MAT122 - Introduction to Calculus

Fall 2013

The aim of this course is to introduce you to the basic ideas of calculus without going into the technical details. We will learn how to solve some simple but useful problems, and the course will be mostly about computation.

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Textbook Applied Calculus by Hughes-Hallet, Gleason et al.(4th edition)

Office Hours My office is 3-105 in the Math Tower, and my office hours are Monday 11-12. Alternatively, you can email me and I can meet you some other time. Your TA will also have office hours. You can also visit the Math Learning Center, open weekdays 10-6pm.

Grading There will be two midterm exams, each counting for 20% of your grade, and one final exam worth 40%. The remaining 20% of your grade will be from homeworks, which will be set on Thursdays and collected in during recitation the following week.

The lowest two homework grades will be dropped. That means you can get away with doing no homework at all on two separate weeks before it starts to affect your grade. Hopefully this will cover any missed buses, dying pets, terrible hangovers, and other small problems. If you are worried that you might be falling behind, or if something comes up that might affect your grade, then please come and see me - we can certainly work something out.

If you haven't spoken to me personally, however, then there will be **no late homeworks accepted**. And telling me about your problems *after* the final is no good: once I've submitted the grades to the department, they cannot be changed. So please talk to me in advance.

Syllabus

Week of:
Aug 27: Functions, graphs, and slope *(No homework this week)*
Sep 5: [No class Sep 3] Power functions *(Homework: Section 1.1 #2, 4, 6; Section 1.2 #2, 4, 6, 8, 12, 22, 26)*
Sep 10: Exponential functions; growth and decay; logarithms *(Homework: Section 1.3 #12. 18; Section 1.5 #2, 4, 8, 12; Section 1.6 #2, 8, 10, 32)*
Sep 17: Combining functions; polynomials and their graphs

Midterm: Monday Sep 23, 8:45pm-10:15pm in Javits 102
The practice midterm is avilable here and some solutions here.

Sep 24: Rate of change and the derivative; concavity *(Homework: Section 3.1 # 2, 4, 6, 8, 14, 20, 24, 26, 40, 42)*
Oct 1: Chain rule, product rule, and quotient rule *(Homework: Section 3.3 # 2, 6, 8, 14, 18; Section 3.4 # 2, 14, 16, 18, 28)*
Oct 8: Optimization problems *(Homework: Section 4.1 # 2, 4, 8, 16, 20, 22, 32)*
Oct 15: Local and global extrema *(Homework: here)<*
Oct 22: Introduction to integration; review *(No homework will be collected, but here are some practice problems and solutions.)*
Oct 29: Review

Midterm: Tuesday Oct 29, 8:45pm-10:15pm in LIGHT ENGINEERING 102

Oct 31: Introduction to integration
Nov 5: Integrals; position, speed and acceleration *(Homework here.)*
Nov 12: Area and definite integrals *(Homework here.)*
Nov 19: Integration by substitution *(Homework here)*
Nov 26: [No class Nov 28] Applications
Practice problems (*not* graded) are available here and solutions here.
Dec 3: Review

Final exam: Wednesday Dec 11, 11:15am-1:45pm in ESS 01

MAT122 - Sample Midterm 1

1 The function f is given by the formula

$$f(x) = x^2 + 3^x - 7$$

- What is $f(0)$?
- What is $f(2)$?

2 A function g has $g(2) = 4$ and $g(7) = 19$. Its graph is a straight line.

- What is the average rate of change of g between these two points?
- What is the equation of g?
- Draw the graph of g.

3

- Draw the graph of the function x^3.
- Use this to draw the graph of the function $(x-2)^3 - 1$

4 The population of a town is monitored. We believe that the population after t years can be approximated by

$$p(t) = 500 \cdot (1.04)^t$$

- What was the population of the town at the start of the experiment?
- What is the percentage growth rate?
- According to this model, how many years will it take before the town has 700 people? Give your answer to the nearest whole year.

5 Suppose we have a function

$$p(t) = a^t$$

where a is some fixed number.

- Suppose first that a is a number bigger than 1. Describe the behavior of the function as t gets very large, and draw a graph.
- Now do the same supposing that a is a number between 0 and 1.

6 Consider the function

$$h(x) = 2x^2 + 3x - 7$$

- Use the quadratic formula to find the solutions to $h(x) = 0$. *[Give an exact answer in terms of square roots, without rounding decimals]*
- Use these solutions to draw the graph of h. *[You will probably want to put your solutions into a calculator, so you know roughly what the numbers are]*

7 Write the function $f(x) = \sqrt{3 - x^3}$ as the composition of two other functions. That is, write $f(x) = g(h(x))$ for two functions g and h that you should determine.

MAT122 - Sample Midterm 1 Solutions

1

- $f(0) = -6$
- $f(2) = 6$

2

- 3
- $g(x) = 3x - 2$

3 The graph of $(x-2)^3 - 1$ should be the graph of x^3 shifted right by two units and down by one.

4

- 500 people
- 4% per year
- 9 years

5 The first graph should grow exponentially, and the second should decay.

6 $\frac{-3\pm\sqrt{65}}{4}$

7 One possible solution is to take $h(x) = x^3$ and $g(x) = \sqrt{3-x}$

MAT122 - Homework

October 17th

1 Differentiate the following functions:

- $6^{(4x+4)}$
- $\frac{4+5x}{8x-3}$
- $\log_5(x^3+x)$

2 Alice and Bob are standing in a field. Alice is 10 yards North of Bob. Alice begins to walk East in a straight line, and every second she travels 2 yards. Bob begins to walk North-East; every second he travels one yard North and one yard East. Find the time at which Alice and Bob are closest, and determine what the minimum distance between them.

3 Find the critical points of the function $(x^2 - 7x + 13)2^x$. Use the second derivative test to decide whether each critical point is a minimum or a maximum. Draw the graph of this function.

4 I am building the four walls of a rectangular barn. I can choose the length of the sides, and I want the area inside to be as large as possible. The North wall has to be made of a special type of brick, and costs \$200 per yard; the other three walls cost only \$100 per yard. If I have \$15,000 to spend on the construction of the walls, what is the largest area that I can enclose?

5 The population of a town after t years is given by

$$P(t) = 9000\left(\frac{3}{4}\right)^{t/5}$$

After three years, is the populaton increasing or decreasing? At what rate?

MAT122 - Midterm 2 Practice Questions

October 22, 2013

1 Differentiate the following functions:

- $42x^{50} - 17x^3 + 80 + \sqrt{x}$
- $\frac{1}{x^3} + \frac{1}{x}$
- $30^{(3x)}$
- $\sqrt{\log_e x}$
- $\frac{x^2+6x+9}{x^2-9}$
- $\log_4(x^4) + \log_9(1 + \log_5(x))$

2 Two particles move on a straight line. Their positions are measured from a fixed point on the line, in nanometers. The position of particle A after t seconds is given by the formula $10 + 3t$. The position of particle B is given by $15 + t^2$.

Write down an expression for the distance between the two particles. How far apart are they at the start of the experiment?

A chemical reaction occurs if the particles come within 3 nanometers of each other. Do you expect this reaction to occur?

3 For each of the following functions, find the local maximums and minimums. Use the second derivative to determine whether each point is maximum or minimum. Give the global maximum and global minimum, if they exist.

- $2x^3 - 3x^2 - 12x - 14$
- $x^3 + x^2 - x - 1$
- $(2x + 2)e^{(-x^2)}$
- $x^2 - 6x + 3$ when x is between 0 and 5
- $x \log_e(x)$ with $\frac{1}{10} \leq x \leq 1$

4 What is the greatest possible area of a right-angled triangle whose hypotenuse is 10cm long?

5 I throw a ball in the air. Its height after t seconds is given by $18t - t^2$. How high does it go?

6 Let f be the function given by

$$f(x) = x^3 - 3x^2 - 4x + 12$$

Compute the first and second derivatives of this function. Determine the critical points of f, the regions on which f is increasing or decreasing, and the regions on which f is concave up or concave down. Use all of this information to draw the graph of f.

MAT122 - Solutions to Practice Questions

1

- $2100x^{49} - 51x^2 + \frac{1}{2}x^{(-1/2)}$
- $-3x^{-4} - x^{-2}$
- $(3\log_e 30)30^{3x}$
- $\frac{(\log_e x)^{-1/2}}{2x}$
- $\frac{(x^2-9)(2x+6)-(x^2+6+9)(2x)}{(x^2-9)^2}$
- $\frac{4}{(\log_e 4)x} + \frac{1}{(\log_e 9)(\log_e 5)(1+\log_e x)x}$

2 The distance between them is given by $t^2 - 3t + 5$; at $t = 0$, they are 5nm apart. Their closest approach is $2\frac{3}{4}$nm, at $t = 1\frac{1}{2}$.

3 For each of the following functions, find the local maximums and minimums. Use the second derivative to determine whether each point is maximum or minimum. Give the global maximum and global minimum, if they exist.

- Local maximum at $x = -1$. Local minimum at $x = 2$. No global maximum or minimum.
- Local maximum at $x = -1$. Local minimum at $x = \frac{1}{3}$. No global maximum or minimum.
- Global minimum at $x = \frac{-1-\sqrt{3}}{2}$. Global maximum at $x = \frac{-1+\sqrt{3}}{2}$.
- Global minimum at $x = 3$. Global maximum at $x = 0$, and another local maximum at $x = 5$.
- Global minimum at $x = \frac{1}{e}$. Global maximum at $x = 1$, and another local maximum at $x = \frac{1}{10}$.

4 25cm^2

5 81m

6 The first derivative is $3x^2 - 6x - 4$, and the second derivative is $6x - 6$. The critical points are at $x = 1 \pm \frac{\sqrt{21}}{3}$. The function is decreasing between the two critical points, and increasing outside. It is concave down for $x < 1$ and concave up for $x > 1$.

MAT122 - Homework

November 7, 2013

1 For each of the following functions, we are given its derivative and its value at a single point. Use this information to find the function.

- $\frac{df}{dx} = -3 \qquad f(0) = 6$
- $\frac{df}{dx} = x^2 + 2 \qquad f(0) = 1$
- $\frac{df}{dx} = x^2 + 2 \qquad f(0) = 2$
- $\frac{df}{dx} = x^3 + 2x + 1 \qquad f(1) = 4$
- $\frac{df}{dx} = -10x - 9 \qquad f(3) = -20$
- $\frac{df}{dx} = \sqrt{x} \qquad f(4) = 5$
- $\frac{df}{dx} = x^{-2} \qquad f(5) = 0$
- $\frac{df}{dx} = \frac{1}{x^3} + \frac{1}{x^4} \qquad f(1) = -1$

2 In each of the following cases, use the two equations given to work out the values of x and y

- $x + y = 2 \qquad x - y = 0$
- $2x + 3y = 8 \qquad 2x + 2y = 6$
- $x + y = 5 \qquad 2x + 5y = 16$
- $x - 2y = -3 \qquad 4x + 3y = -1$
- $-3x + 2y = 11 \qquad x - 5y = 5$

MAT122 - Homework

November 14, 2013

1 Let $f(x)$ be the function $x^3 + x^{-2}$. Let $A(x)$ be the area under the graph of f between 1 and x. Find a formula for $A(x)$.

2 Let $g(x)$ be the function $4x^2 - 17x + e^x$.

- Find a formula for the area under g measured from a starting point of 4.
- Now find a formula for the area under g from a starting point of 5.

3 Compute the following definite integrals.

- $\int_1^3 x^3 dx$
- $\int_0^5 -3x^2 - 4dx$
- $\int_5^9 \sqrt{x}dx$
- $\int_{-4}^{-1} -2x^{-2}dx$
- $\int_{-1}^{1} x^2 - 4dx$

MAT122 - Homework

November 22, 2013

1 Use a substitution to compute the following integrals:

$$\int (4x + 10)^9 \text{ dx}$$

$$\int e^{-x} \text{ dx}$$

$$\int \sqrt{(2 - 3x)} \text{ dx}$$

$$\int (5x)^{1/3} + (5x - 1)^{2/3} \text{ dx}$$

$$\int (4x - 10)^{-1} \text{ dx}$$

2 a) Use the substitution $u = 3x - 3$ to compute the following integral:

$$\int (3x - 3)^2 \text{ dx}$$

b) Now compute the integral again, this time by expending out the bracket $(3x - 3)^2$. Check that your two answers agree.

3 Find the area between the curves $y = x^2$ and $y = 3x^3$, between $x = 1$ and $x = 2$.

4 Find the area between the curves $y = x^{-1}$ and $y = e^x$, between $x = 5$ and $x = 8$.

5 Find the area between the curves $y = x^{-2}$ and $y = 7x^2 - 4$, between $x = 2$ and $x = 3$.

MAT122 - Practice Problems

December 1, 2013

1 Compute the following indefinite integrals

1. $\int x^2$ dx
2. $\int 3x$ dx
3. $\int 7x^5 - 9$ dx
4. $\int \frac{1}{3}x^3$ dx
5. $\int x + x^2 - x^3$ dx
6. $\int x^{-4} - 6x^{-2} + 1$ dx
7. $\int \frac{1}{2}x^{-3} - 2x + 3$ dx
8. $\int \sqrt{x} - x^{1/3}$ dx
9. $\int x^{-1}$ dx
10. $\int e^x - 14x^4$ dx
11. $\int x^{7/3} + x^e + 2$ dx
12. $\int -x^{-10} + 70x + 70$ dx
13. $\int -29e^x + x^7 + x^{-7} + 2$ dx

2 For each of the following functions, use the information given about the derivative to determine the function exactly.

1. $\frac{df}{dx} = x^2 \qquad f(0) = 1$
2. $\frac{df}{dx} = x^3 + 1 \qquad f(0) = 2$
3. $\frac{df}{dx} = x^3 + 1 \qquad f(1) = 9$
4. $\frac{df}{dx} = \sqrt{x} \qquad f(4) = 1$
5. $\frac{df}{dx} = x^{-1} \qquad f(1) = 0$
6. $\frac{df}{dx} = e^x - x + 1 \qquad f(0) = 0$
7. $\frac{d^2f}{dx^2} = x \qquad f(0) = 1 \qquad \frac{df}{dx}(0) = 3$
8. $\frac{d^2f}{dx^2} = x \qquad f(1) = 1 \qquad f(2) = 2$
9. $\frac{d^2f}{dx^2} = e^x + \sqrt{x} \qquad f(1) = 1 \qquad f(2) = 2$
10. $\frac{d^2f}{dx^2} = x^{-2} \qquad f(-1) = 1 \qquad f(2) = 2$
11. $\frac{d^2f}{dx^2} = x^{-1} \qquad f(1) = 1 \qquad f(2) = 2$

3 Use integration by substitution to compute the antiderivatives of these functions.

1. $\int (3x+7)^2$ dx
2. $\int e^{(2-x)}$ dx
3. $\int (x+1)^3 - (x+1)^4$ dx
4. $\int (7-2x)^{-1}$ dx
5. $\int (\frac{1}{5}x - 10)^4$ dx
6. $\int e^{-x}$ dx
7. $\int e^{(-2x-7)}$ dx
8. $\int \sqrt{-4x+1}$ dx
9. $\int (2x+3)^3 + \sqrt{-x-1}$ dx

   [Hint: Integrate the two halves separately]
10. $\int e^{2x} + e^{-2x}$ dx
11. $\int \frac{-3}{(-3x-1)^2}$ dx
12. $\int \sqrt{17x}$ dx
13. $\int -9e^{(x/4)} + 2\sqrt{x/3}$ dx

4 Compute the area under each of the following graphs

1. The area under $e^{(2x)}$ between $x = 0$ and $x = 11$
2. The area under x^{-1} between $x = 1$ and $x = 2$
3. The area under $4x^2 - 1$ between $x = -2$ and $x = -1$
4. The area under $(4-3x)^7$ between $x = 0$ and $x = 3$
5. The area under $\frac{5}{(5x+5)}$ between $x = 5$ and $x = 6$
6. $\int_{-4}^{4} x$ dx
7. $\int_{-4}^{4} x^2$ dx
8. $\int_{-1}^{6} 2x + 2$ dx
9. $\int_{0}^{100} 1 - x$ dx
10. $\int_{1}^{3} e^x - e^{3x}$ dx

5 In each of the following cases, find the area between the curves $f(x)$ and $g(x)$ that lies between the two given x values.

1. $f(x) = x \qquad g(x) = -2 \qquad 0 \le x \le 3$
2. $f(x) = x \qquad g(x) = -x \qquad 0 \le x \le 3$
3. $f(x) = x^2 + 1 \qquad g(x) = 0 \qquad -1 \le x \le 2$
4. $f(x) = e^x \qquad g(x) = e^{-x} \qquad 0 \le x \le 4$
5. $f(x) = \frac{1}{(x+1)} \qquad g(x) = -x \qquad 0 \le x \le 5$
6. $f(x) = x^3 + x^2 \qquad g(x) = 2x + 1 \qquad 2 \le x \le 5$
7. $f(x) = 10 \qquad g(x) = x^4 + 3x - 1 \qquad 0 \le x \le 1$
8. $f(x) = e^{(-5x)} + x^2 \qquad g(x) = 3 - 2x \qquad 1 \le x \le 10$

6 A company earns money at a rate of 1000 dollars per day. If they start with \$3500, how much money do they have after 20 days?

Suppose instead that the company's income after t days is $3t^2 + 200$. If they start with \$3500, how much money do they have after 20 days?

7 I drop a ball from the top of a cliff. The acceleration due to gravity is a constant 10 ms^{-2}. If the ball takes 7 seconds to hit the ground, how high is the cliff? *[Hint: Find a formula for the speed of the ball after t seconds, and use this to find a formula for the distance that the ball has traveled after t seconds. Use this to find out how far it travels in 7 seconds; this distance must be the height of the cliff.]*

8 Suppose I am on the moon, where gravity is $\frac{3}{2}$ ms^{-2}. If I throw a ball in the air from ground level at an initial speed of 4, how long does it take to come back down?

Now suppose I throw the ball up and it takes 10 seconds to come back down. How fast did I initially throw it?

9 A tank of water is filling up. The rate at which water is being added after t seconds is $t + \frac{1}{t+1}$. If the tank begins empty, find a formula for the amount of water in the tank after t seconds.

MAT122 - Practice Problems

December 7, 2013

1

1. $\frac{1}{3}x^3 + c$
2. $\frac{3}{2}x^2 + c$
3. $\frac{7}{6}x^6 - 9x + c$
4. $\frac{1}{12}x^4 + c$
5. $\frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + c$
6. $-\frac{1}{3}x^{-3} + 6x^{-1} + x + c$
7. $-\frac{1}{4}x^{-2} - x^2 + 3x + c$
8. $\frac{2}{3}x^{3/2} - \frac{3}{4}x^{4/3} + c$
9. $\log_e x + c$
10. $e^x - \frac{14}{5}x^5 + c$
11. $\frac{3}{10}x^{10/3} + \frac{1}{e+1}x^{e+1} + 2x + c$
12. $\frac{1}{9}x^{-9} + 35x^2 + 70x + c$
13. $-29e^x + \frac{1}{8}x^8 - \frac{1}{6}x^{-6} + 2x + c$

2

1. $f(x) = \frac{1}{3}x^3 + 1$
2. $f(x) = \frac{1}{4}x^4 + x + 2$
3. $f(x) = \frac{1}{4}x^4 + x + \frac{31}{4}$
4. $f(x) = \frac{2}{3}x^{3/2} - \frac{13}{3}$
5. $f(x) = \log_e x$
6. $f(x) = e^x - \frac{1}{2}x^2 + x - 1$
7. $f(x) = \frac{1}{6}x^3 + 3x + 1$
8. $f(x) = \frac{1}{6}x^3 - \frac{1}{6}x + 1$
9. $f(x) = e^x + \frac{4}{15}x^{5/2} + cx + d$
   $c = 2 + e - e^2 - \frac{11}{15} - \frac{16}{15}\sqrt{2})$
   $d = \frac{11}{15} - e - c$
10. (Ignore questions 10 and 11 - they were miscopied. Apologies!)

3

1. $\frac{1}{9}(3x+7)^3 + c$
2. $-e^{(2-x)} + c$
3. $\frac{1}{4}(x+1)^4 - \frac{1}{5}(x+1)^5 + c$
4. $-\frac{1}{2}\log_e(7-2x) + c$
5. $(\frac{1}{5}x - 10)^5 + c$

6. $-e^{-x} + c$
7. $-\frac{1}{2}e^{(-2x-7)} + c$
8. $-\frac{1}{6}(-4x+1)^{3/2} + c$
9. $\frac{1}{8}(2x+3)^4 - \frac{2}{3}(-x-1)^{3/2} + c$
10. $\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + c$
11. $\frac{1}{3x+1} + c$
12. $\frac{2}{51}(17x)^{3/2} + c$
13. $-36e^{(x/4)} + 4(x/3)^{3/2} + c$

4

1. $\frac{1}{2}e^{22} - \frac{1}{2}$
2. $\log_e(2)$
3. $\frac{25}{3}$
4. $\frac{108363}{8}$
5. $\log_e(\frac{7}{6})$
6. 0
7. $\frac{128}{3}$
8. 49
9. -4900
10. $-e + \frac{4}{3}e^3 - \frac{1}{3}e^9$

5

1. $\frac{21}{2}$
2. 9
3. 6
4. $e^4 + e^{-4} - 2$
5. $\frac{25}{2} + \log_e(6)$
6. $\frac{669}{4}$
7. $\frac{93}{10}$
8. $405 + \frac{1}{5}e^{-5} - \frac{1}{5}e^{50}$

6 \$23500
\$15500

7 245m

8 $5\frac{1}{3}$ seconds
$\frac{15}{2}$m/s

9 $\frac{1}{2}t^2 + \log_e(1+t)$