am and Th 10:00am-11:00am (zoom), see here for the most up-to-date information.
Office hour: Tu 10:00am-11:00am (zoom), see here for the most up-to-date information.

Textbook: Peter J. Eccles, *An Introduction to Mathematical Reasoning*, Cambridge University Press.

Homework: will be assigned biweekly through Brightspace. The emphasis of the course is on writing clear and correct proofs, so please try to write legibly and explain your reasoning carefully and completely. Points may be deducted for work that is not presented clearly. You are encouraged to discuss the homework problems with others, but your write-up must be your own work.
Homework should be submitted to Brightspace; homework sent as an email won't be accepted. Late homework will not be accepted, but under documented extenuating circumstances the grade may be dropped.

Quizzes: there will be periodic 5-10 minute quizzes during class throughout the semester. Dates will be announced in advance through Brightspace. Grading will be based on completion.

Examinations: There will be two midterms (taken during class time) and a final exam. Make-up exams will not be given without a serious and documented reason.

Final exam: Thursday December 21, 8:00-10:45 am.
Midterms: (preliminary) Thursday 10/5 (Midterm 1)*, Thursday 11/9 (Midterm 2)

***MAT 250 (Introduction to Advanced Mathematics)**: This is a 4-credit alternative to MAT 200. It covers the same material, but at an advanced level. The goal of MAT 250 is to prepare for advanced mathematical courses and for the challenges of a graduate or professional school. The lectures for MAT 250 are held on Tuesdays and Thursdays at 2:30- 3:50pm. Students who would like to be considered for MAT 250 must take midterm 1 in Julia Viro's lecture on Thursday 9/14 at 2:30pm-3:50pm in ESS 069 and Physics P127. Note that the content covered in Julia Viro's lectures may be somewhat different, so any students wishing to take this midterm should request a copy of the lecture notes.

Grading system:

- Final exam: 40%
- Midterm 1: 15%
- Midterm 2: 25%
- Homework: 15%
- Quizzes: 5%

Student Accessibility Support Center (SASC) statement: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact SASC (631) 632-6748 or at sasc@stonybrook.edu. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.
Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and SASC. For procedures and information go to the website and search Fire Safety and Evacuation and Disabilities.

Academic integrity statement: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website.

Critical incident management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial

Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.

Student Absences Statement: Students are expected to attend every class, report for examinations and submit major graded coursework as scheduled. If a student is unable to attend lecture(s), report for any exams or complete major graded coursework as scheduled due to extenuating circumstances, the student must contact the instructor as soon as possible. Students may be requested to provide documentation to support their absence and/or may be referred to the Student Support Team for assistance. Students will be provided reasonable accommodations for missed exams, assignments or projects due to significant illness, tragedy or other personal emergencies. In the instance of missed lectures or recitations, the student is responsible for review posted slides, recorded lectures, and notes. Please note, all students must follow Stony Brook, local, state and Centers for Disease Control and Prevention (CDC) guidelines to reduce the risk of transmission of COVID.

Schedule (tentative)

- **Week 1:** (Tu 8/29, Th 8/31) Propositional logic, implication.

  Students should be able to:

  - identify propositions and predicates;
  - understand five basic logical connectives (negation, conjunction, disjunction, implication, and equivalence);
  - recognize logical connectives in colloquial expressions: and, but, though, nevertheless, either . . . or, etc.;
  - operate with truth tables, analyze and construct propositional forms involving connectives;
  - determine equivalent propositional forms;
  - define what a tautology and contradiction mean;
  - use logical symbols, formulate and prove logical identities, including tautology and contradiction, de Morgan's laws, the law of excluded middle.

- **Week 2:** (Tu 9/5, Th 9/7) Propositional logic continued

  Students should be able to:

  - recognize conditional and biconditional sentences and use appropriately various colloquial expressions associated with conditionals and biconditionals (sufficient, necessary, sufficient and necessary, whenever, if and only if etc.);

  - understand the difference between implication in mathematics and causation in language/everyday life;
  - construct useful denials of propositional forms;
  - construct the contrapositive, the converse, and the inverse of a conditional statement.

- **Week 3:** (Tu 9/12, Th 9/14) First-order logic, quantifiers

  Students should be able to:

  - understand and use quantifiers (universal, existential, and unique existential) and use different colloquial expressions associated with quantifiers;
  - translate propositions formulated in plain English into symbolic forms and vice-versa;
  - analyze and construct sentences involving several quantifiers;
  - understand when quantifiers commute and when they do not;
  - construct useful denials of propositional forms and quantified sentences;

- **Week 4:** (Tu 9/19, Th 9/21) Logical structure of definitions and theorems

  Students should be able to:

  - understand the logical structure of a definition; in particular, understand mathematical definitions as biconditional sentences;
  - understand the logical structure of a mathematical theorem and distinguish the formulation (statement) of a theorem from its proof;
  - distinguish a definition from a theorem and example using the logical criteria;
  - identify definitions, theorems, and examples in an unknown mathematical text;
  - express known definitions and theorems in appropriate logical forms, both in words and symbols;
  - comprehend the structure of a mathematical theory: identify the basic objects, axioms and theorems;
  - explain the role of proofs in mathematics;
  - distinguishing the formulation (statement) of a theorem and its proof and recognize the difference between motivation and proof;
  - understand the roles of examples and counterexamples;
  - recognize incorrect proofs;
  - work with actual mathematical text (excerpts).

- **Week 5:** (Tu 9/26, Th 9/28) Techniques of proof

  Students should be able to:

- understand, construct and write proofs of different types: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion;
- evaluate pros and contra of different proof techniques;
- make comparative analysis of various proofs of the same fact;
- recognize and avoid typical logical mistakes of affirming the consequent and denying the antecedent.

- **Week 6:** (Tu 10/3, Th 10/5) Proof by induction, **midterm 1**

  Students should be able to:

  - understand, construct and write proofs using the principle of mathematical induction in different forms (induction, strong induction, well-ordering principle);
  - identify situations when a proof by induction is suitable and situations when it is not;
  - conduct proofs by induction of various elementary statements.

- **Week 7:** (Th 10/12, no classes Tu 10/10) Set theory

  Students should be able to:

  - fluently operate with basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement;
  - understand the role of Venn diagrams as illustrations and counterexamples;
  - relate logical and set-theoretical operations, like negation and complement, conjunction and intersection etc.;
  - formulate and prove set-theoretical identities;
  - give definition of the power set and list several properties of the power set.

- **Week 8:** (Tu 10/17, Th 10/19) Functions between sets

  Students should be able to:

  - define a map from one set to another, know synonyms for the word map;
  - understand and fluently use basic terminology associated with maps: domain, codomain, range, image and preimage of a set, graph of a map;
  - provide examples of maps from different parts of mathematics;
  - be familiar with special maps: identity map and constant map;
  - define a composition of maps and list its properties;
  - define inclusion map, restrictions of a map;
  - define and list properties of the characteristic function of a set;

- provide definitions of injections, surjections and bijections and list synonyms for these words;
- provide definitions of inverse map;
- list basic examples of functions and their inverse: exponential and logarithm, tangent and arctangent, etc.;
- state and prove equivalence of invertibility and bijectivity.

- **Week 9:** (Tu 10/24, Th 10/26) Functions between sets, continued

  Students should be able to:

  - know the definition and properties of the Cartesian product of sets;
  - describe coordinate projections, fibers;
  - define graph as a map;
  - define and give examples of a relation from one set to another and a relation on a set
  - discuss properties associated with a binary relation on a set: reflexivity, irreflexivity, symmetry, antisymmetry, transitivity;
  - define strict partial order, non-strict partial order, and linear order;
  - provide basic examples of sets with strict partial order, non-strict partial order, and linear order;
  - state three equivalent formulations of the axiom of choice, and use it for problem solving.

- **Week 10:** (Tu 10/31, Th 11/2) Number systems and quotients

  Students should be able to:

  - define equivalence relation on a set;
  - provide five examples of equivalence relations;
  - define partition of a set and establish connection between equivalence relations and partitions of a set;
  - describe equivalence classes, the quotient set, and the quotient map;
  - use Peano's axioms to define arithmetic operations on the natural numbers;
  - explain the construction of the integers, rational, real and complex numbers as quotient sets;

- **Week 11:** (Tu 11/7, Th 11/9) Modular arithmetic, **midterm 2**

  Students should be able to:

  - define congruence modulo m and prove that this is an equivalence relation;

  - define operations of addition and multiplication on congruence classes;
  - use modular arithmetic for solving various divisibility problems and control of calculations.

- **Week 12:** (Tu 11/14, Th 11/16) Cardinality of infinite sets

  Students should be able to:

  - define the cardinality of a set, explain some simple examples;
  - explain why natural numbers and integers have the same cardinality;
  - define which sets are called finite and infinite;
  - explain which sets are called denumerable, countable and uncountable;
  - understand why $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and their subsets, disjoint unions and direct products are denumerable;
  - state and prove Cantor's theorem about uncountability of $\mathbb{R}$;
  - prove that an open interval in uncountable;
  - state the Continuum hypothesis;
  - state and prove Cantor's theorem about cardinalities of a set and its power set.

- **Week 13:** (Tu 11/21, no classes Th 11/23) Cardinality continued, combinatorics

  Students should be able to:

  - formulate and prove basic facts about arithmetics of finite sets (addition, multiplication, inclusion-exclusion);
  - count the number of permutations of a finite set;
  - formulate and prove the pigeonhole principle and its corollaries;
  - solve problems using the pigeonhole principle;

- **Week 14:** (Tu 11/28, Th 11/30) Non-classical logics

  Students should be able to:

  - describe some of the criticisms of classical propositional logic;
  - give examples of the paradoxes of the material conditional;
  - explain the significance of the law of excluded middle and the principle of explosion;
  - identify examples of constructive and non-constructive proofs;
  - define the interpretation of possibility and necessity in possible-worlds semantics;
  - prove simple logical identities in modal logic;

– critically examine the notion of mathematical truth.

- **Week 15:** (Tu 12/5, Th 12/7) Review
- **Final exam** (Th 12/21)