MAP-103: Proficiency Algebra (Summer-II 2018)

location: MoWeTh 6:00pm-8:15pm at Melville Library E-4310
office hour: Mo 4:30pm-5:30pm at Math Tower S-240A
email: jin-cheng.guu@stonybrook.edu
instructor: Jin-Cheng Guu

<- teaching / general advice / course information / schedule and assignments / download links

General Advice

While quizzes may be tough, they are useful exercises that help make sure if we really learn/understand the subject. On one hand, lets take it easy because there will be more quizzes and failing a single one affects little on your final grade. On the other hand, however, please take it serious because wer need to build fluency and the foundation to move on. Here, let me quote Thurston's word:

> One feature of mathematics which requires special care in education is its height, that is, the extent to which concepts build on previous concepts. Reasoning in mathematics can be very clear and certain, and, once a principle is established, it can be relied on. This means it is possible to build conceptual structures which are at once very tall, very reliable, and extremely powerful.
>
> The structure is not like a tree, but **more like a scaffolding**, with many interconnected supports. Once the scaffolding is solidly in place, it is not hard to build it higher, but it is impossible to build a layer before previous layers are in place.

Course Information

While you can find the full course information in the course syllabus , here are some critical points:
No curving, no make-ups.
Calculators are not permitted during the quizzes, and the students are encouraged not to rely on calculators for homework.
Homework will not be counted into your final grade. However, the quizzes will be counted, and whose content will be very similar to the homework.
Quizzes will be given in each class (15 quizzes totally); each quiz has two parts. The first part (75%) will be similar to the homework given last time, and the second part (25%) (open book) will be similar to the content of that class.
Textbooks are not required. Please refer to the resources in the download links below.
Distribution of Grades
10% Midterm (I)

10% Midterm (II)
10% Final
10% Participation
60% Quizzes

Letter Grades

[00, 55): F
[55, 60): C- [60, 65): C [65, 70): C+
[70, 75): B- [75, 80): B [80, 85): B+
[85, 90): A- [90, 100]: A

Schedule of a lecture day

6:00 - 6:30pm:	Quiz	(Part I: review)
6:30 - 6:35pm:	Rest	
6:35 - 7:05pm:	Lecture	
7:05 - 7:10pm:	Rest	
7:10 - 7:40pm:	Lecture	
7:40 - 7:45pm:	Rest	
7:45 - 8:15pm:	Quiz	(Part II: openbook)

Schedule and Assignments

The following is a tentative schedule for the course.

Week	Date	Topic(s) Covered	Reading	Homework
1	7/9 Mo	Numbers, Operations, Numerical Expressions Variables, and Algebraic Expressions	Lecture 1, 2, 3	HW 1, 2, 3
	7/11 We	Addition, Multiplication, Subtraction, and Division	Lecture 4, 5	HW 5: 9 - 14, Review quiz 2-2
	7/12 Th	Distributivity Powers	Lecture 6, 7	Review quiz 3-2
2	7/16 Mo	Powers Rules Polynomials	Lecture 8, 9	Review quiz 4-2
	7/18 We	Operations with Polynomials Rational Expressions	Lecture 10, 11	Review quiz 5-2
	7/19 Th	Operations with Rational Expressions Composing Algebraic Expressions	Lecture 12, 13	Prepare for midterm I
3	7/23 Mo	Midterm I (Lecture 1 - 13)		Review quiz 6-2
	7/25 We	Equalities, Identities, and Equations Linear Equations and applications	Lecture 14, 15, 16	Review quiz 7-2
	7/26 Th	Linear Inqualities Absolute Value	Lecture 17, 18	Review quiz 8-2

4	7/30 Mo	Lines on a Plane	Lecture 19, 20	Review quiz 9-2
	8/1 We	Linear Systems	Lecture 21, 22, 23	Review quiz 10-2
	8/2 Th	Radicals (as Powers with Rational Exponents)	Lecture 24, 25	Prepare for midterm II
5	8/6 Mo	Midterm II (Lecture 14 - 25)		Review quiz 11-2
	8/8 We	Quadratic Equations, Equations Reducible to Quadratics	Lecture 26, 27, 28, 29	Review quiz 12-2
	8/9 Th	Parabolas	Lecture 30	Review quiz 13-2
6	8/13 Mo	Quadratic Inequalities	Lecture 31	Prepare for the final
	8/15 We	Final (Lecture 26 - 31)		

Download Links

Quiz and solutions
Reading and Lecture Note
Resources from an older course:
Videos / Homework / Solutions

MAP-103: Proficiency Algebra

Instructor: Jin-Cheng Guu (not a Professor)

Summer-II, Jul.09 - Aug.18, 2018

Office Hours: Check Online	Classroom: Melville Library E4310
Office: Math Tower - S240A	Course Web: Instructor's Webpage
E-mail: jin-cheng.guu@stonybrook.edu	Class Hours: Mo/We/Th 6:00-8:15pm

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Course Description

The goal of the course is to build an algebraic foundation for pre-calculus/calculus study. We will discuss basic number operations, exponents, polynomials, radicals, and rational expressions. We will learn how to solve linear and quadratic equations, draw graphs of linear and quadratic functions, solve linear systems in two variables, solve linear and quadratic inequalities.

Note: This course is not for credit and does not count towards one's cumulative GPA, but the grade does appear on one's transcript, counts towards the semester GPA, and counts towards credit enrollment. It is necessary to pass this course with a grade of C or better to move onto MAT 118, 122, 123 or AMS 101 (you may also enter AMS 101 with a 2+ on the placement exam, but admittance into other courses mentioned requires a 3 or a passing grade in MAP 103). This course does NOT satisfy the DEC C requirement but does satisfy the S1 skills requirement.

Required Materials

- No textbooks are required.
- A pencil, an eraser, and some neat paper.
- Course lecture notes are available on the course webpage.

Prerequisites

- Level 2 on the mathematics placement examination or MAP-101.
- Skills of keeping your written work clean, neat, and organized.

Course Structure

Among the 18 days of class, there will be two midterms and a final. Also, quizzes will be given on every other lecture day, and each quiz has two parts. The first part will be similar to the homework given last time, and the second part (open book) will be similar to the content of that class. The following is the schedule of a lecture day.

- 6:00-6:30pm: Quiz (Part I: review)
- 6:30-6:35pm: Rest
- 6:35-7:05pm: Lecture
- 7:05-7:10pm: Rest
- 7:10-7:40pm: Lecture
- 7:40-7:45pm: Rest
- 7:45-8:15pm: Quiz (Part II: openbook)

Schedule and weekly learning goals

Check the course webpage.

Course Policies

No calculators

Calculators will not be permitted during the quizzes, and the students are encouraged not to rely on calculators for homework / in class.

Grading Policy

- 10% Midterm (I)
- 10% Midterm (II)
- 10% Final
- 10% Class Performance
- 60% Quizzes (equally distributed to the 15 quizzes)
- Letter grade: [0,55) - F;
  [55,60) - C-, [60,65) - C, [65,70) - C+;
  [70,75) - B-, [75,80) - B, [80,85) - B+;
  [85,90) - A-, [90,100] - A.
- No curving, no make-ups.

Math Learning Center (MLC)

The Math Learning Center is a place where you can get free tutoring help with any of your math concerns. No appointment is required, just come in and ask for help. The MLC is located in the basement of the Math Tower. Check the website: www.math.sunysb.edu/MLC/index.html.

Homework Assignments

Homework will be assigned each day, and the solutions are provided online. Notice that the homework will not be counted in your final grade, but it will be very similar to the quizzes. Please come to the office hour for help.

Attendance Policy

The instructor will never judge a student by her/his/their attendance. So please feel free to walk out the classroom for a slight rest if it helps. Your classmates have their right to learn, so please do not bother them.

Academic integrity statement

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at www.stonybrook.edu/uaa/academicjudiciary

Disability support services (DSS) statement

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services (631) 6326748 or http://studentaffairs.stonybrook.edu/dss/. They will de- termine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website: www.stonybrook.edu/ehs/fire/disabilities/asp.

Critical incident management

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

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Solution to Quiz 1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/09 7:45-8:45pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select the answer that best completes the given statement.

The ______ are $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$:
2. integers

Select the answer that best completes the given statement.

The number $\sqrt{5}$ is a(n) ______
3. irrational number

Select the answer that best completes the given statement.

The number $\frac{5}{7}$ is a(n) ______
2. rational number

List the elements in the set $\{x|x$ is a natural number less than $2\}$.

(Ignore this question.)

Subtract

$$11 - 13 = -2$$

Subtract (simplify your answer)

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \frac{9}{6} = \frac{3}{2}$$

Simplify the expression.

$$-14\left(-\frac{2}{7}\right) - 14 = -10$$

Simplify the expression.

$$4 - [\,(7-6) + (9-19)\,] = 13$$

Simplify the expression.

$$4\{-5 + 3[3 - 5(-3+1)]\} = 4 \times 34 = 136$$

Evaluate the expression when $x = 5$ and $y = -6$.

$$5x - 3y = 5 \times 5 - 3 \times (-6) = 25 + 18 = 43$$

Quiz 1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/09 7:45-8:45pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select the answer that best completes the given statement.

The ______ are $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$:
1. rational numbers
2. integers
3. natural numbers
4. irrational numbers

Select the answer that best completes the given statement.

The number $\sqrt{5}$ is a(n) ______
1. natural number
2. rational number
3. irrational number
4. whole number

Select the answer that best completes the given statement.

The number $\frac{5}{7}$ is a(n) ______
1. natural number
2. rational number
3. irrational number
4. whole number

List the elements in the set $\{x | x$ is a natural number less than $2\}$.

(Use a comma to separate answers as needed.)

$$\{\underline{\hspace{8cm}}\}$$

Subtract

$$11 - 13 = \_\_\_\_\_\_$$

Subtract (simplify your answer)

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \_\_\_\_\_\_$$

Simplify the expression.

$$-14\left(-\frac{2}{7}\right) - 14 = \_\_\_\_\_\_$$

Simplify the expression.

$$4 - [\,(7-6) + (9-19)\,] = \_\_\_\_\_\_$$

Simplify the expression.

$$4\{-5 + 3[3 - 5(-3+1)]\} = \_\_\_\_\_\_$$

Evaluate the expression when $x = 5$ and $y = -6$.

$$5x - 3y = \_\_\_\_\_\_\_\_$$

Solution to Quiz 2-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/11 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Select the answer that best completes the given statement.

The number $\sqrt{4}$ is a(n) ______
2. rational number

Select the answer that best completes the given statement.

The number $\frac{4}{7}$ is a(n) ______
2. rational number

Product

$$(-4)(-1)(-8) = (-1)(-1)(-1)\cdot 4\cdot 8 = -32$$

Divide

$$\frac{-10}{-5} = \frac{(-1)\cdot 10}{(-1)\cdot 5} = \frac{10}{5} = 2$$

Subtract (simplify your answer)

$$-\frac{4}{5} - \left(-\frac{7}{15}\right) = -\frac{4}{5} + (-1)\left((-1)\frac{7}{15}\right) = -\frac{12}{15} + \left(\frac{7}{15}\right) = \frac{-5}{15} = \frac{-1}{3}$$

Subtract (simplify your answer)

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \frac{7}{6} + (-1)\left((-1)\frac{1}{3}\right) = \frac{7}{6} + \left(\frac{1}{3}\right) = \frac{7}{6} + \left(\frac{2}{6}\right) = \frac{9}{6} = \frac{3}{2}$$

Simplify the expression.

$$-14\left(-\frac{2}{7}\right) - 14 = (-1)(-1)14\cdot\frac{2}{7} - 14 = 2\cdot 2 - 14 = -10$$

Simplify the expression.

$$4\{-5+3[3-5(-3+1)]\} = 4\{-5+3[3+(-1)\cdot 5\cdot(-2)]\} = 4\{-5+3[3+10]\} = 4(-5+3\cdot 13) = 4\cdot(-34) = -136$$

Simplify the expression.

$$\frac{\frac{1}{2}\cdot 4 - 7}{5+\frac{1}{3}\cdot 9} = \frac{2-7}{5+3} = \frac{-5}{8}$$

Evaluate the expression when $x = 25$ and $y = -6$.

$$\frac{\sqrt{x}}{y} - \frac{y}{x} = \frac{\sqrt{25}}{-6} - \frac{(-1)\cdot 6}{25} = \frac{5}{-6} + \frac{6}{25} = \frac{5\cdot 25}{-6\cdot 25} + \frac{6\cdot 6}{25\cdot 6} = \frac{-125}{150} + \frac{36}{150} = \frac{-89}{150}$$

Quiz 2-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/11 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select the answer that best completes the given statement.

The number $\sqrt{4}$ is a(n) ______
1. negative number
2. rational number
3. irrational number

Select the answer that best completes the given statement.

The number $\frac{4}{7}$ is a(n) ______
1. natural number
2. rational number
3. irrational number
4. whole number

Product

$$(-4)(-1)(-8) = \underline{\qquad}$$

Divide

$$\frac{-10}{-5} = \underline{\qquad}$$

Subtract (simplify your answer)

$$-\frac{4}{5} - \left(-\frac{7}{15}\right) = \underline{\qquad}$$

Subtract (simplify your answer)

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \underline{\qquad}$$

Simplify the expression.

$$-14\left(-\frac{2}{7}\right) - 14 = \underline{\qquad}$$

Simplify the expression.

$$4\{-5 + 3[3 - 5(-3 + 1)]\} = \underline{\qquad}$$

Simplify the expression.

$$\frac{\frac{1}{2} \cdot 4 - 7}{5 + \frac{1}{3} \cdot 9} = \underline{\qquad}$$

Evaluate the expression when $x = 25$ and $y = -6$.

$$\frac{\sqrt{x}}{y} - \frac{y}{x} = \underline{\qquad}$$

Solution to Quiz 2-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/11 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Choose the fraction(s) equivalent to $\frac{1}{5}$ (select all that apply).

b. $\frac{1}{5}$
d. $\frac{-1}{-5}$

Choose the fraction(s) equivalent to $\frac{8}{-(p+r)}$ (select all that apply).

a. $-\frac{8}{p+r}$ c. $\frac{-8}{p+r}$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

d. $-\frac{8r}{-9s}$

Add

$$1+2+3+\cdots+50=\frac{(1+50)\cdot 50}{2}=51\cdot 25=1275$$

Multiply

$$4\cdot 53\cdot 25=4\cdot 25\cdot 53=100\cdot 53=5300$$

Find the reciprocal of π

$$\pi^{-1}=\frac{1}{\pi}$$

Give an example to establish why subtraction is not commutative.

$$1-2\neq 2-1$$

Give an example to establish why subtraction is not associative.

$$(1-1)-1\neq 1-(1-1)$$

Give an example to establish why division is not commutative.

$$1\div 2\neq 2\div 1$$

Give an example to establish why division is not associative.

$$(1\div 2)\div 2\neq 1\div(2\div 2)$$

Quiz 2-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/11 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Choose the fraction(s) equivalent to $\frac{1}{5}$ (select all that apply).

a. $\frac{1}{-5}$ b. $\frac{1}{5}$
c. $\frac{-1}{5}$ d. $\frac{-1}{-5}$

Choose the fraction(s) equivalent to $\frac{8}{-(p+r)}$ (select all that apply).

a. $-\frac{8}{p+r}$ b. $\frac{8}{p+r}$
c. $\frac{-8}{p+r}$ d. $\frac{-8}{-(p+r)}$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

a. $-\frac{8r}{9s}$ b. $\frac{-8r}{9s}$
c. $\frac{8r}{-9s}$ d. $-\frac{8r}{-9s}$

Add

$$1 + 2 + 3 + \cdots + 50 = \underline{\qquad}$$

Multiply

$$4 \cdot 53 \cdot 25 = \underline{\qquad}$$

Find the reciprocal of π

$$\underline{\qquad}$$

Give an example to establish why subtraction is not commutative.

Give an example to establish why subtraction is not associative.

Give an example to establish why division is not commutative.

Give an example to establish why division is not associative.

Solution to Quiz 3-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/12 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Choose the fraction(s) equivalent to $\frac{8}{p+r}$ (select all that apply).

b. $\frac{8}{p+r}$ d. $\frac{-8}{-(p+r)}$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

d. $-\frac{8r}{-9s}$

Add

$$437 + 13999 + 33 + 1 = 14470$$

Evaluate

$$4 \cdot 53 \cdot 25 = 5300$$

Evaluate

$$(-1)(-2)(-3)(-4)(-5) = -120$$

Evaluate

$$(-1)(-2) \cdot \frac{-3}{-4} \cdot (-5) = -\frac{15}{2}$$

Evaluate

$$2 \div \frac{1 \cdot 3}{2 \cdot 4} = \frac{16}{3}$$

Evaluate

$$-4^2 = -16$$

Find the value of the expression

$$\left(-\frac{1}{10}\right)^3 = -\frac{1}{1000}$$

Give an example to establish why division is not associative.

[See the solution to quiz 2-2.]

Quiz 3-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/12 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Choose the fraction(s) equivalent to $\frac{8}{p+r}$ (select all that apply).

a. $-\frac{8}{p+r}$ b. $\frac{8}{p+r}$
c. $\frac{-8}{p+r}$ d. $\frac{-8}{-(p+r)}$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

a. $-\frac{8r}{9s}$ b. $\frac{-8r}{9s}$
c. $\frac{8r}{-9s}$ d. $-\frac{8r}{-9s}$

Add

$$437 + 13999 + 33 + 1 = \_\_\_\_\_\_$$

Evaluate

$$4 \cdot 53 \cdot 25 = \_\_\_\_\_\_$$

Evaluate

$$(-1)(-2)(-3)(-4)(-5) = \_\_\_\_\_\_$$

Evaluate

$$(-1)(-2) \cdot \frac{-3}{-4} \cdot (-5) = \_\_\_\_\_\_$$

Evaluate

$$2 \div \frac{1 \cdot 3}{2 \cdot 4} = \_\_\_\_\_\_$$

Evaluate

$$-4^2 = \_\_\_\_\_\_$$

Find the value of the expression

$$\left(-\frac{1}{10}\right)^3 = \_\_\_\_\_\_$$

Give an example to establish why division is not associative.

Quiz 3-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/12 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ____________________ Name: ____________________

Compute

$25 \times 9 = 25 \times (10 - 1) = 250 - 25 = 225$

$17 \times 19 = 17 \times (20 - 1) = 17 \times 20 - 17 = 340 - 17 = 323$

Clear the parenthesis in the expression

$-3(-x - y - z) = 3x + 3y + 3z$

$-x(-5 + 2y) = 5x - 2xy$

Expansion (clear the parenthesis)

$(a + b) \cdot (a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$(a + b + c) \cdot (x + y) = ax + ay + bx + by + cx + cy$

Simplify the followings by combining similar terms

$5(ab - 3) + ab + 18 - b^2 = 6ab - b^2 + 3$

$-(n + 1) + (2n - 2) = n - 3$

$8k - (4k - 18) = 4k + 18$

$\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y) = (9x - 6) - (5x - \frac{3y}{4}) = 4x + \frac{3y}{4} - 6$

Factor

$7b + 21ab = 7b(1 + 3a)$

$x^3y + 2x^2y^2 = xy(x^2 + 2xy)$

Compute

$\frac{2^{-5}}{3^{-7}} \times \frac{3^{-2}}{2^3} = \frac{3^7}{2^5} \times \frac{1}{2^3 \cdot 3^2} = \frac{3^7}{2^8 \cdot 3^2} = \frac{3^5}{2^8}$

Compute

$4^{-2} = \frac{1}{16}$ $\quad 4^{-1} = \frac{1}{4}$ $\quad 4^0 = 1$ $\quad 4^1 = 4$ $\quad 4^2 = 16$

Compute

$(-4)^{-2} = \frac{1}{16}$ $\quad (-4)^{-1} = \frac{-1}{4}$ $\quad (-4)^0 = 1$ $\quad (-4)^1 = -4$ $\quad (-4)^2 = 16$

Compute

$(-1)^{-2} = 1$ $\quad (-1)^0 = 1$ $\quad (-1)^3 = -1$ $\quad (-1)^6 = 1$ $\quad (-1)^{777} = -1$

Compute

$(-3)^{777} + 3^{777} = (-1)^{777} \cdot 3^{777} + 3^{777} = (-1) \cdot 3^{777} + 3^{777} = 0$

Quiz 3-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/12 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$25 \times 9 =$ ______ $17 \times 19 =$ ______

Clear the parenthesis in the expression

$-3(-x - y - z) =$ ______ $-x(-5 + 2y) =$ ______

Expansion (clear the parenthesis)

$(a + b) \cdot (a + b) =$ ____________ $(a + b + c) \cdot (x + y) =$ ______________________________

Simplify the followings by combining similar terms

$5(ab - 3) + ab + 18 - b^2 =$ ______ $-(n + 1) + (2n - 2) =$ ______

$8k - (4k - 18) =$ ______ $\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y) =$ ______

Factor

$7b + 21ab =$ ______ $x^3y + 2x^2y^2 =$ ______

Compute

$\frac{2^{-5}}{3^{-7}} \times \frac{3^{-2}}{2^3} =$ ______

Compute

$4^{-2} =$ ______ $4^{-1} =$ ______ $4^0 =$ ______ $4^1 =$ ______ $4^2 =$ ______

Compute

$(-4)^{-2} =$ ______ $(-4)^{-1} =$ ______ $(-4)^0 =$ ______ $(-4)^1 =$ ______ $(-4)^2 =$ ______

Compute

$(-1)^{-2} =$ ______ $(-1)^0 =$ ______ $(-1)^3 =$ ______ $(-1)^6 =$ ______ $(-1)^{777} =$ ______

Compute

$(-3)^{777} + 3^{777} =$ ______

Solution to Quiz 4-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/16 6:00-6:30pm
*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$25 \times 9 = 25 \times (10 - 1) = 250 - 25 = 225$ $\quad$ $17 \times 19 = 17 \times (20 - 1) = 17 \times 20 - 17 = 340 - 17 = 323$

Clear the parenthesis in the expression

$-3(-x - y - z) = 3x + 3y + 3z$ $\quad$ $-x(-5 + 2y) = 5x - 2xy$

Expansion (clear the parenthesis)

$(a + b) \cdot (a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$ $\quad$ $(a + b + c) \cdot (x + y) = ax + ay + bx + by + cx + cy$

Simplify the followings by combining similar terms

$5(ab - 3) + ab + 18 - b^2 = 6ab - b^2 + 3$ $\quad$ $-(n + 1) + (2n - 2) = n - 3$

$8k - (4k - 18) = 4k + 18$ $\quad$ $\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y) = (9x - 6) - (5x - \frac{3y}{4}) = 4x + \frac{3y}{4} - 6$

Factor

$7b + 21ab = 7b(1 + 3a)$ $\quad$ $x^3y + 2x^2y^2 = xy(x^2 + 2xy)$

Compute

$\frac{2^{-6}}{3^{-7}} \times \frac{3^{-2}}{2^3} = \frac{3^7}{2^6} \times \frac{1}{2^3 \cdot 3^2} = \frac{3^7}{2^9 \cdot 3^2} = \frac{3^5}{2^9}$

Compute

$4^{-2} = \frac{1}{16}$ $\quad$ $4^{-1} = \frac{1}{4}$ $\quad$ $4^0 = 1$ $\quad$ $4^1 = 4$ $\quad$ $4^2 = 16$

Compute

$(-4)^{-2} = \frac{1}{16}$ $\quad$ $(-4)^{-1} = \frac{-1}{4}$ $\quad$ $(-4)^0 = 1$ $\quad$ $(-4)^1 = -4$ $\quad$ $(-4)^2 = 16$

Compute

$(-1)^{-2} = 1$ $\quad$ $(-1)^0 = 1$ $\quad$ $(-1)^3 = -1$ $\quad$ $(-1)^6 = 1$ $\quad$ $(-1)^{777} = -1$

Compute

$(-7)^{777} + 7^{777} = (-1)^{777} \cdot 7^{777} + 7^{777} = (-1) \cdot 7^{777} + 7^{777} = 0$

Quiz 4-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/16 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$25 \times 9 =$ ______ $17 \times 19 =$ ______

Clear the parenthesis in the expression

$-3(-x - y - z) =$ ______ $-x(-5 + 2y) =$ ______

Expansion (clear the parenthesis)

$(a + b) \cdot (a + b) =$ __________ $(a + b + c) \cdot (x + y) =$ ______________________

Simplify the followings by combining similar terms

$5(ab - 3) + ab + 18 - b^2 =$ ______ $-(n + 1) + (2n - 2) =$ ______

$8k - (4k - 18) =$ ______ $\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y) =$ ______

Factor

$7b + 21ab =$ ______ $x^3y + 2x^2y^2 =$ ______

Compute

$\frac{2^{-6}}{3^{-7}} \times \frac{3^{-2}}{2^3} =$ ______

Compute

$4^{-2} =$ ______ $4^{-1} =$ ______ $4^0 =$ ______ $4^1 =$ ______ $4^2 =$ ______

Compute

$(-4)^{-2} =$ ______ $(-4)^{-1} =$ ______ $(-4)^0 =$ ______ $(-4)^1 =$ ______ $(-4)^2 =$ ______

Compute

$(-1)^{-2} =$ ______ $(-1)^0 =$ ______ $(-1)^3 =$ ______ $(-1)^6 =$ ______ $(-1)^{777} =$ ______

Compute

$(-7)^{777} + 7^{777} =$ ______

Solution to Quiz 4-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/16 7:45-8:15pm
*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$((-2)^3)^2 = (-2)^{3\cdot 2} = (-2)^6 = (-1)^6 \cdot 2^6 = 64$

$((-2)^3)^{-2} = (-2)^{-6} = \frac{1}{(-2)^6} = \frac{1}{2^6} = \frac{1}{64}$

Simplify the expression

$((-x^3)^2) \cdot x^{-6} = (-1)^2 x^{3\cdot 2} \cdot x^{-6} = x^{6-6} = 1$

Simplify the expression

$x^3 \cdot x^{-8} \cdot x^4 = x^{3-8+4} = x^{-1} = \frac{1}{x}$

Simplify the expression

$\frac{a^4 b^{-2}}{a^{-2} b^2} = \frac{a^{4+2}}{b^{2+2}} = \frac{a^6}{b^4}$

Show

For every negative integer n, show that $a^n \cdot b^n = (ab)^n$:

$$a^n \cdot b^n = \frac{1}{a^{-n} \cdot b^{-n}} = \frac{1}{a \cdots a} \cdot \frac{1}{b \cdots b} = \frac{1}{ab} \cdots \frac{1}{ab} = (\frac{1}{ab})^{-n} = (ab)^n$$

Show

For every positive integer n, show that $\frac{a^n}{b^n} = (\frac{a}{b})^n$:

$$\frac{a^n}{b^n} = \frac{a \cdots a}{b \cdots b} = \frac{a}{b} \cdots \frac{a}{b} = (\frac{a}{b})^n$$

Write the polynomial in standard form and indicate its degree

$2x^2 - x^3 + 3x^4 + 1 - 5x^2 + 6x^6 = 6x^6 + 3x^4 - x^3 - 3x^2 + 1$, degree $= 6$

Write the polynomial in standard form and indicate its degree

$$(x+2)(3x+1)(1-x) = (3x^2+7x+2)(1-x) = (3x^2+7x+2) - x(3x^2+7x+2)$$
$$= (3x^2+7x+2) - (3x^3+7x^2+2x) = -3x^3 - 4x^2 + 5x + 2; \text{ degree} = 3$$

Write the polynomial in standard form and indicate its degree

$x(-x(-2x+1)+4) - 1 = x(2x^2 - x + 4) - 1 = 2x^3 - x^2 + 4x - 1$, degree $= 3$

Choose the one that is not a polynomial

(a) $x + \frac{1}{x^2}$

Choose the one that is not a polynomial

(c) $x^2 + \frac{3x}{2x^2}$

Quiz 4-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/16 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Compute

$((-2)^3)^2 =$ ______ $((-2)^3)^{-2} =$ ______

Simplify the expression

$((-x^3)^2) \cdot x^{-6} =$ ______

Simplify the expression

$x^3 \cdot x^{-8} \cdot x^4 =$ ______

Simplify the expression

$\frac{a^4 b^{-2}}{a^{-2} b^2} =$ ______

Show

For every negative integer n, show that $a^n \cdot b^n = (ab)^n$.

Show

For every positive integer n, show that $\frac{a^n}{b^n} = (\frac{a}{b})^n$.

Write the polynomial in standard form and indicate its degree

$2x^2 - x^3 + 3x^4 + 1 - 5x^2 + 6x^6 =$ ______________________, degree = ______

Write the polynomial in standard form and indicate its degree

$(x+2)(3x+1)(1-x) =$ ______________________, degree = ______

Write the polynomial in standard form and indicate its degree

$x(-x(-2x+1)+4) - 1 =$ ______________________, degree = ______

Choose the one that is not a polynomial

(a) $x + \frac{1}{x^2}$ (b) $\frac{x+3}{2}$
(c) $x + \frac{1}{5}$ (d) $x(x(-2x+1)+4) - 1$

Choose the one that is not a polynomial

(a) 3 (b) $x + \frac{1}{x} \cdot x^2$
(c) $x^2 + \frac{3x}{2x^2}$ (d) $x^2 + 3x$

Solution to Quiz 5-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/18 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$((-2)^3)^2 = (-2)^{3\cdot 2} = (-2)^6 = (-1)^6 \cdot 2^6 = 64$

$((-2)^3)^{-2} = (-2)^{-6} = \frac{1}{(-2)^6} = \frac{1}{2^6} = \frac{1}{64}$

Simplify the expression

$((-x^3)^2) \cdot x^{-6} = (-1)^2 x^{3\cdot 2} \cdot x^{-6} = x^{6-6} = 1$

Simplify the expression

$x^3 \cdot x^{-8} \cdot x^4 = x^{3-8+4} = x^{-1} = \frac{1}{x}$

Simplify the expression

$\frac{a^4 b^{-2}}{a^{-2} b^2} = \frac{a^{4+2}}{b^{2+2}} = \frac{a^6}{b^4}$

Simplify the expression

$\frac{a^6 b^{-2}}{a^{-4} b^2} = \frac{a^{10}}{b^4}$

Simplify the expression

$[(-1)^{(-1)}]^{(-1)} = -1$

Write the polynomial in standard form and indicate its degree

$2x^2 - x^3 + 3x^4 + 1 - 5x^2 + 6x^6 = 6x^6 + 3x^4 - x^3 - 3x^2 + 1$, degree $= 6$

Write the polynomial in standard form and indicate its degree

$$(x+2)(3x+1)(1-x) = (3x^2+7x+2)(1-x) = (3x^2+7x+2) - x(3x^2+7x+2)$$
$$= (3x^2+7x+2) - (3x^3+7x^2+2x) = -3x^3 - 4x^2 + 5x + 2; \text{ degree} = 3$$

Write the polynomial in standard form and indicate its degree

$x(-x(-2x+1)+4) - 1 = x(2x^2 - x + 4) - 1 = 2x^3 - x^2 + 4x - 1$, degree $= 3$

Choose the one that is not a polynomial

(a) $x + \frac{1}{x^2}$

Choose the one that is not a polynomial

(c) $x^2 + \frac{3x}{2x^2}$

Quiz 5-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/18 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Compute

$((-2)^3)^2 =$ ______ $((-2)^3)^{-2} =$ ______

Simplify the expression

$((-x^3)^2) \cdot x^{-6} =$ ______

Simplify the expression

$x^3 \cdot x^{-8} \cdot x^4 =$ ______

Simplify the expression

$\frac{a^4 b^{-2}}{a^{-2} b^2} =$ ______

Simplify the expression

$\frac{a^6 b^{-2}}{a^{-4} b^2} =$ ______

Simplify the expression

$[(-1)^{(-1)}]^{(-1)} =$ ______

Write the polynomial in standard form and indicate its degree

$2x^2 - x^3 + 3x^4 + 1 - 5x^2 + 6x^6 =$ ____________________, degree = ______

Write the polynomial in standard form and indicate its degree

$(x+2)(3x+1)(1-x) =$ ____________________, degree = ______

Write the polynomial in standard form and indicate its degree

$x(-x(-2x+1)+4) - 1 =$ ____________________, degree = ______

Choose the one that is not a polynomial

(a) $x + \frac{1}{x^2}$ (b) $\frac{x+3}{2}$
(c) $x + \frac{1}{5}$ (d) $x(x(-2x+1)+4) - 1$

Choose the one that is not a polynomial

(a) 3 (b) $x + \frac{1}{x} \cdot x^2$
(c) $x^2 + \frac{3x}{2x^2}$ (d) $x^2 + 3x$

Solution to Quiz 5-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/18 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Addition and subtraction

Let $p = x^3 - 4x^2 + x - 1$, and $q = x^3 + 4x^2 - 3x + 1$. Compute

$p + q = 2x^3 - 2x$

$p - q = -8x^2 + 4x - 2$

Multiplication

Let $p = x-1$, and $q = -x^2-3x+1$. Compute $p \cdot q = -(x-1)(x^2+3x-1) = -[(x^3+3x^2-x)-(x^2+3x-1)] = -x^3-2x^2+4x-1$

Compute

$(2x + 3)^2 = 4x^2 + 12x + 9$ $\quad$ $(2x - 3)^2 = 4x^2 - 12x + 9$

$(2x + 3)(2x - 3) = 4x^2 - 9$ $\quad$ $(a + b)^2 = a^2 + 2ab + b^2$

Factor out a monomial

$5x^3 + 4x^2 = x^2(5x + 4)$

$2x^3 + 10x^2 - 4x = x(2x^2 + 10x - 4)$

Evaluation and substitution of a polynomial

Let $p(x) = -2x^2 + 5x$. Find

$p(3) = -18 + 15 = -3$ $\quad$ $p(a) = -2a^2 + 5a$

$p(a - 1) = -2(a - 1)^2 + 5(a - 1) = -2(a^2 - 2a + 1) + (5a - 5) = -2a^2 + 9a - 7$

$p(a^2) = -2(a^2)^2 + 5(a^2) = -2a^4 + 5a^2$

Evaluate of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = 3$. $\frac{x^2+5}{x-3}|_{x=3} = \frac{9+5}{3-3}$: not defined (divided by 0)!

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a - 1$. $\frac{x^2+5}{x-3}|_{x=a-1} = \frac{(a-1)^2+5}{(a-1)-3} = \frac{a^2-2a+6}{a-4}$

Simplify the expression by cancellation.

$\frac{x^2-4}{x^2+2x} = \frac{(x+2)(x-2)}{(x+2)x} = \frac{x-2}{x}$

Evaluation (simplification makes it easier)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = 17$: $\frac{x-2}{x}|_{x=17} = \frac{15}{17}$.

Evaluation (is it well-defined?)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = -2$: not defined (divided by 0)!

Quiz 5-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/18 7:45-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Addition and subtraction

Let $p = x^3 - 4x^2 + x - 1$, and $q = x^3 + 4x^2 - 3x + 1$. Compute

$p + q =$ ______________

$p - q =$ ______________

Multiplication

Let $p = x - 1$, and $q = -x^2 - 3x + 1$. Compute $p \cdot q =$ ______________

Compute

$(2x + 3)^2 =$ ______________ $(2x - 3)^2 =$ ______________

$(2x + 3)(2x - 3) =$ ______________ $(a + b)^2 =$ ______________

Factor out a monomial

$5x^3 + 4x^2 =$ ______________

$2x^3 + 10x^2 - 4x =$ ______________

Evaluation and substitution of a polynomial

Let $p(x) = -2x^2 + 5x$. Find

$p(3) =$ ______________ $p(a) =$ ______________

$p(a - 1) =$ ______________ $p(a^2) =$ ______________

Evaluate of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = 3$. $\frac{x^2+5}{x-3}\Big|_{x=3} =$ ______

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a - 1$. $\frac{x^2+5}{x-3}\Big|_{x=a-1} =$ ______________

Simplify the expression by cancellation.

$\frac{x^2-4}{x^2+2x} =$ ______________ [Hint: use $(a^2 - b^2) = (a + b)(a - b)$.]

Evaluation (simplification makes it easier)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = 17$: ______.

Evaluation (is it well-defined?)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = -2$: ______.

Solution to Quiz 6-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/19 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Addition and subtraction

Let $p = x^3 - 4x^2 + x - 1$, and $q = x^3 + 4x^2 - 3x + 1$. Compute

$p + q = 2x^3 - 2x$

$p - q = -8x^2 + 4x - 2$

Multiplication

Let $p = x - 1$, and $q = -x^2 - 3x + 1$. Compute $p \cdot q = -(x-1)(x^2+3x-1) = -[(x^3+3x^2-x)-(x^2+3x-1)] = -x^3-2x^2+4x-1$

Compute

$(2x-3)^2 = 4x^2 - 12x + 9$ $\quad\quad$ $(2x+3)(2x-3) = 4x^2 - 9$

Factor out a monomial

$2x^3 + 10x^2 - 4x = x(2x^2 + 10x - 4)$

Evaluation and substitution of a polynomial

Let $p(x) = -2x^2 + 5x$. Find

$p(3) = -18 + 15 = -3$ $\quad\quad$ $p(a-1) = -2(a-1)^2 + 5(a-1) = -2(a^2 - 2a + 1) + (5a - 5) = -2a^2 + 9a - 7$

Evaluate of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = 3$. $\frac{x^2+5}{x-3}|_{x=3} = \frac{9+5}{3-3}$: not defined (divided by 0)!

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a - 1$. $\frac{x^2+5}{x-3}|_{x=a-1} = \frac{(a-1)^2+5}{(a-1)-3} = \frac{a^2-2a+6}{a-4}$

Simplify the expression by cancellation.

$\frac{x^2-9}{x^2+3x} = \frac{(x+3)(x-3)}{(x+3)x} = \frac{x-3}{x}$

Evaluation (simplification makes it easier)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = 18$: $\frac{x-2}{x}|_{x=18} = \frac{8}{9}$.

Evaluation (is it well-defined?)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = -2$: not defined (divided by 0)!

Quiz 6-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/19 6:00-6:30

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Addition and subtraction

Let $p = x^3 - 4x^2 + x - 1$, and $q = x^3 + 4x^2 - 3x + 1$. Compute

$p + q =$ ______________

$p - q =$ ______________

Multiplication

Let $p = x - 1$, and $q = -x^2 - 3x + 1$. Compute $p \cdot q =$ ______________

Compute

$(2x - 3)^2 =$ ______________ $(2x + 3)(2x - 3) =$ ______________

Factor out a monomial

$2x^3 + 10x^2 - 4x =$ ______________

Evaluation and substitution of a polynomial

Let $p(x) = -2x^2 + 5x$. Find

$p(3) =$ ______________ $p(a - 1) =$ ______________

Evaluate of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = 3$. $\frac{x^2+5}{x-3}\big|_{x=3} =$ ____

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a - 1$. $\frac{x^2+5}{x-3}\big|_{x=a-1} =$ ______________

Simplify the expression by cancellation.

$\frac{x^2-9}{x^2+3x} =$ ______________

Evaluation (simplification makes it easier)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = 18$: ____.

Evaluation (is it well-defined?)

Evaluate $\frac{x^2-4}{x^2+2x}$ for $x = -2$: ____.

Solution to Quiz 6-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/19 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x} = \frac{-1}{x(x+4)}$

Simplify the expression

$\frac{x^2}{x^2-2x+1} \div \frac{x^3}{x-1} = \frac{1}{x(x-1)}$

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} - x^2 = \frac{-x^2-1}{x}$

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-2x+1} + \frac{1}{x^2-1} = \frac{2x}{(x-1)^2(x+1)}$

Perimeter and area of a rectangle

In a rectangle, one side is x feet long. The other side is y feet longer. Compose an algebraic expression in terms of x and y for the perimeter and the area of the rectangle. Perimeter $= 4x + 2y$ (feet). Area $= x(x + y)$ (feet^2).

Counting money

Jin receives some coins as a street performer. He saves one quarter and n dimes everyday in his piggy bank.
1. Compose an algebraic expression for the total amount of money in the piggy bank after d days in terms of n and d: $25d + 10nd$ (cents).
2. How many dimes does Jin have to save if he plans to save \$195 in 300 days: 4 dimes.

Counting money

Jin works in a fast food restaurant, the wage being \$$x$ per hour. He pays his rent daily, which amounts to \$30. If he moves into a flat with the monthly rent being \$750, how much time could he save from not working monthly? Express the answer in terms of x: $\frac{150}{x}$ (hours). Evaluate the answer at $x = 10$: 15 (hours).

Counting money

Suppose the inflation rate is 3% per year, the market pays you 7% per year, and the other factors do not affect. How much should Jin invest in the market so that he does not have to work for his \$$x$ annual expense? Express the answer in terms of x: \$$25x$. Evaluate the answer at $x = 22000$: \$550000.

Uniform motion

Jin drives from Lawrence to a friend's house in Kansas City, the total distance being 40 miles. For the last 10 miles, he has to slow down to x miles per hour. At least how fast should he drive before slowing down in order to be there in an hour? Express the answer in terms of x: $\frac{30}{1-\frac{10}{x}}$ (miles per hour).

Uniform motion

Jin drives from Chicago to Stony Brook, the total distance being 840 miles. He wants to drive as slow as possible. He also wants to be there in two days, while he can only drive for 6 hours a day. What is the slowest average speed possible for him in order to fulfill all of his wishes: 70 (miles per hour).

Quiz 6-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/19 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x}$ ______________

Simplify the expression

$\frac{x^2}{x^2-2x+1} \div \frac{x^3}{x-1}$ ______________

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} - x^2$ ______________

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-2x+1} + \frac{1}{x^2-1}$ ______________

Perimeter and area of a rectangle

In a rectangle, one side is x feet long. The other side is y feet longer. Compose an algebraic expression in terms of x and y for the perimeter and the area of the rectangle. Perimeter = ______________. Area = ______________.

Counting money

Jin receives some coins as a street performer. He saves one quarter and n dimes everyday in his piggy bank.
1. Compose an algebraic expression for the total amount of money in the piggy bank after d days in terms of n and d: ______________.
2. How many dimes does Jin have to save if he plans to save \$195 in 300 days: ______________?

Counting money

Jin works in a fast food restaurant, the wage being \$$x$ per hour. He pays his rent daily, which amounts to \$30. If he moves into a flat with the monthly rent being \$750, how much time could he save from not working monthly? Express the answer in terms of x: ______________. Evaluate the answer at $x = 10$: ______________.

Counting money

Suppose the inflation rate is 3% per year, the market pays you 7% per year, and the other factors do not affect. How much should Jin invest in the market so that he does not have to work for his \$$x$ annual expense? Express the answer in terms of x: ______________. Evaluate the answer at $x = 22000$: ______________.

Uniform motion

Jin drives from Lawrence to a friend's house in Kansas City, the total distance being 40 miles. For the last 10 miles, he has to slow down to x miles per hour. At least how fast should he drive before slowing down in order to be there in an hour? Express the answer in terms of x: ______________.

Uniform motion

Jin drives from Chicago to Stony Brook, the total distance being 840 miles. He wants to drive as slow as possible. He also wants to be there in two days, while he can only drive for 6 hours a day. What is the slowest average speed possible for him in order to fulfill all of his wishes?

Solution to Quiz 7-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/25 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________________ Name: ________________________________

Clear the parenthesis

$(x-1)(x+1)(x^2+1) = x^4 - 1$

Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x} = \frac{-1}{x(x+4)}$

Simplify the expression

$\frac{x}{16-x^2} \cdot \frac{x-4}{x^2+x} = \frac{-1}{(x+1)(x+4)}$

Simplify the expression

$\frac{x^2}{x^2-2x+1} \div \frac{x^3}{x-1} = \frac{1}{x(x-1)}$

Simplify the expression

$\frac{x^2}{x^2+4x+4} \div \frac{x^3}{x+2} = \frac{1}{x(x+2)}$

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} - x^2 = \frac{-x^2-1}{x}$

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} + x = \frac{x^3-1}{x}$

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-2x+1} + \frac{1}{x^2-1} = \frac{2x}{(x-1)^2(x+1)}$

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-4x+4} + \frac{1}{x^2-4} = \frac{2x}{(x+2)(x-2)^2}$

Perimeter and area of a rectangle

In a rectangle, one side is x feet long. The other side is y feet longer. Compose an algebraic expression in terms of x and y for the perimeter and the area of the rectangle. Perimeter $= 4x + 2y$ (feet). Area $= x(x+y)$ (feet^2).

Quiz 7-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/25 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Clear the parenthesis

$(x-1)(x+1)(x^2+1) =$ ______________

Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x} =$ ______________

Simplify the expression

$\frac{x}{16-x^2} \cdot \frac{x-4}{x^2+x} =$ ______________

Simplify the expression

$\frac{x^2}{x^2-2x+1} \div \frac{x^3}{x-1} =$ ______________

Simplify the expression

$\frac{x^2}{x^2+4x+4} \div \frac{x^3}{x+2} =$ ______________

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} - x^2 =$ ______________

Present the following as a single fraction

$\frac{x^3-x^2-1}{x} + x =$ ______________

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-2x+1} + \frac{1}{x^2-1} =$ ______________

Perform the operations and simplify the resulting expression

$\frac{1}{x^2-4x+4} + \frac{1}{x^2-4} =$ ______________

Perimeter and area of a rectangle

In a rectangle, one side is x feet long. The other side is y feet longer. Compose an algebraic expression in terms of x and y for the perimeter and the area of the rectangle. Perimeter = ______________. Area = ______________.

Solution to Quiz 7-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/25 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select all that apply.

(a) $x = x + 2$ is always false.
(c) $(x+1)^2 - 1 = x(x+2)$ is always true.

Select all that apply.

(a) $x + y = 1$ has infinitely many solutions.
(b) $x^2 = 0$ has only one solution.
(d) $x + 1 = 0$ has only one solution.

Prove the identity

$(x-1)^3 = (x-1)(x^2 - 2x + 1) = x^3 - 3x^2 + 3x - 1$.

Find all solutions to the equation.

$x + 2 = 3x \Leftrightarrow x = 1$

Find all solutions to the equation.

$3x - 1 = 5 + x \Leftrightarrow x = 3$

Find all solutions to the equation.

$2x + 3 = 4x + 5 \Leftrightarrow x = -1$

Find all solutions to the equation.

$9x - 5 = 5 + 109x \Leftrightarrow x = -\frac{1}{10}$

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x = 28$

Perimeter of a triangle

In a rectangle, one side is 6 feet longer than the other side. Suppose the perimeter is 24 feet. Find the lengths of the sides: $2(x + (x+6)) = 24$, so $x = 3$ and $x + 6 = 9$, the lengths being 3 and 9 feet.

Angles in a triangle

In a triangle, two angles are same, and the third angle is twice as large as the others. Find the angles: $x + x + 2x = 180$, so $x = 45$ and $2x = 90$, the angles being $45, 45, 90$ degrees.

Quiz 7-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/25 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Select all that apply.

(a) $x = x + 2$ is always false.
(b) $x + 2 = 5$ is always false.
(c) $(x + 1)^2 - 1 = x(x + 2)$ is always true.
(d) $x^2 - y^2 = (x - y)^2$ is always true.

Select all that apply.

(a) $x + y = 1$ has infinitely many solutions.
(b) $x^2 = 0$ has only one solution.
(c) $x^2 = 1$ has only one solution
(d) $x + 1 = 0$ has only one solution.

Prove the identity

$(x - 1)^3 = x^3 - 3x^2 + 3x - 1$.

Find all solutions to the equation.

$x + 2 = 3x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$3x - 1 = 5 + x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$2x + 3 = 4x + 5 \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$9x - 5 = 5 + 109x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x =$ ______________________

Perimeter of a triangle

In a rectangle, one side is 6 feet longer than the other side. Suppose the perimeter is 24 feet. Find the lengths of the sides: ______________________.

Angles in a triangle

In a triangle, two angles are same, and the third angle is twice as large as the others. Find the angles: ______________________.

Solution to Quiz 8-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/26 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Select all that apply.

(a) $x = x + 2$ is always false.

Select all that apply.

(a) $x + y = 1$ has infinitely many solutions.
(c) $x^2 = 1$ has only two solutions.
(d) $x + 1 = 0$ has only one solution.

Prove the identity

$(x-1)^3 = (x-1)(x^2 - 2x + 1) = x^3 - 3x^2 + 3x - 1.$

Find all solutions to the equation.

$x + 2 = 3x \Leftrightarrow x = 1$

Find all solutions to the equation.

$3x - 1 = 5 + x \Leftrightarrow x = 3$

Find all solutions to the equation.

$2x + 3 = 4x + 5 \Leftrightarrow x = -1$

Find all solutions to the equation.

$9x - 5 = 5 + 109x \Leftrightarrow x = -\frac{1}{10}$

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x = 28$

Perimeter of a triangle

In a rectangle, one side is 8 feet longer than the other side. Suppose the perimeter is 24 feet. Find the lengths of the sides: $2(x + (x + 8)) = 24$, so $x = 2$ and $x + 8 = 10$, the lengths being 2 and 10 feet.

Angles in a triangle

In a triangle, two angles are same, and the third angle is three times as large as the others. Find the angles: $x+x+3x = 180$, so $x = 36$ and $3x = 108$, the angles being $36, 36, 108$ degrees.

Quiz 8-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/26 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Select all that apply.

(a) $x = x + 2$ is always false.
(b) $x + 2 = 5$ is always false.
(c) $(x + 1)^2 - 1 = x(x + 2)$ is always false.
(d) $x^2 - y^2 = (x - y)^2$ is always true.

Select all that apply.

(a) $x + y = 1$ has infinitely many solutions.
(b) $x^2 = 0$ has only two solutions.
(c) $x^2 = 1$ has only two solutions.
(d) $x + 1 = 0$ has only one solution.

Prove the identity

$(x - 1)^3 = x^3 - 3x^2 + 3x - 1$.

Find all solutions to the equation.

$x + 2 = 3x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$3x - 1 = 5 + x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$2x + 3 = 4x + 5 \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$9x - 5 = 5 + 109x \Leftrightarrow x =$ ______________________

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x =$ ______________________

Perimeter of a triangle

In a rectangle, one side is 8 feet longer than the other side. Suppose the perimeter is 24 feet. Find the lengths of the sides: ______________________.

Angles in a triangle

In a triangle, two angles are same, and the third angle is three times as large as the others. Find the angles: ______________________

Solution to Quiz 8-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/26 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the inequality.

$x - 4 \leqslant 5 : x \leqslant 9$.

Solve the inequality.

$3x - 1 \geqslant 5 + x : x \geqslant 3$.

Solve the inequality.

$-3x < 6 \Leftrightarrow x > -2$.

Solve the inequality.

$-\frac{1}{3}x + 4 > x \Leftarrow x < 3$.

Solve the system.

$x - 2 \leqslant 4$ and $-x + 2 < 4 : -2 < x \leqslant 6$.

Absolute value

Calculate $|-6 + |2 - 3|| = |-6 + 1| = |-5| = 5$.

Absolute value

Solve the equation: $|2x| = 2 \Leftrightarrow x = \pm 1$.

Absolute value

Solve the equation: $|3x - 3| = 3 \Leftrightarrow x = 0$ or 2.

Absolute value

Solve the inequality: $|x - 1| < 2 \Leftrightarrow -1 < x < 3$.

Absolute value

Solve the inequality: $|1 - x| \geqslant 5 \Leftrightarrow x \geqslant 6$ or $x \leqslant -4$.

Quiz 8-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/26 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________ Name: ________________________

Solve the inequality.

$x - 4 \leqslant 5$: ____________.

Solve the inequality.

$3x - 1 \geqslant 5 + x$: ____________.

Solve the inequality.

$-3x < 6$: ____________.

Solve the inequality.

$-\frac{1}{3}x + 4 > x$: ____________.

Solve the system.

$x - 2 \leqslant 4$ and $-x + 2 < 4$: ____________.

Absolute value

Calculate $|-6 + |2 - 3|| =$ ____________.

Absolute value

Solve the equation: $|2x| = 2 \Leftrightarrow x =$ ____________.

Absolute value

Solve the equation: $|3x - 3| = 3 \Leftrightarrow x =$ ____________.

Absolute value

Solve the inequality: $|x - 1| < 2 \Leftrightarrow$ ____________.

Absolute value

Solve the inequality: $|1 - x| \geqslant 5 \Leftrightarrow$ ____________.

Solution to Quiz 9-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/30 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the inequality.

$x - 4 \leqslant 5 : x \leqslant 9$.

Solve the inequality.

$-3x - 1 \geqslant 5 + x : x \leqslant \frac{-3}{2}$.

Solve the inequality.

$-3x < 9 \Leftrightarrow x > -3$.

Solve the inequality.

$-\frac{1}{3}x + 4 > x \Leftarrow x < 3$.

Solve the system.

$x - 2 \leqslant 4$ and $-x + 2 < 4 : -2 < x \leqslant 6$.

Absolute value

Calculate $|-9 + |2 - 3|| = |-9 + 1| = |-8| = 8$.

Absolute value

Solve the inequality: $|1 - x| \geqslant 5 \Leftrightarrow x \geqslant 6$ or $x \leqslant -4$.

Absolute value

Solve the equation: $|2x| = 2 \Leftrightarrow x = \pm 1$.

Absolute value

Solve the equation: $|3x - 3| = 3 \Leftrightarrow x = 0$ or 2.

Absolute value

Solve the inequality: $|x - 1| < 2 \Leftrightarrow -1 < x < 3$.

Quiz 9-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/30 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________ Name: ________________________

Solve the inequality.

$x - 4 \leqslant 5$: ________________.

Solve the inequality.

$-3x - 1 \geqslant 5 + x$: ________________.

Solve the inequality.

$-3x < 9$: ________________.

Solve the inequality.

$-\frac{1}{3}x + 4 > x$: ________________.

Solve the system.

$x - 2 \leqslant 4$ and $-x + 2 < 4$: ________________.

Absolute value

Calculate $|-9 + |2 - 3|| =$ ________________.

Absolute value

Solve the inequality: $|1 - x| \geqslant 5 \Leftrightarrow$ ________________________.

Absolute value

Solve the equation: $|2x| = 2 \Leftrightarrow x =$ ________________.

Absolute value

Solve the equation: $|3x - 3| = 3 \Leftrightarrow x =$ ________________.

Absolute value

Solve the inequality: $|x - 1| < 2 \Leftrightarrow$ ________________________.

Solution to Quiz 9-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/30 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Draw the line $y = 3$, and indicate its slope and intercepts.

Slope = 0
$x-$intercept = (not defined)
$y-$intercept = 3

Draw the line $x = -3$, and indicate its slope and intercepts.

Slope = (not defined)
$x-$intercept = -3
$y-$intercept = (not defined)

Draw the line $x + y = 1$, and indicate its slope and intercepts.

Slope = -1
$x-$intercept = 1
$y-$intercept = 1

Draw the line $y = 3x + 2$, and indicate its slope and intercepts.

Slope = 3
$x-$intercept = $-\frac{2}{3}$
$y-$intercept = 2

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope = $-\frac{3}{4}$
$x-$intercept = $-\frac{2}{3}$
$y-$intercept = $-\frac{1}{2}$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(0, -2)$: $y = 2x - 2$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(2, 3)$: $y = 3x - 3$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ with slope = 3: $y = 3x - 3$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and is parallel to the line $y = 2x + 3$: $y = 2x - 2$

Find the linear equation of a given line.

The line that passes through $(0, 0)$ and is perpendicular to the line $y = 2x + 3$: $y = -\frac{1}{2}x$

Quiz 9-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/30 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Draw the line $y = 3$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $x = -3$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $x + y = 1$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $y = 3x + 2$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(0, -2)$: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(2, 3)$: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ with slope $= 3$: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and is parallel to the line $y = 2x + 3$: ______________

Find the linear equation of a given line.

The line that passes through $(0, 0)$ and is perpendicular to the line $y = 2x + 3$: ______________

Solution to Quiz 10-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/01 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Draw the line $y = 3x + 2$, and indicate its slope and intercepts.

Slope $= 3$
$x-$intercept $= -\frac{2}{3}$
$y-$intercept $= 2$

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope $= -\frac{3}{4}$
$x-$intercept $= -\frac{2}{3}$
$y-$intercept $= -\frac{1}{2}$

Draw the line $y = 3$, and indicate its slope and intercepts.

Slope $= 0$
$x-$intercept = (not defined)
$y-$intercept $= 3$

Draw the line $x = -3$, and indicate its slope and intercepts.

Slope = (not defined)
$x-$intercept $= -3$
$y-$intercept = (not defined)

Draw the line $x + y = 1$, and indicate its slope and intercepts.

Slope $= -1$
$x-$intercept $= 1$
$y-$intercept $= 1$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ with slope $= 3$: $y = 3x - 3$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and is parallel to the line $y = 2x + 3$: $y = 2x - 2$

Find the linear equation of a given line.

The line that passes through $(0, 0)$ and is perpendicular to the line $y = 2x + 3$: $y = -\frac{1}{2}x$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(0, -2)$: $y = 2x - 2$

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(2, 3)$: $y = 3x - 3$

Quiz 10-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/01 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________________ Name: ________________________________

Draw the line $y = 3x + 2$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $y = 3$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $x = -3$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Draw the line $x + y = 1$, and indicate its slope and intercepts.

Slope = ______
$x-$intercept = ______
$y-$intercept = ______

Find the linear equation of a given line.

The line that passes through $(1, 0)$ with slope = 3: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and is parallel to the line $y = 2x + 3$: ______________

Find the linear equation of a given line.

The line that passes through $(0, 0)$ and is perpendicular to the line $y = 2x + 3$: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(0, -2)$: ______________

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and $(2, 3)$: ______________

Solution to Quiz 10-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/01 7:45-8:15

*A problem worths 20 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} x = 2 \\ y = -4 \end{cases}$ (($2, -4$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} x - 3y = 2 \\ y = 2 \end{cases}$ (($8, 2$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x + 3y = 8 \\ 2x - y = 0 \end{cases}$ (($2, 4$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x + y = 3 \\ -4x + 2y = 0 \end{cases}$ (There are no solutions. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x + y = 3 \\ -4x + 2y = 6 \end{cases}$ (There are infinitely many solutions. The graph is omitted.)

Quiz 10-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/01 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} x = 2 \\ y = -4 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} x - 3y = 2 \\ y = 2 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + 3y = 8 \\ 2x - y = 0 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + y = 3 \\ -4x + 2y = 0 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + y = 3 \\ -4x + 2y = 6 \end{cases}$$

Solution to Quiz 11-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/02 6:00-6:30pm

*A problem worths 20 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x+3y=3 \\ 3x+2y=6 \end{cases}$ (($\frac{12}{13}, \frac{21}{13}$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x+y=3 \\ -4x+2y=6 \end{cases}$ (There are infinitely many solutions. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} x-3y=2 \\ y=2 \end{cases}$ (($8, 2$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x+3y=8 \\ 2x-y=0 \end{cases}$ (($2, 4$) is the only solution. The graph is omitted.)

Draw the lines, indicate the amount of solutions, and solve the system if any.

$\begin{cases} -2x+y=3 \\ -4x+2y=0 \end{cases}$ (There are no solutions. The graph is omitted.)

Quiz 11-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/02 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + 3y = 3 \\ 3x + 2y = 6 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + y = 3 \\ -4x + 2y = 6 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} x - 3y = 2 \\ y = 2 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + 3y = 8 \\ 2x - y = 0 \end{cases}$$

Draw the lines, indicate the amount of solutions, and solve the system if any.

$$\begin{cases} -2x + y = 3 \\ -4x + 2y = 0 \end{cases}$$

Solution to Quiz 11-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/02 7:45-8:15
*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Compute

$\sqrt{3^2} = 3, \sqrt{(-3)^2} = 3, \sqrt{-3^2} = \text{(not defined)}, -\sqrt{3}^2 = -3$

Simplify the following expressions

$\sqrt{5} \cdot \sqrt{125} = 25,\ \sqrt{21} \cdot \sqrt{3} = 3\sqrt{7}$

Simplify the following expressions

$\sqrt{\frac{21}{12}} = \frac{\sqrt{7}}{2},\ \sqrt{\frac{-21}{-12}} = \frac{\sqrt{7}}{2}$

Simplify the following expressions

$\frac{2}{6-\sqrt{5}} = \frac{2(6+\sqrt{5})}{31},\ \frac{\sqrt{7}-2}{6-\sqrt{7}} = \frac{(\sqrt{7}-2)(6+\sqrt{7})}{29} = \frac{-5+4\sqrt{7}}{29},$

Simplify the following expressions

$\sqrt{7}(\sqrt{21} - \sqrt{35}) = 7(\sqrt{3} - \sqrt{5})$

Compute

$\sqrt{64} = 8, \sqrt[3]{64} = 4, \sqrt[4]{64} = 2\sqrt{2}, \sqrt[6]{64} = 2$

Compute

$\sqrt{-64} = \text{(not defined)}, \sqrt[3]{-64} = -4, \sqrt[4]{-64} = \text{(not defined)}, \sqrt[6]{64} = 2$

Compute

$\sqrt[5]{4} \cdot \sqrt[5]{8} = 2$

Compute

$27^{\frac{4}{3}} = 81, 27^{\frac{-4}{3}} = \frac{1}{81}, 27^{\frac{4}{-3}} = \frac{1}{81}, 27^{\frac{-4}{-3}} = 81$

Compute

$2^{\frac{7}{4}} \cdot 2^{\frac{4}{7}} \cdot 2^{\frac{-1}{2}} = 2^{\frac{51}{28}}.$

Quiz 11-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/02 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Compute

$\sqrt{3^2} = \_\_\_, \sqrt{(-3)^2} = \_\_\_, \sqrt{-3^2} = \_\_\_, -\sqrt{3}^2 = \_\_\_$

Simplify the following expressions

$\sqrt{5} \cdot \sqrt{125} = \_\_\_, \sqrt{21} \cdot \sqrt{3} = \_\_\_$

Simplify the following expressions

$\sqrt{\frac{21}{12}} = \_\_\_, \sqrt{\frac{-21}{-12}} = \_\_\_$

Simplify the following expressions

$\frac{2}{6-\sqrt{5}} = \_\_\_, \frac{\sqrt{7}-2}{6-\sqrt{7}} = \_\_\_,$

Simplify the following expressions

$\sqrt{7}(\sqrt{21} - \sqrt{35}) = \_\_\_$

Compute

$\sqrt{64} = \_\_\_, \sqrt[3]{64} = \_\_\_, \sqrt[4]{64} = \_\_\_, \sqrt[6]{64} = \_\_\_$

Compute

$\sqrt{-64} = \_\_\_, \sqrt[3]{-64} = \_\_\_, \sqrt[4]{-64} = \_\_\_, \sqrt[6]{64} = \_\_\_$

Compute

$\sqrt[5]{4} \cdot \sqrt[5]{8} = \_\_\_$

Compute

$27^{\frac{4}{3}} = \_\_\_, 27^{\frac{-4}{3}} = \_\_\_, 27^{\frac{4}{-3}} = \_\_\_, 27^{\frac{-4}{-3}} = \_\_\_$

Compute

$2^{\frac{7}{4}} \cdot 2^{\frac{4}{7}} \cdot 2^{\frac{-1}{2}} = \_\_\_\_\_\_$

Solution to Quiz 12 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/08 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the equation.

$x^2 = 1 \Leftrightarrow x = 1, -1$

Solve the equation.

$x^2 = -1 \Leftrightarrow x =$ (no solution)

Solve the equation.

$x^2 - 4 = 0 \Leftrightarrow x = 2, -2$

Solve the equation.

$4x^2 - 9 = 0 \Leftrightarrow x = \frac{3}{2}, \frac{-3}{2}$

Solve the equation.

$2x^2 - 9 = 0 \Leftrightarrow x = \frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}$

Solve the equation.

$x^2 + 6x + 9 = 0 \Leftrightarrow x = -3$

Solve the equation.

$(x-3)(x-4)(x+\frac{7}{2}) = 0 \Leftrightarrow x = 3, 4, \frac{-7}{2}$

Factor the polynomial if possible

$2x^2 - 11x - 21 = (2x+3)(x-7)$

Factor the polynomial if possible

$x^2 + x - 1 = (x - \frac{-1+\sqrt{5}}{2})(x - \frac{-1-\sqrt{5}}{2})$

Factor the polynomial if possible

$x^2 + x + 1$ is not factorizable because $b^2 - 4ac < 0$.

Quiz 12 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/08 7:45-8:15

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________ Name: ________________________

Solve the equation.

$x^2 = 1 \Leftrightarrow x =$ ____________

Solve the equation.

$x^2 = -1 \Leftrightarrow x =$ ____________

Solve the equation.

$x^2 - 4 = 0 \Leftrightarrow x =$ ____________

Solve the equation.

$4x^2 - 9 = 0 \Leftrightarrow x =$ ____________

Solve the equation.

$2x^2 - 9 = 0 \Leftrightarrow x =$ ____________

Solve the equation.

$x^2 + 6x + 9 = 0 \Leftrightarrow x =$ ____________

Solve the equation.

$(x - 3)(x - 4)(x + \frac{7}{2}) = 0 \Leftrightarrow x =$ ____________

Factor the polynomial if possible

$2x^2 - 11x - 21 =$ ____________

Factor the polynomial if possible

$x^2 + x - 1 =$ ____________

Factor the polynomial if possible

$x^2 + x + 1 =$ ____________

Solution to Quiz 13-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/09 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________________ Name: ________________________________

Solve the equation.

$x^2 = 16 \Leftrightarrow x = 4, -4$

Solve the equation.

$2x^2 - 9 = 0 \Leftrightarrow x = \frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}$

Solve the equation.

$x^2 - 6x + 8 = 0 \Leftrightarrow x = 2, 4$

Solve the equation.

$x^2 - 6x + 9 = 0 \Leftrightarrow x = 3$

Solve the equation.

$(x-3)(x-\pi)(x+\frac{7}{2}) = 0 \Leftrightarrow x = 3, \pi, \frac{-7}{2}$

Solve the equation.

$x^2 = -1 \Leftrightarrow x =$ (no solution)

Solve the equation.

$4x^2 - 9 = 0 \Leftrightarrow x = \frac{3}{2}, \frac{-3}{2}$

Factor the polynomial if possible

$3x^2 + 2x - 21 = (3x-7)(x+3)$

Factor the polynomial if possible

$x^2 + x - 1 = (x - \frac{-1+\sqrt{5}}{2})(x - \frac{-1-\sqrt{5}}{2})$

Factor the polynomial if possible

$x^2 + x + 1$ is not factorizable because $b^2 - 4ac < 0$.

Quiz 13-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/09 6:00-6:30pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Solve the equation.

$x^2 = 16 \Leftrightarrow x =$ ______________

Solve the equation.

$2x^2 - 9 = 0 \Leftrightarrow x =$ ______________

Solve the equation.

$x^2 - 6x + 8 = 0 \Leftrightarrow x =$ ______________

Solve the equation.

$x^2 - 6x + 9 = 0 \Leftrightarrow x =$ ______________

Solve the equation.

$(x-3)(x-\pi)(x+\frac{7}{2}) = 0 \Leftrightarrow x =$ ______________

Solve the equation.

$x^2 = -1 \Leftrightarrow x =$ ______________

Solve the equation.

$4x^2 - 9 = 0 \Leftrightarrow x =$ ______________

Factor the polynomial if possible

$3x^2 + 2x - 21 =$ ______________

Factor the polynomial if possible

$x^2 + x - 1 =$ ______________

Factor the polynomial if possible

$x^2 + x + 1 =$ ______________

Solution to Quiz 13-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/09 7:45-8:15

*A problem worths 50 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Draw the parabola $y = x^2 - 9$, indicate all its intercepts, vertex, and axis of symmetry

The parabola is

$$y = (x + 3)(x - 3).$$

x-intercepts: $(3, 0)$, $(-3, 0)$
y-intercepts: $(0, -9)$
vertex $= (0, -9)$
axis of symmetry: $x = 0$

Draw the parabola $y = -2x^2 + 11x + 21$, indicate all its intercepts, vertex, and axis of symmetry

The parabola is

$$y = -(2x + 3)(x - 7) = -2(x^2 - \frac{11}{2}x) + 21 = -2(x - \frac{11}{4})^2 + (21 + 2 \cdot \frac{11^2}{4^2}) = -2(x - \frac{11}{4})^2 + (\frac{289}{8}).$$

x-intercepts: $= (\frac{-3}{2}, 0)$, $(7, 0)$
y-intercepts: $= (0, 21)$
vertex $= (\frac{11}{4}, \frac{289}{8})$
axis of symmetry: $x = \frac{11}{4}$

Quiz 13-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/09 7:45-8:15

*A problem worths 50 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ________________________ Name: ________________________

Draw the parabola $y = x^2 - 9$, indicate all its intercepts, vertex, and axis of symmetry

x-intercepts = ____________

y-intercepts = ____________

vertex = ____________

axis of symmetry: ____________

Draw the parabola $y = -2x^2 + 11x + 21$, indicate all its intercepts, vertex, and axis of symmetry

x-intercepts = ____________

y-intercepts = ____________

vertex = ____________

axis of symmetry: ____________

Solution to Quiz 14-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/13 6:00-6:30pm

*A problem worths 50 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Draw the parabola $y = x^2 + 9$, indicate all its intercepts, vertex, and axis of symmetry

The parabola has no roots because "$b^2 - 4ac < 0$".
x-intercepts: none.
y-intercepts: $(0, 9)$
vertex $= (0, 9)$
axis of symmetry: $x = 0$

Draw the parabola $y = 3x^2 + 5x + 2$, indicate all its intercepts, vertex, and axis of symmetry

The parabola is

$$y = (3x+2)(x+1) = 3(x^2 + \frac{5}{3}x) + 2 = 3(x + \frac{5}{6})^2 + (2 - (\frac{5}{6})^2) = 3(x + \frac{5}{6})^2 + \frac{47}{36}.$$

x-intercepts: $= (\frac{-2}{3}, 0)$, $(-1, 0)$
y-intercepts: $= (0, 2)$
vertex $= (-\frac{5}{6}, \frac{47}{36})$
axis of symmetry: $x = -\frac{5}{6}$

Quiz 14-1 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/13 6:00-6:30pm

*A problem worths 50 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Draw the parabola $y = x^2 + 9$, indicate all its intercepts, vertex, and axis of symmetry

x-intercepts = ______________

y-intercepts = ______________

vertex = ______________

axis of symmetry: ______________

Draw the parabola $y = 3x^2 + 5x + 2$, indicate all its intercepts, vertex, and axis of symmetry

x-intercepts = ______________

y-intercepts = ______________

vertex = ______________

axis of symmetry: ______________

Solution to Quiz 14-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/13 7:45-8:15

*A problem worths 25 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the quadratic inequality

$x^2 - 4x + 3 > 0 \Leftrightarrow x < 1$ or $x > 3$.

Solve the quadratic inequality

$x^2 + 4x + 4 \leqslant 0 \Leftrightarrow x = -2$

Solve the quadratic inequality

$x^2 + 4x + 4 < 0 \Leftrightarrow$ (no solution).

Solve the quadratic inequality

$x^2 + x + 1 > 0 \Leftrightarrow x \in R$

A letter to all:

Congratulations! The course is almost done! This is not at all an easy one, especially it was condensed into a 6-week course. The first moment when I read the syllabus, I thought that was absolutely crazy. I could not have learnt that fast, so how then should I teach this course? When I turned to someone who has taught MAP-103, they brought bad news to me and wished me good luck. However, you guys are so different, and I really appreciate all the efforts you made.

For those who do not have to deal with math course, congrats! For those who still have to, I wish you good luck and would like to share the trick about learning (at least for math) again: try to find the most basic principles not just for the tests but for math itself. This really helps improve your math. As far as I know, no one good at math I know does NOT use this trick! So I guess that is the ultimate secret of learning math. Though you'll find it hard in the beginning, it will become easier and easier every time you try.

I was not that kind of math genius, so I really understand the pain of math; however, every time when I got the ah-ha moment, I feel so rewarding. So I sincerely hope this course did not only bring you painful memories, but also some of the ah-ha moments. Guess this is the very most I can do, and I hope I did.

To be honest, this is the first time I teach on stage, and you guys made me enjoy it. Anyways, thank you all very much! I feel grateful! Wish you have a good rest of the summer.

Best, Jin

Quiz 14-2 (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/13 7:45-8:15

*A problem worths 25 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the quadratic inequality

$x^2 - 4x + 3 > 0 \Leftrightarrow$ ______________________

Solve the quadratic inequality

$x^2 + 4x + 4 \leqslant 0 \Leftrightarrow$ ______________________

Solve the quadratic inequality

$x^2 + 4x + 4 < 0 \Leftrightarrow$ ______________________

Solve the quadratic inequality

$x^2 + x + 1 > 0 \Leftrightarrow$ ______________________

Solution to Final (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/15 6:00-7:00pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the equation.

$(x-\sqrt[3]{3})(x-\pi)(x+\frac{7}{2})=0 \Leftrightarrow x=\sqrt[3]{3}, \pi, \frac{-7}{2}$

Factor the polynomial if possible

$3x^2+2x-21=(3x-7)(x+3)$

Factor the polynomial if possible

$x^2+x-1=(x-\frac{-1+\sqrt{5}}{2})(x-\frac{-1-\sqrt{5}}{2})$

Factor the polynomial if possible

$x^2+0.8x+1$ is not factorizable because $b^2-4ac<0$.

The Vertex of the parabola $y=-2x^2+11x+21$ is vertex $=(\frac{11}{4},\frac{289}{8})$

The Vertex of the parabola $y=3x^2+5x+2$ is vertex $=(-\frac{5}{6},\frac{47}{36})$

Solve the quadratic inequality

$x^2-4x+3>0 \Leftrightarrow x<1$ or $x>3$.

Solve the quadratic inequality

$x^2+4x+4\leqslant 0 \Leftrightarrow x=-2$

Solve the quadratic inequality

$x^2+4x+4<0 \Leftrightarrow$ (no solution).

Solve the quadratic inequality

$x^2+x+1>0 \Leftrightarrow x\in R$

Final (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/15 6:00-7:00pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Solve the equation.

$(x-\sqrt[3]{3})(x-\pi)(x+\frac{7}{2})=0 \Leftrightarrow x=$ ____________________

Factor the polynomial if possible

$3x^2+2x-21=$ ____________________

Factor the polynomial if possible

$x^2+x-1=$ ____________________

Factor the polynomial if possible

$x^2+0.8x+1=$ ____________________

The Vertex of the parabola $y=-2x^2+11x+21$ is = ____________________

The Vertex of the parabola $y=3x^2+5x+2$ is vertex = ____________________

Solve the quadratic inequality

$x^2-4x+3>0 \Leftrightarrow$ ______________________________

Solve the quadratic inequality

$x^2+4x+4\leqslant 0 \Leftrightarrow$ ______________________________

Solve the quadratic inequality

$x^2+4x+4<0 \Leftrightarrow$ ______________________________

Solve the quadratic inequality

$x^2+x+1>0 \Leftrightarrow$ ______________________________

Solution to Midterm I (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/23 6:30-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Simplify the expression

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \frac{9}{6} = \frac{3}{2}$$
$$-14\left(-\frac{2}{7}\right) - 14 = -10$$

Simplify the expression

$$4 - [\,(7-6) + (9-19)\,] = 13$$
$$4\{-5 + 3[3 - 5(-3+1)]\} = 4 \times 34 = 136$$

Add and multiply

$$1 + 2 + 3 + \cdots + 50 = \frac{(1+50)\cdot 50}{2} = 51 \cdot 25 = 1275$$
$$4 \cdot 53 \cdot 25 = 4 \cdot 25 \cdot 53 = 100 \cdot 53 = 5300$$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

d. $-\frac{8r}{-9s}$

Expansion (clear the parenthesis)

$(a+b)\cdot(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$ $(a+b+c)\cdot(x+y) = ax + ay + bx + by + cx + cy$

Compute

$(-4)^{-2} = \frac{1}{16}$ $(-4)^{-1} = \frac{-1}{4}$ $(-4)^0 = 1$ $(-4)^1 = -4$ $(-4)^2 = 16$

Simplify the expression

$$x^3 \cdot x^{-8} \cdot x^4 = x^{3-8+4} = x^{-1} = \frac{1}{x}$$
$$\frac{a^4 b^{-2}}{a^{-2} b^2} = \frac{a^{4+2}}{b^{2+2}} = \frac{a^6}{b^4}$$

Write the polynomial in standard form and indicate its degree

$$(x+2)(3x+1)(1-x) = (3x^2+7x+2)(1-x) = (3x^2+7x+2) - x(3x^2+7x+2)$$
$$= (3x^2+7x+2) - (3x^3+7x^2+2x) = -3x^3 - 4x^2 + 5x + 2;\ \text{degree} = 3$$

$x(-x(-2x+1)+4) - 1 = x(2x^2 - x + 4) - 1 = 2x^3 - x^2 + 4x - 1$, degree $= 3$

Multiplication

Let $p = x-1$, and $q = -x^2-3x+1$. Compute $p\cdot q = -(x-1)(x^2+3x-1) = -[(x^3+3x^2-x)-(x^2+3x-1)] = -x^3-2x^2+4x-1$
$(2x+3)(2x-3) = 4x^2 - 9$

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a-1$. $\frac{x^2+5}{x-3}\big|_{x=a-1} = \frac{(a-1)^2+5}{(a-1)-3} = \frac{a^2-2a+6}{a-4}$

Bonus: Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x} = \frac{-1}{x(x+4)}$

Midterm I (MAP-103 Proficiency Algebra Summer-II 2018) 2018/07/23 6:30-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________ Name: ______________________

Simplify the expression

$$\frac{7}{6} - \left(-\frac{1}{3}\right) = \_\_\_\_\_\_$$

$$-14\left(-\frac{2}{7}\right) - 14 = \_\_\_\_\_\_$$

Simplify the expression

$$4 - [\,(7-6) + (9-19)\,] = \_\_\_\_\_\_$$

$$4\{-5 + 3[3 - 5\,(-3+1)]\} = \_\_\_\_\_\_$$

Add and multiply

$$1 + 2 + 3 + \cdots + 50 = \_\_\_\_\_\_$$

$$4 \cdot 53 \cdot 25 = \_\_\_\_\_\_$$

Choose the fraction(s) equivalent to $\frac{-8r}{-9s}$ (select all that apply).

a. $-\frac{8r}{9s}$ b. $\frac{-8r}{9s}$

c. $\frac{8r}{-9s}$ d. $-\frac{8r}{-9s}$

Expansion (clear the parenthesis)

$(a+b)\cdot(a+b) =$ ____________ $(a+b+c)\cdot(x+y) =$ ______________________

Compute

$(-4)^{-2} =$ ______ $(-4)^{-1} =$ ______ $(-4)^{0} =$ ______ $(-4)^{1} =$ ______ $(-4)^{2} =$ ______

Simplify the expression

$$x^3 \cdot x^{-8} \cdot x^4 = \_\_\_\_\_\_$$

$$\frac{a^4 b^{-2}}{a^{-2} b^2} = \_\_\_\_\_\_$$

Write the polynomial in standard form and indicate its degree

$(x+2)(3x+1)(1-x) =$ ____________________, degree = ______

$x(-x(-2x+1)+4) - 1 =$ ____________________, degree = ______

Multiplication

Let $p = x - 1$, and $q = -x^2 - 3x + 1$. Compute $p \cdot q =$ ____________________

$(2x+3)(2x-3) =$ ____________________

Substitution of a rational expression

Find the value of the expression $\frac{x^2+5}{x-3}$ for $x = a - 1$. $\frac{x^2+5}{x-3}\Big|_{x=a-1} =$ ______________________

Bonus: Simplify the expression

$\frac{x+1}{16-x^2} \cdot \frac{x-4}{x^2+x}$ ____________________

Solution to Midterm II (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/06 6:00-8:15pm
*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select all that apply.

(a) $x = x + 2$ is always false.
(c) $(x+1)^2 - 1 = x(x+2)$ is always true.

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x = 28$

Solve the inequality.

$-3x < 6 \Leftrightarrow x > -2.$

$|1-x| \geqslant 5 \Leftrightarrow x \geqslant 6$ or $x \leqslant -4.$

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope $= -\frac{3}{4}$
$x-$intercept $= -\frac{2}{3}$
$y-$intercept $= -\frac{1}{2}$

Find the linear equation of a given line.

The line that passes through $(1,0)$ and is parallel to the line $y = 2x+3$: $y = 2x - 2$

Solve the linear system.

$\begin{cases} -2x + 3y = 8 \\ 2x - y = 0 \end{cases}$ ($(2,4)$ is the only solution.)

Solve the linear system.

$\begin{cases} -2x + y = 3 \\ -4x + 2y = 6 \end{cases}$ (There are infinitely many solutions.)

Solve the linear system.

$\begin{cases} -2x + 3y = 3 \\ 3x + 2y = 6 \end{cases}$ ($(\frac{12}{13}, \frac{21}{13})$ is the only solution.

Simplify the following expressions

$\frac{2}{6-\sqrt{5}} = \frac{2(6+\sqrt{5})}{31}$, $\frac{\sqrt{7}-2}{6-\sqrt{7}} = \frac{(\sqrt{7}-2)(6+\sqrt{7})}{29} = \frac{-5+4\sqrt{7}}{29}$, $\sqrt{7}(\sqrt{21} - \sqrt{35}) = 7(\sqrt{3} - \sqrt{5}))$

Compute

$27^{\frac{4}{3}} = 81, 27^{\frac{-4}{3}} = \frac{1}{81}, 27^{\frac{4}{-3}} = \frac{1}{81}, 27^{\frac{-4}{-3}} = 81$, $2^{\frac{7}{4}} \cdot 2^{\frac{4}{7}} \cdot 2^{\frac{-1}{2}} = 2^{\frac{51}{28}}$.

Midterm II (MAP-103 Proficiency Algebra Summer-II 2018) 2018/08/06 6:00-8:15pm

*A problem worths 10 points. If you answer a choice problem incorrectly, 5 points will be taken away. So it is better to leave a choice problem blank if you are not certain of your answer.

ID: ______________________________ Name: ______________________________

Select all that apply.

(a) $x = x + 2$ is always false.
(b) $x + 2 = 5$ is always false.
(c) $(x+1)^2 - 1 = x(x+2)$ is always true.
(d) $x^2 - y^2 = (x-y)^2$ is always true.

Find all solutions to the equation.

$\frac{2}{3}x - 5 = \frac{1}{6}x + 9 \Leftrightarrow x =$ ____________________

Solve the inequality.

$-3x < 6$: ____________________.

$|1 - x| \geqslant 5 \Leftrightarrow$ ______________________________.

Draw the line $3x + 4y + 2 = 0$, and indicate its slope and intercepts.

Slope = _____
$x-$intercept = _____
$y-$intercept = _____

Find the linear equation of a given line.

The line that passes through $(1, 0)$ and is parallel to the line $y = 2x + 3$: _______________

Solve the linear system.

$$\begin{cases} -2x + 3y = 8 \\ 2x - y = 0 \end{cases}$$

Solve the linear system.

$$\begin{cases} -2x + y = 3 \\ -4x + 2y = 6 \end{cases}$$

Solve the linear system.

$$\begin{cases} -2x + 3y = 3 \\ 3x + 2y = 6 \end{cases}$$

Simplify the following expressions

$\frac{2}{6-\sqrt{5}} =$ ___, $\frac{\sqrt{7}-2}{6-\sqrt{7}} =$ ___, $\sqrt{7}(\sqrt{21} - \sqrt{35}) =$ ___

Compute

$27^{\frac{4}{3}} =$ ___, $27^{\frac{-4}{3}} =$ ___, $27^{\frac{4}{-3}} =$ ___, $27^{\frac{-4}{-3}} =$ ___, $2^{\frac{7}{4}} \cdot 2^{\frac{4}{7}} \cdot 2^{\frac{-1}{2}} =$ _____

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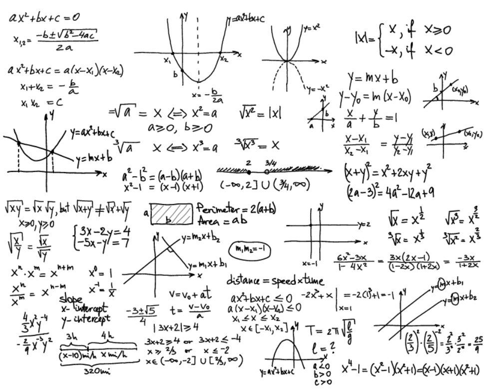
Lecture 1

Numbers and Operations

From chaos to harmony

For many of us, mathematics is a messy jumble of incomprehensible formulas:,
just the way musical scores are for someone who can't read music.



Both formulas and scores hide meaning and harmony.

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What this course is about

This a proficiency course in Algebra.

We will learn the basics of **algebraic literacy**:

- how to read, understand and manipulate algebraic expressions,
- how to solve simple equations and inequalities,
- how to visualize formulas by drawing graphs.

Enjoy the course!

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What Algebra studies

• **Numbers:** $1,\ \frac{5}{7},\ -27.4,\ 0,\ \sqrt{2},\ \pi,\ \ldots$

• **Operations** with numbers:
addition $1+2=3$,
subtraction $3-1=2$,
multiplication $3\cdot 2=6$,
division $6\div 3=2$,
exponentiation $2^3=8$,
taking the radical $\sqrt{49}=7$,
and their combinations $-5+2^3\cdot(3-\sqrt{4})=3$.

Often numbers are denoted by symbols (letters):

$$a,\ b,\ c,\ \ldots,\ x,\ y,\ z,\ A,\ B,\ C,\ \ldots,\ X,\ Y,\ Z,\ \alpha,\ \beta,\ \gamma,\ \ldots$$

Symbols are connected by operations into **formulas**:

$$1+2x\,,\quad x-3y\,,\quad x^2-x+1\,,\quad x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\,,\ \ldots$$

Numbers

Positive integers: $1,\ 2,\ 3,\ 4,\ 5,\ldots$

Integers: $\ldots -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots$

Rational numbers are quotients $\frac{p}{q}$, where p and q are integers and $q\neq 0$.

For example, $\frac{1}{2},\ \frac{2}{1},\ \frac{6}{3},\ \frac{-4}{7}$ are rational numbers.

Any integer is a rational number. For example, $3=\frac{3}{1}$.

Irrational numbers are numbers which cannot be represented as a quotient of two integers.

For example, $\sqrt{2}\,,\ \sqrt[3]{5}\,,\ \sqrt{2}+\sqrt{3}\,,\ \pi\,.$

Decimal presentations

Rational numbers can be represented as **decimals**:

$$\frac{9}{2} = 4.5, \quad \frac{1}{3} = 0.333\ldots = 0.\overline{3}$$

Any rational number is presented as either a **finite** decimal, like $\frac{9}{2} = 4.5$ or $\frac{7}{8} = 0.875$,

or a **repeating** decimal, like $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$ or $\frac{168}{11} = 15.272727\ldots = 15.\overline{27}$.

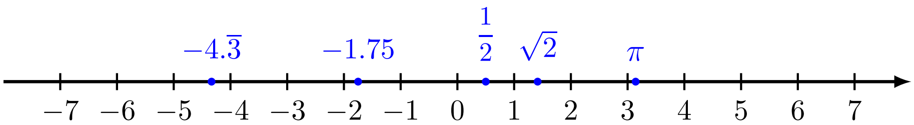
Irrational numbers also have decimal representations. They are infinite and not repeating.

For example, $\sqrt{2} = 1.41421356\ldots$ and $\pi = 3.14159265\ldots$.

Real numbers and the real line

Both rational and irrational numbers are called **real** numbers.

Real numbers live on the **real line**:



Summary

In this lecture, we have learned

- ☑ what this course is about
- ☑ what Algebra studies (**numbers, operations, formulas**)
- ☑ what kinds of numbers we are going to deal with (**integers, rational and irrational numbers**)
- ☑ what is a decimal representation of a number
- ☑ what real numbers are
- ☑ what the real line is

Lecture 2

Numerical Expressions

Numerical expressions

A **numerical expression** consists of
numbers, symbols of operations and parentheses,
and describes an algorithm (a set of instructions) for calculation.

For example, $3 - 8 \div 4 \cdot (1 + 2)$.

The result of the calculation is called the **value** of the numerical expression.
The process of calculation is called **evaluation**.

In this lecture we will learn **how to evaluate** a numerical expression.
For example, here is the evaluation of the numerical expression given above:

$$\begin{aligned}&3 - 8 \div 4 \cdot (1 + 2) =\\&3 - 8 \div 4 \cdot 3 =\\&3 - 2 \cdot 3 =\\&3 - 6 =\\&-3.\end{aligned}$$

In particular, we will learn in which **order** to perform the arithmetic operations.

Without parentheses

Multiplication and division have to be done **before** addition and subtraction,
if the formula does not contain parentheses.

By this rule, $1 + 2 \cdot 3 = 1 + 6 = 7$.

If the formula contains several multiplications and divisions(and still no parentheses),
the multiplications and divisions are performed in order **from left to right.**

For example, $6 \div 3 \cdot 5 = \boxed{\boxed{6 \div 3} \cdot 5} = 2 \cdot 5 = 10$,

$6 \div 3 + 4 \cdot 5 = \boxed{6 \div 3} + \boxed{4 \cdot 5} = 2 + \boxed{4 \cdot 5} = 2 + 20 = 22$.

Additions and subtractions are done after all multiplications and divisions, also **from left to right**:

$5 - 4 \div 2 + 3 \cdot 2 \div 6 = 5 - \boxed{4 \div 2} + \boxed{\boxed{3 \cdot 2} \div 6} = 5 - 2 + 1 = \boxed{5 - 2} + 1 = 3 + 1 = 4$.

Two kinds of parentheses

In expressions, parentheses play **two** different roles.

• First, they help describe the **order of operations**:

$$(1+2)\cdot 3 = 3\cdot 3 = 9\,.$$

Notice that the expression above **without** parentheses has a different value:

$$1+2\cdot 3 = 1+6 = 7\,.$$

• Second, parentheses have to surround a **negative** number,

when the number comes after the sign of an arithmetic operation, as in

$$2+(-3)\,, \quad \text{or} \quad 2\cdot(-3)\,.$$

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Parentheses around a negative number

Parentheses around a negative number **do not matter** for the order of operations.

If all parentheses in a formula are of that kind,

then calculations should be performed as if there were no parentheses:

first, all multiplications and divisions from left to right,

then all additions and subtractions from left to right:

$$(-4)\div 2+3\cdot(-5) = \boxed{(-4)\div 2}+\boxed{3\cdot(-5)} = -2+(-15) = -17\,.$$

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Parentheses rule

If a formula contains parentheses which surround more than one number, then

1. find the innermost parentheses of this kind,
2. evaluate the formula within the parentheses,
3. and continue if needed.

For example,

$$\begin{aligned}(3-1)\cdot(1+4\div(3-5)) &=\\ 2\cdot(1+4\div(3-5)) &=\\ 2\cdot(1+4\div(-2)) &=\\ 2\cdot(1+(-2)) &=\\ 2\cdot(-1) &=\\ -2\,.\end{aligned}$$

Summary

In this lecture, we have learned

- ☑ what a **numerical expression** is
- ☑ what the **value** and **evaluation** of a numerical expression are
- ☑ how parentheses are used in a numerical expression
- ☑ in which order arithmetic operations are performed

Lecture 3

Variables and Algebraic Expressions

Variables

A **variable** is a letter representing a number.

Why do we need letters?

- Some numbers are **special**,
  but don't have any convenient representation, like π.
- Some numbers are given by a formula, which is too **bulky** to deal with,
  like the golden ratio $\varphi = \dfrac{1+\sqrt{5}}{2}$.
- Sometimes we don't know the number, but want to **find** it.
  For example, when we are solving the equation $2x+1=7$.
- Sometimes we want to express a **relationship** between quantities,
  like $d = v \cdot t$, where d is distance, v is speed and t is time.

Variables for numbers are like **names** (or nicknames) for people.

Algebraic expressions

We already know (from Lecture 2) that a **numerical expression**
consists of numbers, symbols for operations and parentheses,
and describes an algorithm for calculation.

For example, $1 \cdot 2 - 3 \cdot (1+2) \div 4$ is a numerical expression.

An **algebraic expression** (or simply "an expression")
consists of numbers, **variables**, symbols for operations and parentheses,
and becomes a numerical expression when we substitute (plug in)
a numerical value for each variable.

Example 1. $3 \cdot x - 4 \cdot (x+1)$ is an algebraic expression. It involves the numbers $3, 4, 1$, the variable x, and the operations multiplication, addition and subtraction. How many operations are there in this expression? Four.

Example 2. $x \cdot y - \dfrac{5 \cdot (x+y)}{4}$ is an algebraic expression. It involves the numbers $5, 4$, the variables x, y, and the operations multiplication, division, addition and subtraction. How many operations are there in this expression? Five.

When can the multiplication dot be omitted?

It is customary **not** to write the multiplication dot in front of a variable or parenthesis:

$a \cdot b$ is written as ab,

$2 \cdot x$ is written as $2x$,

but the dot has to be present in $x \cdot 2$ and $2 \cdot 2$,

$a \cdot (b + c)$ is written as $a(b + c)$,

$(a + b) \cdot (c + d)$ is written as $(a + b)(c + d)$.

Evaluation of expressions

An algebraic expression becomes a numerical expression
if we **substitute** (plug in) a numerical value for each variable.

For example, if we plug $x = 2$ into the expression $3x - 4(x + 1)$, we get

$$3x - 4(x + 1)\Big|_{x=2} = 3 \cdot 2 - 4(2 + 1),$$

which is a numerical expression. Its value is

$$3 \cdot 2 - 4(2 + 1) = 6 - 4 \cdot 3 = 6 - 12 = -6.$$

This process is called **evaluation** at $x = 2$.

A numerical expression is a **special kind** of algebraic expression.

A **numerical** expression is an **algebraic** expression which contains **no** variables.

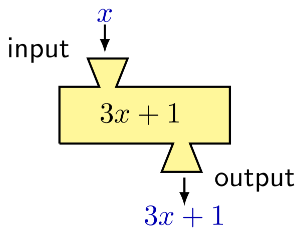
An expression as a program

An expression may be understood as a **program**
(or algorithm, or set of instructions) describing a calculation.

For example, the expression $3x+1$ represents the following procedure:

$$x \xrightarrow{\text{multiply by } 3} 3x \xrightarrow{\text{add } 1} 3x+1.$$

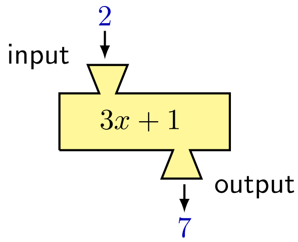
For each value of the variable x (for each input), this program delivers an output, which is called the **value** of the expression $3x+1$.



Evaluating an expression is running the program

When we **evaluate** an expression at a number, we run the corresponding program.

For example, to evaluate the expression $3x+1$ at the number 2,
we need to plug $x=2$ into $3x+1$:



We denote this evaluation as follows:

$$3x+1\Big|_{x=2} = 3\cdot 2+1 = 7.$$

Examples of evaluations

Example 1. Evaluate the expression $\frac{2x-1}{x+1}$ at $x=-3$.

Solution.

$$\left.\frac{2x-1}{x+1}\right|_{x=-3} = \frac{2(-3)-1}{(-3)+1} = \frac{-6-1}{-2} = \frac{-7}{-2} = \frac{7}{2}.$$

Example 2. Find the value of the expression $3(x-1)+2y$ at $x=1$, $y=-2$.

Solution.

$$\left.3(x-1)+2y\right|_{x=1,\,y=-2} = 3(1-1)+2(-2) = 3\cdot 0-4 = 0-4 = -4.$$

Why algebraic expressions are important

Algebraic expressions and operations with them are fundamentally **involved** in all parts of Algebra.

So far, we have met only the **simplest** of them.

Later in the course we will study **more complex** expressions and operations.

Fluency in operating with algebraic expressions is crucial for your success in the course.

Summary

In this lecture, we have learned

- ☑ what a variable is
- ☑ what an algebraic expression is
- ☑ how to evaluate an expression at a number
- ☑ how to understand an expression as a program
- ☑ why algebraic expressions are important

Lecture 4

Addition and Multiplication

Properties of operations

Addition and multiplication are **basic** arithmetic operations.

They share two useful **properties**.

These properties are

- **commutativity**
- **associativity**

In this lecture, we will study these properties

and learn how to make use of them.

Commutativity of addition

When adding two numbers, the order of the numbers doesn't matter.

For example, $2 + 3 = 3 + 2$.

This property of addition can be written using variables:

$$\boxed{a + b = b + a \quad \text{for any } a \text{ and } b}$$

Since a and b can represent **any** numbers, this formula represents infinitely many equalities.
For example, if $a = 8$ and $b = 5$, then $a + b = b + a$ becomes
$8 + 5 = 5 + 8$.
If $a = x$ and $b = 5$, then $a + b = b + a$ becomes
$x + 5 = 5 + x$.

This property of addition is called **commutativity**.

Commutativity of multiplication

Multiplication is also **commutative**.

When multiplying two numbers, the order of the numbers doesn't matter.

For example, $2 \cdot 3 = 3 \cdot 2$.

This property is expressed using variables as follows:

$$\boxed{a \cdot b = b \cdot a \quad \text{for any } a \text{ and } b}$$

Since a and b represent **any** numbers, this formula represents infinitely many equalities.

For example, if $a = 4$ and $b = 7$, then $a \cdot b = b \cdot a$ becomes
$4 \cdot 7 = 7 \cdot 4$,

if $a = 2$ and $b = x$, then $a \cdot b = b \cdot a$ becomes
$2 \cdot x = x \cdot 2$.

Associativity of addition

When we add **three** numbers, the result does **not** depend on the order of operations:

$$(1 + 2) + 3 = 3 + 3 = 6$$
$$1 + (2 + 3) = 1 + 5 = 6 .$$

That is, $(1 + 2) + 3 = 1 + (2 + 3)$.

In general,

$$\boxed{(a + b) + c = a + (b + c) \ \text{ for any } a , \ b \text{ and } c}$$

This property of addition is called **associativity**.

Associativity helps to make calculations easier. Compare:

$428 + 13999 + 1 = (428 + 13999) + 1 = 14427 + 1 = 14428$ and

$428 + 13999 + 1 = 428 + (13999 + 1) = 428 + 14000 = 14428$.

Associativity of multiplication

Multiplication is also **associative**:

$$\boxed{(ab)c = a(bc) \text{ for any } a\,,\ b \text{ and } c}$$

Associativity of multiplication is useful:

$53 \cdot 25 \cdot 4 = 53 \cdot (25 \cdot 4) = 53 \cdot 100 = 5300.$

In the next examples, **both** associativity and commutativity are used:

$5 \cdot 97 \cdot 20 = (5 \cdot 97) \cdot 20 = (97 \cdot 5) \cdot 20 = 97 \cdot (5 \cdot 20) = 97 \cdot 100 = 9700,$

$2x \cdot 3y = 2(x \cdot 3)y = 2(3x)y = (2 \cdot 3)xy = 6xy.$

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When can we leave out parentheses?

Due to **associativity**,
when we perform either additions only, or multiplications only,
the result does **not** depend on the order of operations:

$$((1+2)+3)+4 = (1+(2+3))+4 = 1+((2+3)+4)$$
$$((2\cdot 3)\cdot 4)\cdot 5 = (2\cdot(3\cdot 4))\cdot 5 = 2\cdot((3\cdot 4)\cdot 5)\,.$$

Therefore, we do **not** use parentheses in a formula
which involves additions **only** or multiplications **only**, like this

$$1+2+3+4\,, \qquad 2\cdot 3\cdot 4\cdot 5$$

Moreover, due to **commutativity**, the order of numbers **doesn't** matter:

$$1+2+3+4 = 2+3+4+1 = 4+2+1+3 = \dots$$

$$2\cdot 3\cdot 4\cdot 5 = 2\cdot 3\cdot 5\cdot 4 = 4\cdot 2\cdot 5\cdot 3 = \dots$$

Recall that if **both** addition and multiplication are present,
then the order **does** matter: $(1+2)\cdot 3 \neq 1+2\cdot 3$

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Special numbers: 0 and 1

$$\boxed{a + 0 = a \quad \text{for any } a}$$

Numbers a and $-a$ are called **opposite** to each other.

For example, -2 is opposite to 2, and 2 is opposite to -2.

$$\boxed{a + (-a) = 0 \quad \text{for any } a}$$

The product of any number by 0 equals 0:

$$\boxed{a \cdot 0 = 0 \quad \text{for any } a}$$

The product of any number by 1 equals this number:

$$\boxed{a \cdot 1 = a \quad \text{for any } a}$$

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Reciprocals

Numbers a and b are called **reciprocals** if $a \cdot b = 1$.

For example, 2 and $\frac{1}{2}$ are reciprocals, since $2 \cdot \frac{1}{2} = 1$.

Numbers a and $\frac{1}{a}$ are reciprocals for any non-zero a.

$$\boxed{a \cdot \frac{1}{a} = 1 \quad \text{for any non-zero } a}$$

0 has **no** reciprocal, because there is **no** number b such that $0 \cdot b = 1$.

Indeed, $0 \cdot b = 0$ for any b.

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Summary

In this lecture, we have learned

- ☑ commutativity of addition: $a + b = b + a$
- ☑ commutativity of multiplication: $ab = ba$
- ☑ associativity of addition: $(a + b) + c = a + (b + c)$
- ☑ associativity of multiplication: $(ab)c = a(bc)$
- ☑ when parentheses are not needed
- ☑ identities involving 0 and 1: $a + 0 = a,\ a \cdot 1 = a,\ a \cdot 0 = 0$
- ☑ opposite numbers
- ☑ reciprocal numbers

Lecture 5

Subtraction and Division

Subtraction is the opposite of addition

Subtraction is the operation which is **opposite** to addition:

$$\begin{tikzcd} 3 \arrow[r, bend left, "+2"] & 5 \arrow[l, bend left, "-2"] \end{tikzcd}$$

This means that $(3+2)-2=3$ and $(5-2)+2=5$.

Recall that numbers a and $-a$ are called **opposite** to each other.
For example, -2 is opposite to 2, and 2 is opposite to -2.

Subtraction of a number is addition of its opposite:

$$5-2=5+(-2)=3 \quad \text{and} \quad 5-(-2)=5+2=7\,.$$

Therefore, we can **express** any subtraction as addition of the opposite quantity:

$$a-b=a+(-b) \quad \text{for any } a,\ b\,.$$

No commutativity for subtraction

We know that addition is **commutative**: $a+b=b+a$ for any $a,\ b$.

Subtraction is **not** commutative: it is **not** true that $a-b=b-a$ unless $a=b$.

Indeed, take $a=1$ and $b=2$. Then $a-b=1-2=-1$,
but $b-a=2-1=1$.

In general, $a-b$ and $b-a$ are opposite to each other: $b-a=-(a-b)$.

So subtraction is **not** commutative.
But expressing subtraction $a-b$ in terms of addition $a+(-b)$, we may apply the commutativity of addition to get:

$$a-b=a+(-b)=-b+a \quad \text{for any } a,\ b\,.$$

No associativity for subtraction

We know that addition is **associative**:

$$(a+b)+c = a+(b+c) \quad \text{for any } a,\ b,\ c\,.$$

Subtraction is **not** associative:

$$(a-b)-c \neq a-(b-c)\,.$$

For example, if $a=3$, $b=1$ and $c=1$, then

$$(a-b)-c = (3-1)-1 = 2-1 = 1\,,$$
$$\text{but } a-(b-1) = 3-(1-1) = 3-0 = 3\,.$$

So subtraction is **not** associative.

But expressing subtraction $(a-b)-c$ in terms of addition $(a+(-b))+(-c)$, we may apply the associativity of addition to get:

$$(a-b)-c = (a+(-b))+(-c) = a+((-b)+(-c)) = a+(-b-c)\,.$$

Recall that $a-b-c$ has to be understood as $(a-b)-c$.

Division is the opposite of multiplication

Division is the operation which is **opposite** to multiplication:

$$\begin{tikzcd} 3 \arrow[r, bend left, "\times 2"] & 6 \arrow[l, bend left, "\div 2"] \end{tikzcd}$$

This means that $\quad (3\cdot 2)\div 2 = 3 \quad$ and $\quad (6\div 2)\cdot 2 = 6.$

Recall that numbers a and $1/a$ are called **reciprocals**.

For example, 2 and $1/2$ are reciprocals.

Division by a **non-zero** number is multiplication by its **reciprocal**:

$$6\div 2 = 6\cdot\frac{1}{2} = 3 \quad \text{and} \quad 6\div\frac{1}{2} = 6\cdot 2 = 12\,.$$

(Keep in mind that the reciprocal of $\frac{1}{2}$ is 2.)

In general: $\quad a\div b = a\cdot\frac{1}{b} \quad$ for any a and non-zero b.

Negative one

The reciprocal of -1 is -1, that is $\frac{1}{-1} = -1$. Indeed, $(-1)(-1) = 1$.

Sometimes negative one is slightly hidden: $-a = (-1)a$.

It is helpful to keep this in mind.

For example, $\frac{-a}{-b} = \frac{a}{b}$, because $\frac{-a}{-b} = \frac{(-1)a}{(-1)b} = \frac{a}{b}$.

Another example: $\frac{a}{-b} = \frac{a}{(-1)b} = \frac{1}{-1}\frac{a}{b} = (-1)\frac{a}{b} = -\frac{a}{b} = \frac{-a}{b}$.

Why division by zero does not make sense

Let us try to divide some number, say 1, by 0.
We do not know what result will be. Let us call it x: $\quad 1 \div 0 = x$.

$$\begin{tikzcd} x \arrow[r, bend left, "\times 0"] & 1 \arrow[l, bend left, "\div 0"] \end{tikzcd}$$

If $1 \div 0 = x$, then x is a number such that $\quad x \cdot 0 = 1$.
Which is **impossible** since $\quad x \cdot 0 = 0 \quad$ for any x.

Never divide by zero! It doesn't make sense.

No commutativity for division

We know that multiplication is **commutative**: $ab = ba$ for any a, b.

Division is **not** commutative:

in general, it is **not** true that $a \div b = b \div a$.

For example, if $a = 2$ and $b = 1$, then $a \div b = 2 \div 1 = 2$,

but $b \div a = 1 \div 2 = \frac{1}{2}$.

The expressions $a \div b$ and $b \div a$ are **reciprocal** to each other.

Indeed, $a \div b = a \cdot \frac{1}{b}$ and $b \div a = b \cdot \frac{1}{a}$. Therefore

$$(a \div b)(b \div a) = \left(a \cdot \frac{1}{b}\right) \cdot \left(b \cdot \frac{1}{a}\right) = a \left(\frac{1}{b} \cdot b\right) \frac{1}{a} = a \cdot 1 \cdot \frac{1}{a} = a \cdot \frac{1}{a} = 1$$

In fractional notation, this may be written as $\frac{b}{a} = \frac{1}{a/b}$.

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No associativity for division

We know that multiplication is **associative**:

$$(ab)c = a(bc) \quad \text{for any } a, b, c.$$

Division is **not** associative: $(a \div b) \div c \neq a \div (b \div c)$.

Or, in fractional notation, $\frac{a/b}{c} \neq \frac{a}{b/c}$.

For example, if $a = 8$, $b = 4$ and $c = 2$, then

$$(a \div b) \div c = (8 \div 4) \div 2 = 2 \div 2 = 1,$$

but $a \div (b \div c) = 8 \div (4 \div 2) = 8 \div 2 = 4$.

So division is **not** associative.

But expressing division $(a \div b) \div c$ in terms of multiplication $\left(a \cdot \frac{1}{b}\right) \cdot \frac{1}{c}$,

we may apply the associativity of multiplication to get:

$$(a \div b) \div c = \left(a \cdot \frac{1}{b}\right) \cdot \frac{1}{c} = a \cdot \left(\frac{1}{b} \cdot \frac{1}{c}\right) = a \cdot \frac{1}{b \cdot c} = a \div (b \cdot c).$$

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Summary

In this lecture, we have learned that

- ☑ subtraction is the **opposite** of addition
- ☑ subtraction can be **expressed** as addition of the opposite: $a - b = a + (-b)$
- ☑ subtraction is **neither** commutative **nor** associative
- ☑ division is the **opposite** of multiplication
- ☑ division can be **expressed** as multiplication by the reciprocal: $a \div b = a \cdot \dfrac{1}{b}$
- ☑ division by zero **does not make sense**
- ☑ division is **neither** commutative **nor** associative

Lecture 6

Distributivity

Properties of operations

There are five important **properties** of the basic arithmetic operations of addition and multiplication.

These properties are

- **commutativity** of addition and multiplication
- **associativity** of addition and multiplication
- **distributivity** of multiplication over addition.

Commutativity and associativity (which we studied in Lecture 4)
refer either to addition or multiplication.

Distributivity connects addition and multiplication.

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Distributivity of multiplication over addition

Multiplication **distributes** over addition :

$$\boxed{a(b+c) = ab + ac \quad \text{for any } a\,,\ b \text{ and } c}$$

Example 1. If $a = 2,\ b = 3,\ c = 4,$ then the distributive property reads

$$2(3+4) = 2\cdot 3 + 2\cdot 4.$$

Distributivity means that
we may calculate the value of the expression $2(3+4)$ in **two** different ways:

$$2(3+4) = 2\cdot 7 = 14 \quad \text{or} \quad 2\cdot 3 + 2\cdot 4 = 6 + 8 = 14.$$

Which way is better (easier, faster)? The first one!

Example 2. Calculate the value of the expression $25(4+10)$.
Direct calculation gives

$$25(4+10) = 25\cdot 14 = ?\ \text{(need calculator?)}$$

If we use distributivity instead, then

$$25(4+10) = 25\cdot 4 + 25\cdot 10 = 100 + 250 = 350.$$

Distributivity gives us a choice. Use it!

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Distributivity with variables

Example 3. If in the distributive formula $a(b+c) = ab + ac$ we put $a = 2,\ b = x,\ c = 3y,$ then we get

$$2(x+3y) = 2x + 2 \cdot 3y = 2x + 6y.$$

Example 4. Eliminate the parentheses in the expression $x(1-2y)$.

In order to eliminate the parentheses, we need to distribute x over $1-2y$:

$$x(1-2y) = x(1+(-2y)) =$$
$$x \cdot 1 + x(-2y) = x + x(-2)y = x + (-2)xy = x - 2xy.$$

This problem may be solved faster! Because multiplication distributes over subtraction, too.

Distributivity over subtraction

Distributivity is valid for **subtraction** also:

$$\boxed{a(b-c) = ab - ac \quad \text{for any } a,\ b \text{ and } c}$$

Indeed, since $b - c = b + (-c)$, we have

$$a(b-c) = a(b+(-c)) = ab + a(-c) = ab - ac.$$

Example (the same as before).

Get rid of the parentheses in the expression $x(1-2y)$.

$$x(1-2y) = x \cdot 1 - x(2y) = x - 2xy.$$

Example. Clear parentheses in the expression $x(-1-2y)$.

$$x(-1-2y) = x \cdot (-1) - x(2y) = -x - 2xy.$$

Another look at distributivity

$$\boxed{(a+b)c = ac+bc \quad \text{for any } a\,,\ b \text{ and } c}$$

Indeed, by **commutativity** of multiplication,

$$(a+b)c = c(a+b)\,.$$

By **distributivity**,

$$c(a+b) = ca+cb\,.$$

By commutativity,

$$ca+cb = ac+bc\,.$$

Overall,

$$(a+b)c = ac+bc\,.$$

Example. Clear parentheses in the expression $(2x+3y)z$.

Solution. $(2x+3y)z = 2xz+3yz$.

Similarly, $(a-b)c = ac-bc$ for any a, b and c.

Distribution of a negative quantity

Be careful in application of the formula $a(b+c) = ab+ac$, when a is **negative**!

Example 1. Clear parentheses in the expression $-2(x+y)$.

Solution.

$$-2(x+y) = (-2)x+(-2)y = -2x-2y\,.$$

Example 2. Clear parentheses in the expression $-2(x-y)$.

Solution.

$$-2(x-y) = -2(x+(-y)) = (-2)x+(-2)(-y) = -2x+2y\,.$$

Example 3. Clear parentheses in the expression $-2(-x-y)$.

Solution.

$$-2(-x-y) = -2((-x)+(-y)) = (-2)(-x)+(-2)(-y) = 2x+2y\,.$$

Negative sign in front of parentheses

What is the meaning of the expression $-x$? It represents a quantity **opposite** to x.
For example, if $x = 3$, then $-x = -3$.
If $x = -3$, then $-x = -\underbrace{(-3)}_{x} = 3$.

You can always check if one number is the opposite of another: their sum must be zero.

In some cases, it may be convenient to represent $-x$ as $(-1)x$.

Example 1. Clear parentheses in the expression $-(x + y)$.
Solution.

$$-(x + y) = (-1)(x + y) = (-1)x + (-1)y = -x - y\,.$$

Example 2. Clear parentheses in the expression $-(x - y)$.
Solution.

$$-(x - y) = (-1)(x - y) = (-1)x - (-1)y = -x + y\,.$$

Expansion

Problem. Clear parentheses in the expression $(a + b)(c + d)$.

Solution. How to distribute $a + b$ over $c + d$?

We may think of $c + d$ as a **single** entity.

For this, denote $c + d$ by x. Then

$$(a + b)\underbrace{(c + d)}_{x} = (a + b)x = ax + bx = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd\,.$$

Expansion

Our result in distribution of $a+b$ over $c+d$ is

$$(a+b)(c+d) = ac + ad + bc + bd\,.$$

It is convenient to understand this formula in the following way:

First, we distribute a over $(c+d)$, the result is $ac+ad$.
Then, we distribute b over $(c+d)$, the result is $bc+bd$.
Overall, $(a+b)(c+d) = ac + ad + bc + bd$.

Observe that the right hand side contains **no** parentheses.

This procedure is called **expansion** or **clearing the parentheses**.

Similar arguments are valid when the parentheses contain **any** number of terms.

For example,

$$(a+b)(x+y+z) = ax + ay + az + bx + by + bz\,.$$

Examples of expansion

Example 1. Expand the expression $(2+x)(3+y)$.

Solution. Expand means clear parentheses using distribution.

$$(2+x)(3+y) = 2\cdot 3 + 2y + x\cdot 3 + xy = 6 + 2y + 3x + xy\,.$$

Example 2. Clear parentheses in the expression $(1-2x)(-3+y)$.

Solution. In this example, we have to be careful about the **negative** signs in the expression.

For this reason, we rewrite the expression as follows

$$(1-2x)(-3+y) = (1+(-2x))((-3)+y)\,.$$

Now we distribute:

$$\begin{aligned}(1+(-2x))((-3)+y) &= 1\cdot(-3) + 1\cdot y + (-2x)\cdot(-3) + (-2x)y \\ &= -3 + y + 6x - 2xy\,.\end{aligned}$$

Factoring

Rewriting the distributivity formula $a(b+c) = ab + ac$ backwards, we get

$$ab + ac = a(b+c)\,.$$

This formula is called **factoring**.
When we add two terms, ab and ac, containing a **common factor** of a,
we may **factor out** a from the parentheses.

Example. Factor the expression $6x + 9xy$.

Solution. Both terms, $6x$ and $9xy$, have a **common factor** of $3x$:

$$6x = 3x \cdot 2, \quad 9xy = 3x \cdot 3y.$$

Factoring out $3x$, we get

$$6x + 9xy = 3x \cdot 2 + 3x \cdot 3y = 3x(2 + 3y).$$

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Combining similar terms

Distributivity and factoring helps us to **combine similar terms**:

$$2x + 3x = (2+3)x = 5x.$$

Example. Simplify the expression $2x + 3y + x + 4y$.

Solution. First, we use **commutativity and associativity of addition**:

$$2x + 3y + x + 4y = \underbrace{(2x + x)}_{x-\text{terms}} + \underbrace{(3y + 4y)}_{y-\text{terms}}.$$

Then we combine **similar terms**:

$$(2x + x) + (3y + 4y) = 3x + 7y\,.$$

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Summary

In this lecture, we have learned

- ☑ what distributivity is: $a(b+c) = ab + ac$
  $a(b-c) = ab - ac$
- ☑ how to clear parentheses (expand expressions)
- ☑ what factoring is
- ☑ how to factor expressions using distributivity
- ☑ how to combine similar terms

Lecture 7

Powers

Multiplying repeated factors

Abbreviations are common and useful.
For example, instead of $2+2+2+2+2$ we can write $2 \cdot 5$:

$$2 \cdot 5 = 2+2+2+2+2\,.$$

The sum of several equal numbers can be abbreviated to a product.

The product of several equal numbers can be **abbreviated** similarly:
instead of $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, we can write 2^5:

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\,.$$

In general, $$x^n = \underbrace{x \cdot x \cdot x \cdots x \cdot x}_{n \text{ times}}.$$

Here x is any number and n is a positive integer.

Examples. $x^4 = x \cdot x \cdot x \cdot x$

$10^2 = 10 \cdot 10 = 100$

$2^{10} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{10 \text{ times}} = 1024.$

Exponential notation

$$x^n$$

exponent

base

We read x^n as "x **to the** nth."

$n=2$ and $n=3$ are special:

x^2 is read as "x **squared**",

x^3 as "x **cubed**".

Do you see why $x^1 = x$ for any x?

and why $1^n = 1$ for any positive integer n?

When the base is negative

Example 1. $(-1)^2 = (-1)\cdot(-1) = 1$

$(-1)^3 = (-1)\cdot(-1)\cdot(-1) = -1$.

In general, if n is **even** ($n = 0, 2, 4, 6, \dots$) then $(-1)^n = 1$ and
if n is **odd** ($n = 1, 3, 5, 7, \dots$) then $(-1)^n = -1$.

Example 2. $(-2)^3 = (-2)\cdot(-2)\cdot(-2) = -8$
$(-2)^4 = (-2)\cdot(-2)\cdot(-2)\cdot(-2) = 16$

In general, $$(-x)^n = \underbrace{(-x)\cdot(-x)\dots(-x)}_{n\text{ times}} = \underbrace{(-1)x\cdot(-1)x\dots(-1)x}_{n\text{ times}}$$
$$= \underbrace{(-1)\cdot(-1)\dots(-1)}_{n\text{ times}}\cdot\underbrace{x\cdot x\dots x}_{n\text{ times}} = (-1)^n x^n.$$

Recall: if n is **even**, then $(-1)^n = 1$, and if n is **odd**, then $(-1)^n = -1$.
Therefore, $(-x)^n = x^n$ if n is **even**, and $(-x)^n = -x^n$ if n is **odd**.

Warning: $(-x)^n \neq -x^n$ when n is even.

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Zero and negative exponents

What is 2^0, or 2^{-1}, 2^{-2}, $2^{-3}, \dots$?

To answer this question, let us have a look at the process of consecutive multiplication by 2:

$$\cdots \xrightarrow{\times 2} \frac{1}{8} \xrightarrow{\times 2} \frac{1}{4} \xrightarrow{\times 2} \frac{1}{2} \xrightarrow{\times 2} 1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} \cdots$$

We understand this as an infinite sequence of **powers** of 2:

$$\cdots \xrightarrow{\times 2} \underbrace{2^{-3}}_{\frac{1}{8}} \xrightarrow{\times 2} \underbrace{2^{-2}}_{\frac{1}{4}} \xrightarrow{\times 2} \underbrace{2^{-1}}_{\frac{1}{2}} \xrightarrow{\times 2} \underbrace{2^{0}}_{1} \xrightarrow{\times 2} \underbrace{2^{1}}_{2} \xrightarrow{\times 2} \underbrace{2^{2}}_{4} \xrightarrow{\times 2} \underbrace{2^{3}}_{8} \xrightarrow{\times 2} \underbrace{2^{4}}_{16} \xrightarrow{\times 2} \cdots$$

We see that $2^0 = 1$, $2^{-1} = \dfrac{1}{2} = \dfrac{1}{2^1}$, $2^{-2} = \dfrac{1}{4} = \dfrac{1}{2^2}$, $2^{-3} = \dfrac{1}{8} = \dfrac{1}{2^3}$, and so on.

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Zero and negative exponents

We define $x^0 = 1$ for any non-zero x and

$$x^{-n} = \frac{1}{x^n}$$ for any non-zero x and any **positive integer** n.

Examples. $7^0 = 1$, $\left(\frac{2}{3}\right)^0 = 1$, $(-5)^0 = 1$, $(-1)^0 = 1$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}, \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}, \quad x^{-2} = \frac{1}{x^2}.$$

Observe that the formula $x^{-n} = \frac{1}{x^n}$ means that x^n and x^{-n} are **reciprocals**. Therefore, $x^n = \frac{1}{x^{-n}}$. A power can be **moved** from numerator to denominator (or the other way around) with the **opposite** exponent.

Example. $\frac{3^{-1}}{2^{-4}} = \frac{2^4}{3^1} = \frac{16}{3}$.

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Drill

Here are the exponential **rules** we have learned so far:

$$x^n = \underbrace{x \cdot x \cdot x \cdot \dots \cdot x \cdot x}_{n \text{ times}}$$

$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}, \qquad \frac{1}{x^{-n}} = x^n$$

Let us master these rules.

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

$$(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}, \qquad \frac{1}{2^{-3}} = 2^3 = 8$$

$$(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$$

$$2^0 = 1, \qquad (-2)^0 = 1$$

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Summary

In this lecture, we have learned about

- ☑ powers with **positive** exponents: $x^n = \underbrace{x \cdot x \cdot \dots \cdot x}_{n \text{ times}}$
- ☑ powers with **negative** exponents: $x^{-n} = \dfrac{1}{x^n}$
- ☑ reciprocals of powers with negative exponent: $\dfrac{1}{x^{-n}} = x^n$
- ☑ powers with exponent 0: $x^0 = 1$

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Lecture 8

Power rules

What are powers?

In Lecture 7, we learned about

powers with **positive** exponents: $x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$

powers with **negative** exponents: $x^{-n} = \dfrac{1}{x^n}$

powers with exponent 0: $x^0 = 1$.

In this lecture, we study the **properties** of powers (a.k.a. "power **rules**").

Multiplication of powers with the same base

$$\boxed{x^n \cdot x^m = x^{n+m}}$$

This formula is valid for **any** integers n, m. To **prove** the formula, we consider 4 cases.

Case 1. If n, m are both **positive**, then

$$x^n \cdot x^m = (\underbrace{x \cdot x \cdots x}_{n \text{ times}}) \cdot (\underbrace{x \cdot x \cdots x}_{m \text{ times}}) = \underbrace{x \cdot x \cdots x}_{(n+m) \text{ times}} = x^{n+m}.$$

Case 2. If n, m are both **negative**, then $-n, -m$ are positive and

$$x^n \cdot x^m = \frac{1}{x^{-n}} \cdot \frac{1}{x^{-m}} = \frac{1}{x^{-n}x^{-m}} = \frac{1}{x^{-n-m}} = x^{-(-n-m)} = x^{n+m}.$$

Multiplication of powers with the same base

Case 3: one of the integers is **positive** and the other one is **negative**.

Say, if $n = 5$ and $m = -3$, then

$$x^5 \cdot x^{-3} = \frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2 = x^{5+(-3)}\,.$$

If $n = -5$ and $m = 3$, then

$$x^{-5} \cdot x^3 = \frac{x^3}{x^5} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^2} = x^{-2} = x^{-5+3}\,.$$

For **any other** values of n and m, having opposite signs, the reasoning is the **same** as above.

Case 4: if one of the integers (say, m) is **zero**. Then

$$x^n \cdot x^m = x^n \cdot \underbrace{x^0}_{1} = x^n \cdot 1 = x^n = x^{n+0}\,.$$

We see that in **all** cases, $x^n \cdot x^m = x^{n+m}$.

Examples

Example 1. $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

$$(-1)^9 \cdot (-1)^7 = (-1)^{9+7} = (-1)^{16} = 1$$

$$3^5 \cdot 3^{-8} = 3^{5-8} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$\left(\frac{2}{3}\right)^{-5} \cdot \left(\frac{2}{3}\right)^{7} = \left(\frac{2}{3}\right)^{-5+7} = \left(\frac{2}{3}\right)^{2} = \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) = \frac{4}{9}$$

$$10^{12} \cdot 10^{-12} = 10^{12-12} = 10^0 = 1$$

Example 2. Simplify the expression $x^3 \cdot x^{-8} \cdot x^{-4}$.

Solution.

$$x^3 \cdot x^{-8} \cdot x^{-4} = x^{3-8-4} = x^{-9} = \frac{1}{x^9}\,.$$

Division of powers with the same base

$$\boxed{\frac{x^n}{x^m} = x^{n-m}}$$

This formula is valid for **any** integers n, m.

Indeed, $\dfrac{x^n}{x^m} = x^n \cdot \dfrac{1}{x^m} = x^n \cdot x^{-m} = x^{n-m}$.

Example 1. Find the value of the expression $\dfrac{5^4}{5^6}$.

Solution. $\dfrac{5^4}{5^6} = 5^{4-6} = 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$.

Example 2. Simplify the expression $\dfrac{x^4\,y^{-3}}{x^{-2}\,y^2}$.

Solution.

$$\frac{x^4\,y^{-3}}{x^{-2}\,y^2} = \frac{x^4}{x^{-2}} \cdot \frac{y^{-3}}{y^2} = x^{4-(-2)} \cdot y^{-3-2} = x^6\,y^{-5} = \frac{x^6}{y^5}.$$

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A power of a power

$$\boxed{(x^n)^m = x^{nm}}$$

This formula is valid for **any** integers n, m.
It is proven by cases depending on the signs of the integers.

If n, m are both **positive**, then

$$(x^n)^m = \underbrace{(x^n)\cdot(x^n)\cdot\dots\cdot(x^n)}_{m} =$$

$$\underbrace{\underbrace{(x\cdots x)}_{n}\cdot\underbrace{(x\cdots x)}_{n}\cdot\dots\cdot\underbrace{(x\cdots x)}_{n}}_{m} =$$

$$\underbrace{x\cdot x\cdot\dots\cdot x}_{nm} = x^{nm}.$$

All **other** cases can be reduced to this case using $x^{-n} = \dfrac{1}{x^n}$.

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Examples

Example 1. $(2^3)^4 = 2^{3\cdot 4} = 2^{12} = 4096.$

Example 2. $(2^{-3})^4 = 2^{(-3)\cdot 4} = 2^{-12} = \dfrac{1}{2^{12}} = \dfrac{1}{4096}.$

Example 3. $\left((-2)^{-3}\right)^{-4} = (-2)^{(-3)\cdot(-4)} = (-2)^{12} = 2^{12} = 4096.$

Example 4. $((-1)^{-1})^{-1} = (-1)^{(-1)\cdot(-1)} = (-1)^1 = -1.$

Example 5. Simplify the expression $(x^3)^2 \cdot x^{-4}$.

Solution. $(x^3)^2 \cdot x^{-4} = x^{3\cdot 2} \cdot x^{-4} = x^6 \cdot x^{-4} = x^{6-4} = x^2.$

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Multiplication of powers with the same exponent

$$\boxed{x^n \cdot y^n = (xy)^n}$$

This formula is valid for **any** integer n.

Indeed, if n is **positive**, then

$$x^n \cdot y^n = (\underbrace{x \cdot x \cdot \dots \cdot x}_{n}) \cdot (\underbrace{y \cdot y \cdot \dots \cdot y}_{n}) = \underbrace{(xy) \cdot (xy) \cdot \dots \cdot (xy)}_{n} = (xy)^n.$$

If n is **negative**, then $-n$ is positive and

$$x^n \cdot y^n = \frac{1}{x^{-n}} \cdot \frac{1}{y^{-n}} = \frac{1}{x^{-n}y^{-n}} = \frac{1}{(xy)^{-n}} = (xy)^n .$$

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Examples

Example 1. Simplify the expression $(-x)^9$.

Solution. $(-x)^9 = ((-1)\cdot x)^9 = (-1)^9 \cdot x^9 = (-1)\cdot x^9 = -x^9.$

Example 2. Simplify the expression $(10^{-5}x^2)^{-3}$.

Solution.

$$(10^{-5}x^2)^{-3} = (10^{-5})^{-3}\cdot(x^2)^{-3} = 10^{(-5)\cdot(-3)}\cdot x^{2\cdot(-3)} = 10^{15}x^{-6}\,.$$

Example 3. Simplify the expression $(5x)^2(-3x)^3$.

Solution.

$$(5x)^2(-3x)^3 = 5^2x^2\cdot(-3)^3x^3 = \underbrace{5^2\cdot(-3)^3}_{\text{numbers}}\cdot\underbrace{x^2\cdot x^3}_{\text{variables}} = 25\cdot(-27)x^{2+3} = -675x^5\,.$$

Division of powers with the same exponent

$$\boxed{\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n}$$

This formula is valid for **any** integer n.

Indeed, if n is **positive**, then

$$\frac{x^n}{y^n} = \frac{\overbrace{x\cdot x\cdot x\cdots x}^{n}}{\underbrace{y\cdot y\cdot y\cdots y}_{n}} = \underbrace{\frac{x}{y}\cdot\frac{x}{y}\cdots\frac{x}{y}}_{n} = \left(\frac{x}{y}\right)^n.$$

If n is **negative**, then $-n$ is positive and

$$\frac{x^n}{y^n} = \frac{y^{-n}}{x^{-n}} = \left(\frac{y}{x}\right)^{-n} = \left(\frac{x}{y}\right)^n.$$

Examples

Example 1. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$.

Example 2. $\left(\frac{2}{3}\right)^{-1} = \frac{2^{-1}}{3^{-1}} = \frac{3^1}{2^1} = \frac{3}{2}$.

In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

In particular, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

Summary

In this lecture, we have learned

- ☑ how to **multiply** powers with the **same base**: $x^n \cdot x^m = x^{n+m}$
- ☑ how to **divide** powers with the **same base**: $\frac{x^n}{x^m} = x^{n-m}$
- ☑ how to calculate a **power** of a power: $(x^n)^m = x^{nm}$
- ☑ how to **multiply** powers with the **same exponent**: $x^n \cdot y^n = (xy)^n$
- ☑ how to **divide** powers with the **same exponent**: $\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$

Lecture 9

Polynomials

What is a polynomial?

A **polynomial expression** is an expression that involves only numbers, variables and the operations of addition, subtraction and multiplication. Division by a number is allowed (because it is a multiplication by the reciprocal number), but division by an expression which contains a variable is not allowed.

Example 1. $x^2 + x$ is a polynomial expression. It involves a variable x and operations of multiplication and addition: $x^2 + x = x \cdot x + x$.

Example 2. $x(x + 1)$ is a polynomial expression. It involves a variable x and operations of addition and multiplication.

The polynomial expressions in Examples 1 and 2 are **equal:** $x^2 + x = x(x + 1)$.

We say that they represent the **same polynomial**.

Example 3. 1 is a polynomial expression because it involves a single number 1 , and neither variables nor operations.
In general, any **constant** (number) is a polynomial.

Example 4. x is a polynomial in **one** variable x .

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What is a monomial?

Example. $2x^3$ is a polynomial in **one** variable x , because it is represented by an expression involving the constant 2 , the variable x , and three operations (multiplications):

$$2x^3 = 2 \cdot x \cdot x \cdot x \, .$$

An expression like ax^n , where a is a constant and x^n is a variable x raised to a **non-negative** power n is called a **monomial**.

A monomial is a polynomial with **neither** addition **nor** subtraction involved.

Examples of monomials: $4x$, $-5x^2$, $\dfrac{2}{5}x^3$.

Any constant is a monomial. For example, 3 is a monomial, since $3 = 3 \cdot \underbrace{x^0}_{1}$.

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More examples of polynomials

Example 4. $-2x^3 + x^2 + 4x - 1$ is a polynomial in **one** variable x. It is the sum of four monomials.

• **A sum of several monomials is a polynomial.**

Example 5. $x(x(-2x+1)+4)-1$ is a polynomial in **one** variable x.

Let us clear parentheses in this polynomial:

$$x(x(-2x+1)+4)-1 = x(x(-2x) + x \cdot 1 + 4) - 1$$
$$= x(-2x^2 + x + 4) - 1 = -2x^3 + x^2 + 4x - 1.$$

We see that the polynomial $x(x(-2x+1)+4)-1$ is actually the polynomial from Example 4:

$$x(x(-2x+1)+4)-1 = -2x^3 + x^2 + 4x - 1\,.$$

A polynomial may be presented by different polynomial expressions.

Polynomials in several variables

Example 1. $3xy^2$ is a polynomial in **two** variables x, y. It is a **monomial** (the product of a constant and powers of variables).

Example 2. $3xy(3x+1)(4y-2)+x-1$ is a polynomial in **two** variables x, y.

Example 3. $x + 2y^2 + z^3 - xy - 3xz^7$ is a polynomial in three variables x, y, z. It is the sum of five monomials.

Polynomial or not?

Example 1. $x + \frac{1}{x}$ is **not** a polynomial. This expression involves **division** by a variable. Division by variables is **not** allowed in polynomials.

Example 2. $\frac{x+1}{2}$ is a polynomial. Division by 2 is actually multiplication by $\frac{1}{2}$:

$$\frac{x+1}{2} = \frac{1}{2}(x+1) = \frac{1}{2}x + \frac{1}{2}.$$

Division by any non-zero number is a multiplication by its reciprocal.

Example 3. $x^{-2} + 3x - 1$ is **not** a polynomial.
x^{-2} can't show up in a polynomial, because
a polynomial can't contain a variable with negative exponent.

Simplifying polynomial expressions

Expressions representing polynomials may be simplified.

Example 1. Clear parentheses in the expression $x^3(2x-1)$.

Solution. We distribute x^3 to clear parentheses:

$x^3(2x-1) = x^3 \cdot (2x) + x^3(-1) = 2x^3 \cdot x - x^3 = 2x^4 - x^3.$

Example 2. Clear parentheses and combine similar terms in the expression $(x+2)(3x-1)$.

Solution. We use distributivity to clear parentheses:

$(x+2)(3x-1) = x(3x) + x(-1) + 2(3x) + 2(-1) = 3x^2 - x + 6x - 2 = 3x^2 + 5x - 2.$

Example 3. Clear parentheses: $(-2x^3 + x - 4)(5x+1)$.

Solution. We distribute, and then combine similar terms:

$(-2x^3 + x - 4)(5x+1) = -2x^3 \cdot 5x + (-2x^3) \cdot 1 + x \cdot 5x + x \cdot 1 + (-4)(5x) + (-4) \cdot 1$
$= -10x^4 - 2x^3 + 5x^2 + x - 20x - 4 = -10x^4 - 2x^3 + 5x^2 - 19x - 4.$

The standard form of a polynomial in one variable

No matter how a polynomial in one variable is written, one can use commutativity, associativity and distributivity to put it in **standard form**:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where x is the variable, n is a non-negative number, and $a_0,\ a_1,\ a_2, \ldots, a_{n-1}, a_n$ are constants.

Scared by this "letter monster"? Let us take it apart, to see what it is made of.

As we know, x is a **variable**.
The letter n stands for the non-negative (positive or zero) number, which is the highest power of x in the expression. It is called the **degree** of the polynomial.

The letters $a_0,\ a_1,\ a_2, \ldots, a_{n-1}, a_n$ stand for constants (numbers).
They are called the **coefficients** of the polynomial.

The word "polynomial" means "many parts".

Taming the monster

Let us see how to put a polynomial in **standard form**:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0\,.$$

Example 1. Put the polynomial $(4x-1)(2x+3)$ in standard form.

Solution. We distribute, and combine similar terms:

$(4x-1)(2x+3) = 4x\cdot 2x + 4x\cdot 3 - 1\cdot 2x - 1\cdot 3 = 8x^2 + 12x - 2x - 3 = 8x^2 + 10x - 3.$

The resulting expression, $8x^2 + 10x - 3$, is a polynomial in **standard form**.

Indeed, the highest power of x is $n = 2$. And the long expression

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is reduced in this case to

$$a_2 x^2 + a_1 x + a_0$$

with $n = 2$, $a_2 = 8$, $a_1 = 10$ and $a_0 = -3$: $\underbrace{8}_{a_2} x^2 + \underbrace{10}_{a_1} x + \underbrace{-3}_{a_0}.$

Polynomials in standard form

Example 1. Write the polynomial $2x + 3x^4 - x^3 + 1$ in **standard form**, identify the **coefficients**, and determine the **degree** of the polynomial.

Solution. Rearrange the monomials in **descending** order of exponents:

$$2x + 3x^4 - x^3 + 1 = 3x^4 - x^3 + 2x^1 + 1x^0.$$

The standard form is $3x^4 - x^3 + 0 \cdot x^2 + 2x + 1$. The degree is $n = 4$.
The coefficients are $a_4 = 3,\ a_3 = -1,\ a_2 = 0,\ a_1 = 2,\ a_0 = 1$.
Observe that the term containing x^2 is included with thith the coefficient 0.

Example 2. What is the degree of the polynomial 1?

Solution. As we know, any constant is a polynomial. Actually, it is a monomial. In our case, $1 = 1x^0$. The degree is the **highest** power of x, which is 0.

Answer: the degree of the polynomial 1 is **zero**.

Remark. Any constant is a polynomial of degree **zero**.

Summary

In this lecture, we have learned

- ☑ what **polynomial expressions** are
- ☑ what **polynomials** are
- ☑ what **monomials** are
- ☑ that polynomials may be in **one** or **several** variables
- ☑ how to **simplify** polynomial expressions
- ☑ that the **standard form** of a polynomial is
$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$
- ☑ how to identify the **degree** of a polynomial
- ☑ how to identify the **coefficients** of a polynomial
- ☑ how to bring a polynomial to the standard form

Lecture 10

Operations with Polynomials

Reminder: what is a polynomial?

We learned in Lecture 9 that

A **polynomial** is an expression involving numbers, variables
and operations of addition, subtraction and multiplication.

Any polynomial in one variable can be written in the **standard form**

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0\,,$$

where x is a variable, n is a non-negative integer,
and $a_0,\, a_1,\, a_2, \ldots, a_{n-1}, a_n$ are **coefficients** (constants).

The highest power of x is called the **degree** of the polynomial.

In this lecture, we will learn how to **operate** with polynomials.

Addition and subtraction

If we **add** or **subtract** two polynomials, then the resulting expression is again a polynomial.

Example 1. Let $p = 2x^3 - 4x^2 + x - 1$ and $q = x^3 + 3x^2 - 4x + 2$ be two polynomials. Find $p+q$ and $p-q$ and put them in standard form.

Remark. We have given the polynomials the names, p and q.
It is common in mathematics to give short names to long expressions.

Solution.

$p + q = \underbrace{(2x^3 - 4x^2 + x - 1)}_{p} + \underbrace{(x^3 + 3x^2 - 4x + 2)}_{q}$ This is the sum. Put it in **standard form**:

$$\underset{\text{combine similar terms}}{=} (2x^3 + x^3) + (-4x^2 + 3x^2) + (x - 4x) + (-1 + 2) = \underbrace{3x^3 - x^2 - 3x + 1}_{\text{standard form}}.$$

Subtraction

Now we calculate $p-q$, where $p=2x^3-4x^2+x-1$ and $q=x^3+3x^2-4x+2$ as before.

$$p-q=\underbrace{(2x^3-4x^2+x-1)}_{p}-\underbrace{(x^3+3x^2-4x+2)}_{q}=$$

$$2x^3-4x^2+x-1-x^3-3x^2+4x-2=$$

$$(2x^3-x^3)+(-4x^2-3x^2)+(x+4x)+(-1-2)=$$

$$x^3-7x^2+5x-3.$$

Multiplication

If we multiply two polynomials, then the resulting expression is a polynomial.

Example 1. Let $p=2x-1$ and $q=-x^2+3x+4$ be two polynomials.
Find the polynomial pq, put it in standard form and determine its degree.

Solution.

$$pq=(2x-1)(-x^2+3x+4)=$$

$$2x(-x^2)+(2x)(3x)+(2x)\cdot 4+(-1)(-x^2)+(-1)(3x)+(-1)\cdot 4=$$

$$-2x^3+6x^2+8x+x^2-3x-4=-2x^3+7x^2+5x-4\,.$$

Therefore, $pq=-2x^3+7x^2+5x-4$. The degree of pq is 3.

In general, if p and q are polynomials of degree n and m respectively,
then their product pq has the degree $n+m$.

That is, when we multiply polynomials, their degrees are added.

Short multiplication formulas

$$\boxed{(x+y)^2 = x^2 + 2xy + y^2 \quad \text{for any } x \text{ and } y}$$

Indeed,

$(x+y)^2 = (x+y)(x+y) = x \cdot x + \underbrace{x \cdot y + y \cdot x}_{2xy} + y \cdot y = x^2 + 2xy + y^2\,.$

This formula will save you an enormous amount of time. It's worth memorizing!

Examples. $(x+3)^2 = x^2 + 2x \cdot 3 + 3^2 = x^2 + 6x + 9\,.$

$(3a+4b)^2 = (3a)^2 + 2(3a)\cdot(4b) + (4b)^2 = 9a^2 + 24ab + 16b^2\,.$

A similar formula for the difference:

$$\boxed{(x-y)^2 = x^2 - 2xy + y^2 \quad \text{for any } x \text{ and } y}$$

Examples. $(xz-5)^2 = (xz)^2 - 2(xz)\cdot 5 + 5^2 = x^2z^2 - 10xz + 25\,.$

$(2a-1)^2 = (2a)^2 - 2(2a)\cdot(1) + 1^2 = 4a^2 - 4a + 1\,.$

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Factoring

To **factor** a polynomial means to present the polynomial as a product of non-constant polynomials.

For example, we factor $3x^2 + x$ as follows:

$$3x^2 + x = x(3x+1)\,.$$

Factoring is opposite to multiplication:

$$x(3x+1) \underset{\text{factoring}}{\overset{\text{multiplication}}{\rightleftarrows}} 3x^2 + x$$

Multiplication of polynomials is straightforward:

given two polynomials, you can **always** multiply them.

Factoring may be **difficult** or **impossible**.

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Factoring out a monomial

Example 1. Factor the polynomial $4x^3 + 5x^2$.

Solution. The monomials $4x^3$ and $5x^2$ have the common factor of x^2:

$$4x^3 = x^2 \cdot 4x \quad \text{and} \quad 5x^2 = x^2 \cdot 5 .$$

We factor out x^2:

$$4x^3 + 5x^2 = x^2 \cdot 4x + x^2 \cdot 5 = x^2(4x + 5) .$$

Example 2. Factor the polynomial $10x^3 + 6x^2 - 4x$.

Solution. The monomials $10x^3$, $6x^2$ and $4x$ have the common factor of $2x$:

$$10x^3 = 2x \cdot 5x^2 , \quad 6x^2 = 2x \cdot 3x , \quad \text{and } 4x = 2x \cdot 2 .$$

We factor out $2x$:

$$10x^3 + 6x^2 - 4x = 2x \cdot 5x^2 + 2x \cdot 3x - 2x \cdot 2 = 2x(5x^2 + 3x - 2) .$$

Remark. As we will learn later, the polynomial $5x^2 + 3x - 2$ can be factored further:

$$5x^2 + 3x - 2 = (5x - 2)(x + 1) .$$

Difference of squares

$$\boxed{x^2 - y^2 = (x - y)(x + y) \quad \text{for any } x \text{ and } y}$$

Indeed,

$(x + y)(x - y) = x \cdot x + x(-y) + y \cdot x + y(-y) = x^2 - xy + xy - y^2 = x^2 - y^2.$

Example 1. Factor $x^2 - 1$.

Solution. $x^2 - 1 = x^2 - 1^2 = (x - 1)(x + 1)$.

Example 2. Factor $4 - a^2$.

Solution. $4 - a^2 = 2^2 - a^2 = (2 - a)(2 + a)$.

Example 3. Factor $9x^4 - y^6$.

Solution. $9x^4 - y^6 = (3x^2)^2 - (y^3)^2 = (3x^2 - y^3)(3x^2 + y^3)$.

Example 4. Factor $x^4 - 1$.

Solution. $x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1)$.

Evaluation of a polynomial at a number

Let p be a polynomial in a single variable x. As any expression, p may be evaluated at a number.

"Evaluating p at 2", say, means substituting 2 for every occurrence of x in p. This gives a number, the **value of** p **at** 2, which we denote by $p(2)$.
The polynomial p itself can then also be denoted by $p(x)$.

Example 1. Let $p(x) = 3x^2 - x + 4$. Find $p(0)$, $p(1)$, $p(-2)$.

Solution. We have to **evaluate** the polynomial $p(x)$ at numbers $0, 1, -2$.
For this, we **substitute** (plug in) $x = 0$, $x = 1$, and $x = -2$, into $p(x)$.

$$p(0) = p(x)\Big|_{x=0} = 3x^2 - x + 4\Big|_{x=0} = 3 \cdot 0^2 - 0 + 4 = 4.$$

$$p(1) = p(x)\Big|_{x=1} = 3x^2 - x + 4\Big|_{x=1} = 3 \cdot 1^2 - 1 + 4 = 3 - 1 + 4 = 6.$$

$$p(-2) = p(x)\Big|_{x=-2} = 3x^2 - x + 4\Big|_{x=-2} = 3 \cdot (-2)^2 - (-2) + 4 = 3 \cdot 4 + 2 + 4 = 12 + 2 + 4 = 18.$$

Evaluation of a polynomial at a number

Remark. The polynomial $p(x) = 3x^2 - x + 4$ describes the following algorithm:

$$x \xrightarrow{\text{multiply by } x} x^2 \xrightarrow{\text{multiply by } 3} 3x^2 \xrightarrow{\text{subtract } x} 3x^2 - x \xrightarrow{\text{add } 4} 3x^2 - x + 4$$

Evaluation of $p(x)$ at a given number, say 1, is plugging $x = 1$ into the algorithm:

$$1 \xrightarrow{\text{multiply by } 1} 1^2 \xrightarrow{\text{multiply by } 3} 3 \cdot 1^2 \xrightarrow{\text{subtract } 1} 3 \cdot 1^2 - 1 \xrightarrow{\text{add } 4} \underbrace{3 \cdot 1^2 - 1 + 4}_{6}$$

Note that $p(x)$ does **not** mean $p \cdot (x)$. If p is a polynomial in the variable x, then $p(x)$ is just another notation for p. We do **not** mean to multiply p by x!

Substitution

Example 1. Let $p(x) = -x^2 + 3x$. Find $p(a)$, $p(a-1)$, $p(a^2)$.

Remark. We have to substitute $x = a$, $x = a - 1$, $x = a^2$ into $p(x)$.

This procedure is called a **substitution**. Substitution is like **evaluation**, but instead of a number, we plug in an algebraic **expression**.

Solution. $p(a) = -x^2 + 3x\Big|_{x=a} = -a^2 + 3a.$

$$\begin{aligned} p(a-1) = -x^2 + 3x\Big|_{x=a-1} &= -(a-1)^2 + 3(a-1) \\ &= -(a^2 - 2a + 1) + 3(a-1) \\ &= -a^2 + 2a - 1 + 3a - 3 = -a^2 + 5a - 4. \end{aligned}$$

$$p(a^2) = -x^2 + 3x\Big|_{x=a^2} = -(a^2)^2 + 3a^2 = -a^4 + 3a^2.$$

Summary

In this lecture, we have learned

- ☑ how to **add** and **subtract** polynomials
- ☑ how to **multiply** polynomials
- ☑ formulas for **short multiplication**: $(x+y)^2 = x^2 + 2xy + y^2$
$(x-y)^2 = x^2 - 2xy + y^2$
- ☑ how to **factor out** monomials
- ☑ the formula for **difference of squares**: $x^2 - y^2 = (x-y)(x+y)$
- ☑ how to **evaluate** a polynomial at a number
- ☑ how to **substitute** an expression into a polynomial

Lecture 11

Rational Expressions

What a rational expressions is

A **rational expression** $\frac{p}{q}$ is a quotient of two polynomials p and q, where q is **non**-zero polynomial.

For example, $\frac{x+1}{x^2}$, $\frac{3x^3-x^2+x}{x^2+3x-2}$, $\frac{x}{1}$, $\frac{xy+2}{x^2+y^2}$ are rational expressions.

Any polynomial $p(x)$ is a rational expression whose denominator is 1:

$$p(x) = \frac{p(x)}{1}.$$

In this lecture, we will learn how to:

- **evaluate** a rational expression at a number
- **substitute** an expression into a rational expression
- **simplify** rational expressions

Evaluation

Example. Find the value of the expression $\frac{-x^2+4}{x-3}$ for $x=1$, $x=-1$, $x=3$.

Solution. We have to substitute $x = 1, -1, 3$ into the expression.

$$\left.\frac{-x^2+4}{x-3}\right|_{x=1} = \frac{-(1)^2+4}{(1)-3} = \frac{-1+4}{1-3} = \frac{3}{-2} = -\frac{3}{2}.$$

$$\left.\frac{-x^2+4}{x-3}\right|_{x=-1} = \frac{-(-1)^2+4}{(-1)-3} = \frac{-1+4}{-1-3} = \frac{3}{-4} = -\frac{3}{4}.$$

$$\left.\frac{-x^2+4}{x-3}\right|_{x=3} = \frac{-(3)^2+4}{(3)-3} = \frac{-9+4}{0}$$ Oops! Division by 0 is prohibited!

Therefore, the expression $\frac{-x^2+4}{x-3}$ is **not** defined for $x=3$.

Substitution

Example 1. Find the value of the expression $\dfrac{x-1}{x^2+2x}$ for $x = a-1$.

Solution. We have to substitute $a-1$ for x into the expression $\dfrac{x-1}{x^2+2x}$.
The result should be a new expression involving a, not x.

$$\left.\frac{x-1}{x^2+2x}\right|_{x=a-1} = \frac{(a-1)-1}{(a-1)^2+2(a-1)} = \frac{a-1-1}{a^2-2a+1+2a-2} = \frac{a-2}{a^2-1}.$$

Short multiplication: $(a-1)^2 = a^2 - 2a + 1$

Example 2. Find the value of the expression $\dfrac{1}{xy}$ for $x = a^2$ and $y = a^{-3}$.

Solution. $\left.\dfrac{1}{xy}\right|_{x=a^2,\, y=a^{-3}} = \dfrac{1}{a^2a^{-3}} = \dfrac{1}{a^{2-3}} = \dfrac{1}{a^{-1}} = a$.

Cancellation

Cancellation rule says that
one can cancel out a **common factor** both in numerator and denominator:

$$\frac{ac}{bc} = \frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}.$$

Examples. $\dfrac{(x+1)(x-1)}{x+1} = \dfrac{\cancel{(x+1)}\cdot(x-1)}{\cancel{(x+1)}\cdot 1} = \dfrac{x-1}{1} = x-1.$

$$\frac{x^2\cdot(x+1)^3}{x^5\cdot(x+1)^2} = \frac{x^2\cdot(x+1)^2\cdot(x+1)}{x^2\cdot x^3\cdot(x+1)^2} = \frac{x+1}{x^3}.$$

Warning: It's **incorrect** to cancel out a common summand:

$$\frac{a+c}{b+c} \neq \frac{a}{b}.$$

For example, $\dfrac{4}{5} = \dfrac{1+3}{2+3} \neq \dfrac{1}{2}$.

Cancellation simplifies

Factoring followed by cancellation is used to **simplify** rational expressions.

Example. Simplify the expression $\dfrac{x^2 - x}{x^2 - 1}$.

Solution. Both numerator and denominator may by factored:

In numerator $x^2 - x$, we factor out x:

$$x^2 - x = x(x-1).$$

To factor denominator, we use the **difference of squares** formula $x^2 - y^2 = (x-y)(x+y)$:

$$x^2 - 1 = x^2 - 1^2 = (x-1)(x+1).$$

Therefore,

$$\frac{x^2 - x}{x^2 - 1} = \frac{x(x-1)}{(x-1)(x+1)} = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \frac{x}{x+1}.$$

Simplify before evaluating

Simplify, if you can, before evaluating.

For example, if we need to evaluate $\dfrac{x^2 - x}{x^2 - 1}$ at $x = 14$,

then a straightforward evaluation is cumbersome:

$$\left.\frac{x^2 - x}{x^2 - 1}\right|_{x=14} = \frac{14^2 - 14}{14^2 - 1} = \frac{196 - 14}{196 - 1} = \frac{182}{195},$$

but it gets easier if we simplify first: $\dfrac{x^2 - x}{x^2 - 1} = \dfrac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \dfrac{x}{x+1}$,

then evaluate: $\left.\dfrac{x}{x+1}\right|_{x=14} = \dfrac{14}{14+1} = \dfrac{14}{15}$. Is $\dfrac{182}{195} = \dfrac{14}{15}$?

Yes, because $\dfrac{182}{195} = \dfrac{14 \cdot \cancel{13}}{\cancel{13} \cdot 15} = \dfrac{14}{15}$. Observe that $\left. x - 1\right|_{x=14} = 14 - 1 = 13$.

Something may go wrong

Evaluate the same expression $\dfrac{x^2-x}{x^2-1}$ at $x=1$.

Using the same simplification $\dfrac{x^2-x}{x^2-1}=\dfrac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}=\dfrac{x}{x+1}$, we get

$$\left.\frac{x}{x+1}\right|_{x=1}=\frac{1}{1+1}=\frac{1}{2}$$

Using the original expression $\dfrac{x^2-x}{x^2-1}$, we get

$$\left.\frac{x^2-x}{x^2-1}\right|_{x=1}=\frac{1^2-1}{1^2-1}=\frac{0}{0}$$ Oops! Division by 0 is impossible!

$\left.\dfrac{x^2-x}{x^2-1}\right|_{x=1}$ is not defined, while $\left.\dfrac{x}{x+1}\right|_{x=1}=\dfrac{1}{2}$, although $\dfrac{x^2-x}{x^2-1}=\dfrac{x}{x+1}$!

Why this happens and how to avoid

How could this happen? Let us analyse our calculations:

$$\left.\frac{x^2-x}{x^2-1}\right|_{x=1}=\left.\frac{x(x-1)}{(x-1)(x+1)}\right|_{x=1}=\frac{(1)(1-1)}{(1-1)(1+1)}=\frac{1\cdot 0}{0\cdot 2}$$

It is OK to cancel out $x-1$ in $\dfrac{x(x-1)}{(x-1)(x+1)}$,

but $\left.x-1\right|_{x=1}=1-1=0$, and cancellation by 0 is impossible!

It is useful and safe to simplify a rational expression $\dfrac{p(x)}{q(x)}$ prior to evaluating at $x=a$, if $q(a)\neq 0$.

Summary

In this lecture, we have learned

- ☑ what a **rational expression** is
- ☑ how to **evaluate** a rational expression at a number
- ☑ when a rational expression is **not** defined
- ☑ how to **substitute** an expression into a rational expression
- ☑ how to cancel a **common factor**
- ☑ how to **simplify** a rational expression

Lecture 12

Operations with Rational Expressions

Fractions

A **rational expression** is a quotient of two polynomials.
It is a **fraction** in which numerator and denominator are polynomials.
Therefore rational expressions comply with the same **rules** as fractions:

Cancellation: $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$ for any $c \neq 0$

Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$

Addition: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

These are all the rules that you need to know for operating with fractions,
and with rational expressions.

Multiplying rational expressions

Rational expressions, being fractions, are multiplied as fractions:

$$\boxed{\frac{a}{b} \cdot \frac{c}{d} = \frac{a\,c}{b\,d}}$$

Example. Simplify the expression $\frac{x}{9-x^2} \cdot \frac{x-3}{x^2+x}$.

Solution. The expression is a product of two rational expressions:

$$\frac{x}{9-x^2} \cdot \frac{x-3}{x^2+x} = \frac{x(x-3)}{(9-x^2)(x^2+x)}\,.$$

To simplify the product, we factor the denominator:

$$\underbrace{(9-x^2)}_{3^2-x^2}\underbrace{(x^2+x)}_{x(x+1)} = (3-x)(3+x)x(x+1)\,.$$

Therefore,

$$\frac{x}{9-x^2} \cdot \frac{x-3}{x^2+x} = \frac{\cancel{x}(x-3)}{(3-x)(3+x)\cancel{x}(x+1)} = \frac{-(3-x)}{(3-x)(3+x)(x+1)} = -\frac{1}{(x+3)(x+1)}\,.$$

Dividing rational expressions

Rational expressions, being fractions, are divided as fractions. To **divide** an expression by a fraction, we multiply the expression by the **reciprocal** of the fraction:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a\,d}{b\,c}.$$

Thus,

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a\,d}{b\,c}}$$

Example. Simplify the expression $\dfrac{x^3}{x^2+2x+1} \div \dfrac{x^2}{x+1}$.

Solution. $\dfrac{x^3}{x^2+2x+1} \div \dfrac{x^2}{x+1} = \dfrac{x^3}{x^2+2x+1} \cdot \dfrac{x+1}{x^2} = \dfrac{x(x+1)}{x^2+2x+1}$.

By short multiplication formula, $x^2+2x+1=(x+1)^2$.

So $\dfrac{x(x+1)}{x^2+2x+1} = \dfrac{x(x+1)}{(x+1)^2} = \dfrac{x\cancel{(x+1)}}{(x+1)\cancel{(x+1)}} = \dfrac{x}{x+1}$.

Adding rational expressions

Rational expressions, being fractions, are **added** as fractions: if they have a common denominator, then

$$\boxed{\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}}$$

Otherwise the denominators are made coinciding using the relation $\dfrac{a}{b} = \dfrac{ac}{bc}$ and then the same rule applies.

The product of denominators can always serve as a common denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}.$$

This gives a formula which always works:

$$\boxed{\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}}$$

Often fractions have a common denominator simpler than bd.

Subtracting rational expressions

Subtraction is similar to addition. There are similar formulas:

$$\boxed{\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}}$$

$$\boxed{\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}}$$

Moreover, subtraction is reduced to addition:

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(-\frac{c}{d}\right)$$

Keep in mind that

$$-\frac{c}{d} = \frac{-c}{d} = \frac{c}{-d}$$

Examples of addition

Example 1. Present $\dfrac{1-x^2}{x} + x$ as a single fraction.

Solution. We have to perform addition of fractions. For this, we need to find a **common denominator** of $\dfrac{1-x^2}{x}$ and $x = \dfrac{x}{1}$. The common denominator is $x \cdot 1 = x$. Therefore,

$$\frac{1-x^2}{x} + x = \frac{1-x^2}{x} + \frac{x}{1} = \frac{(1-x^2)\cdot 1 + x \cdot x}{x \cdot 1} = \frac{1-x^2+x^2}{x} = \frac{1}{x}.$$

Example 2. Present $2 + \dfrac{3}{x+1}$ as a single fraction.

Solution.

$$2 + \frac{3}{x+1} = \frac{2}{1} + \frac{3}{x+1} = \frac{2\cdot(x+1) + 1\cdot 3}{1\cdot(x+1)} = \frac{2x+2+3}{x+1} = \frac{2x+5}{x+1}.$$

Examples of addition

Example 3. Perform the operations and simplify the resulting expression:

$$\frac{1}{x+1}+\frac{1}{x^2-1}.$$

Solution. By the universal formula $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$,

$$\frac{1}{x+1}+\frac{1}{x^2-1}=\frac{(x^2-1)+(x+1)}{(x+1)(x^2-1)}=\frac{x^2+x}{(x+1)(x^2-1)}=\frac{x(x+1)}{(x+1)(x^2-1)}=\frac{x}{x^2-1}.$$

Another solution. Since $x^2-1=(x-1)(x+1)$, a common denominator is $(x-1)(x+1)$:

$$\frac{1}{x+1}+\frac{1}{x^2-1}=\frac{1}{x+1}+\frac{1}{(x-1)(x+1)}=\frac{(x-1)+1}{(x-1)(x+1)}=\frac{x}{(x-1)(x+1)}.$$

Summary

In this lecture, we have learned

- ☑ how to **multiply** rational expressions
- ☑ how to **divide** rational expressions
- ☑ how to **add** and **subtract** rational expressions

Lecture 13

Composing Algebraic Expressions

Translating English to Algebra

In this lecture, we will learn how to **compose** algebraic expressions after word descriptions.

Composing algebraic expressions is an important **skill** for solving
"**real-life**" problems that you may encounter in the math classroom and beyond.

To **translate** successfully English phrases into algebraic expressions, we need to understand the **meaning** of each phrase and **express** this meaning algebraically.

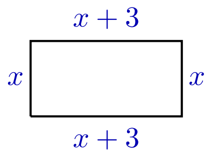
This translation may **require** some basic knowledge from other fields, for example

- geometric formulas for area, volume, and perimeter,
- formula for uniform motion: distance=speed×time ,
- common facts about money system (cents, nickels, dimes, quarters), pricing,
- percentage.

Perimeter of a rectangle

Problem. In a rectangle, one side is x feet long. The other side is 3 feet longer. Compose an algebraic expression (in terms of x) for the **perimeter** of the rectangle. Simplify the expression. Find the value of this expression for $x = 5$ feet.

Solution.



The **perimeter** is the sum of the lengths of all sides.

The perimeter of our rectangle is

$$x + (x + 3) + x + (x + 3)\,.$$

Simplify this expression:

$$x + (x + 3) + x + (x + 3) = \boxed{4x + 6}\,.$$

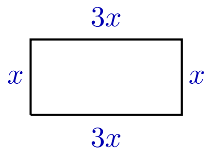
Find the value of the expression at $x = 5$:

$$4x + 6\Big|_{x=5} = 4 \cdot 5 + 6 = 20 + 6 = \boxed{26}\,\text{(feet)}.$$

Area of a rectangle

Problem. In a rectangle, the width is x feet. The length is 3 times as long as the width. Compose an algebraic expression (in terms of x) for the **area** of the rectangle. Simplify this expression. Find the value of this expression for $x = 5$ feet.

Solution.



The **area** of a rectangle is
the product of the width by the length.

The area of our rectangle is $x(3x)$.

Simplify this expression: $x(3x) = \boxed{3x^2}$.

Find the value of the expression at $x = 5$:

$$3x^2\Big|_{x=5} = 3 \cdot 5^2 = 3 \cdot 25 = \boxed{75}\ (\text{ft}^2).$$

Counting money

Problem. In a piggy bank, there are dimes and quarters. The number of quarters is 5 less than the number of dimes. Compose an algebraic expression for the total amount of money in the piggy bank, if the number of dimes is x. Find the value of the expression if $x = 20$.

Solution. There are x dimes in the piggy bank. Their total value is $10x$ cents.

The number of quarters is 5 less than the number of dimes (which is x).
So there are $x - 5$ quarters. Their total value is $25(x - 5)$ cents.

The total money value in the piggy bank is the value of dimes plus the value of quarters:

$$10x + 25(x - 5).$$

Let us simplify the expression:

$$10x + 25(x - 5) = 10x + 25x - 125 = \boxed{35x - 125},$$

and evaluate it at $x = 20$:

$35x - 125\Big|_{x=20} = 35 \cdot 20 - 125 = 700 - 125 = \boxed{575}$ cents.

Uniform motion

Problem 1. A car moved for 4 hours at a constant speed of x mi/h.
Compose an algebraic expression (in terms of x) for the distance covered.

Solution. For **uniform motion** (motion with a constant speed),
the distance, speed and time are related by the formula

$$\text{distance}=\text{speed}\times\text{time}.$$

Therefore, the distance that the car covered traveling for 4 hours at a constant speed of x mi/h is $\boxed{4x}$ (miles).

Problem 2. It took x seconds for an athlete to run the distance of 300 meters. Compose an algebraic expression for the **speed** of the athlete.

Solution. Given: time $= x$ seconds, distance $= 300$ meters. Find the speed.

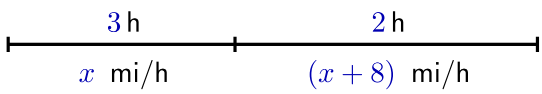
Since $\text{distance}=\text{speed}\times\text{time}$, then $\text{speed} = \dfrac{\text{distance}}{\text{time}}$.

In our case, the **speed** of the athlete is $\dfrac{300}{x}$ (m/s).

Uniform motion

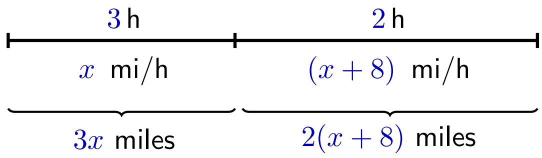
Problem. A car traveled for 3 hours at a constant speed of x mi/h. Then it increased the speed by 8 mi/h and traveled for another 2 hours. Compose an algebraic expression for the **total distance** covered by the car. Simplify the expression. Find the value of the expression for $x = 50$ mi/h.

Solution. Let us show schematically what is given in the problem:



The total distance is the **sum** of two distances.

Uniform motion



The total distance is $3x + 2(x + 8)$ miles.

Simplify the expression:

$$3x + 2(x + 8) = 3x + 2x + 16 = \boxed{5x + 16}.$$

Find the value of the expression for $x = 50$ mi/h:

$$5x + 16\Big|_{x=50} = 5 \cdot 50 + 16 = 250 + 16 = \boxed{266} \text{ miles.}$$

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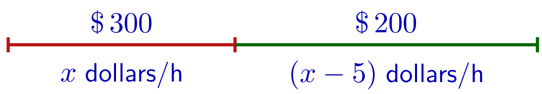
Pay rate

Problem. This week, Rob earned $\$\,300$ while tutoring for x dollars per hour, and $\$\,200$ working at an office, where the pay rate is $\$\,5$ per hour less than for tutoring. Compose an algebraic expression for the **total time** that Rob spent working this week.

Solution. Let us show schematically what is given in the problem:



The **total** time is the time spent on tutoring plus the time spent in office.

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Pay rate

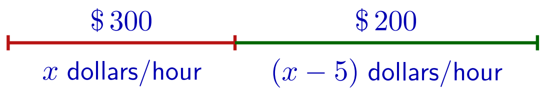
The amount earned, the pay rate, and the time are related by the formula

$$\text{amount earned} = \text{pay rate} \times \text{time}.$$

From which we get

$$\text{time} = \frac{\text{amount earned}}{\text{pay rate}}.$$

Calculate the time spent on each job **separately**:



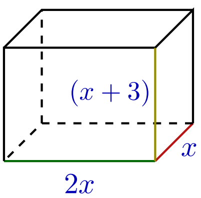
The time spent while tutoring is $\dfrac{300}{x}$. The time spent in office is $\dfrac{200}{x-5}$.

The total time is $\dfrac{300}{x} + \dfrac{200}{x-5}$ (hours).

Volume

Problem. The width of a rectangular aquarium is x inches, the length is twice as long as the width, and the height is 3 inches more than the width. Compose an algebraic expression for the **volume** of the aquarium. Simplify the expression.

Solution.



The **volume** of a rectangular box is width×length×height.

By this, the volume of the aquarium is $x \cdot (2x) \cdot (x+3)$.

Simplify this expression:

$$x \cdot (2x) \cdot (x+3) = 2x^2(x+3) = \boxed{2x^3 + 6x^2}.$$

Electric bill

Problem. An electric company charges a flat rate of $\$\,50$ per month plus $\$\,x$ per kWh. The sales tax is 2.5%. Compose an algebraic expression showing total charges in the electric bill for this month, if 1000 kWh have been consumed. Simplify the expression.

Solution. The charge for consumed 1000 kWh is $1000x$. Adding the flat rate of $\$\,50$, we get the charge before tax: $50 + 1000x$.
Upon the top, we have to add 2.5% tax, which is 2.5% of before-tax amount. Since $2.5\% = \dfrac{2.5}{100} = 0.025$, the tax is $0.025(50 + 1000x)$.

The total charge is

$$\underbrace{50}_{\text{flat rate}} + \underbrace{1000x}_{\text{consumed}} + \underbrace{0.025(50 + 1000x)}_{\text{tax}}.$$

Simplify the expression:
$50 + 1000x + 0.025(50 + 1000x) = 50 + 1000x + 1.25 + 25x = \boxed{51.25 + 1025x}$ (dollars).

Summary

In this lecture, we have learned

- ☑ how to translate English phrases into algebraic language
- ☑ which additional information may be required:
  - formulas for area, volume, perimeter of geometric figures
  - formula for uniform motion
  - percentage
- ☑ how to make schematic drawings for problems

Lecture 14

Equalities, Identities and Equations

Equalities

An (algebraic) **equality** consists of two algebraic expressions connected by the equality sign "$=$".

For example,

$$\begin{aligned} x^2 - 3x + 1 &= x + 2\,, \\ 1 + 1 &= 2\,, \\ 0 &= 1\,, \\ a + b &= b + a\,, \\ (x-y)^2 &= x^2 - 2xy + y^2\,. \end{aligned}$$

An algebraic equality with a variable becomes a numerical one if we **evaluate** the expressions on both sides of the equality at some number.

For example, if we substitute $x = 1$ into both sides of the equality $x^2 = x$, it turns into a numerical equality $1^2 = 1$, which is true.

If we substitute $x = -1$, then we get $(-1)^2 = -1$, which is false.

True or false

equalities

numerical: $1 + 1 = 2$

with variables: $a(b + c) = ab + ac$

numerical equalities

true: $1 + 4 = 2 + 3$

false: $1 + 4 = 2 + 5$

equalities **with variables**

always true (**identities**): $x^2 - y^2 = (x - y)(x + y)$

always false (**contradictions**): $x = x + 1$

true or false depending on values of variables: $x + 2 = 3$

Identities

Here are some important identities that we have learned:

$a + b = b + a$ (commutativity of addition)

$a(b + c) = ab + bc$ (distributive law)

$x^n \cdot x^m = x^{n+m}$ (multiplication rule for powers)

$x^2 - y^2 = (x - y)(x + y)$ (difference of squares)

$(x + y)^2 = x^2 + 2xy + y^2$ (short multiplication)

Proving identities

A **typical** problem about an identity is to **prove** it.

That is, to prove that the equality is true for all values of the variables.

Example. Prove that $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$ for all values of x.

Solution. Work on the left hand side:

$$\begin{aligned}(x + 1)^3 = (x + 1)(x + 1)^2 &= (x + 1)(x^2 + 2x + 1)\\ &= x^3 + 2x^2 + x + x^2 + 2x + 1\\ &= x^3 + 3x^2 + 3x + 1,\end{aligned}$$

which is the right hand side of the identity.

Therefore,
$(x + 1)^3 = x^3 + 3x^2 + 3x + 1$ for all values of x, and the identity is proven.

Equation and its solution

Often we use the word "**equation**" instead of "equality".
This happens when we are interested to **find** the values of variables
at which the equality turns to a **true numerical equality**.

A **solution** of an equation with a single variable
is the value of the variable which turns the equation into a true numerical equality.

Example. Consider the equation $x + 2 = 3x$. At $x = 1$, the equation turns into a **true** numerical equality:

$$1 + 2 = 3 \cdot 1 .$$

If we substitute $x = 0$, then the equation turns into a **false** numerical equality:

$$0 + 2 = 3 \cdot 0 .$$

Therefore, $x = 1$ is a solution of the equation $x + 2 = 3x$, while $x = 0$ is not a solution.

All the solutions

It may happen that an equation has **no** solutions.

For example, the equation $0 \cdot x = 1$ has no solution, since $0 \cdot x \neq 1$ no matter what x is.

Some equations have **infinitely** many solutions.

For example, the equation $0 \cdot x = 0$ has infinitely many solutions. **Any** number is a solution.

To solve an equation means to find **all** its solutions,
that is to find all values of the variable
which turn the equation into a **true** numerical equality.

The variable in the equation is called **unknown**.
To solve an equation means to make this unknown known.

Several unknowns

An equation may have **several** unknowns.

For example, $x + 2y = 7$ is an equation with two unknowns x and y.
The equation turns into a **true** numerical equality if we plug in $x = 1$ and $y = 3$:

$$1 + 2 \cdot 3 = 7 .$$

Plugging in $x = 1$ and $y = 2$ results into a **false** equality:

$$1 + 2 \cdot 2 = 7 .$$

A **solution** of such equation is a **pair** of numbers which turns the equation into a true numerical equality. For example, the pair $x = 1$ and $y = 3$ is a solution.

Another solution is $x = -1$, $y = 4$. Indeed:

$$(-1) + 2 \cdot 4 = 7 .$$

As we will learn later, equations like this have **infinitely many** solutions.

Summary

In this lecture, we have learned

- ☑ what an **equality** is
- ☑ that there are **numerical** equalities and equalities **with variables**
- ☑ what an **identity** is
- ☑ what a **contradiction** is
- ☑ what an **equation** is
- ☑ what a **solution** of an equation is
- ☑ what it means **to solve** an equation

Lecture 15

Linear Equations

Equation and its solutions

Recall that an **equation** is an equality between two algebraic expressions.

The variables in equation are called **unknowns**.

For example, $3x + 1 = 7$ is an equation with one unknown x.

To solve an equation means to find **all** its solutions,
that is all the values of the variables which **satisfy** the equation.
In other words, to find all values of the unknowns
which turn the equation into a true numerical equality.

For example, $x = 2$ is a solution of the equation $3x + 1 = 7$, since it satisfies the equation:

$$3 \cdot 2 + 1 = 7 .$$

Equivalent equations

Some equations are easy.

Example. $x = 2$ is an equation. But it looks like a solution, and it is a **solution** for itself!

Often, more complicated equations are replaced by simpler equations which have the same solutions.

If two equations have the same solutions,

that is if

any solution of the first equation is a solution of the second one

and vice versa:

each solution of the second equation is a solution of the first one

then we call the equations **equivalent**,
and write the equivalence sign " $\iff$ " between them, like this:

$$x + 1 = 3 \iff x = 2 .$$

How to **transform** an equation into an equivalent equation?

To this end, we will use two **elementary** transformations.

Add the same to both sides

> Any equation is equivalent to the equation obtained from it by **adding** the same expression to **both** sides.

Example 1. Consider the equation $x - 1 = 2$. If we add 1 to both sides, then we get an equavalent equation:

$$x - 1 = 2 \iff x - 1 + 1 = 2 + 1 \iff \boxed{x = 3}$$

Example 2. $5 - x = 0 \iff 5 - x + x = 0 + x \iff 5 = x \iff \boxed{x = 5}$

Example 3.

$$5 - x = 2 \iff 5 - x + (x - 2) = 2 + (x - 2) \iff$$
$$5 - x + x - 2 = 2 + x - 2 \iff 3 = x \iff \boxed{x = 3}$$

Similarly, **subtracting** the same expression from both sides of an equation gives rise to an equivalent equation:

$$x + 2 = 6 \iff x + 2 - 2 = 6 - 2 \iff \boxed{x = 4}$$

Example

$$\begin{tikzcd}
2x - 1 = 5 + x \arrow[r, "\text{subtract } x"] & 2x - 1 - x = 5 + x - x \arrow[d, "\text{simplify}"'] & \\
& x - 1 = 5 \arrow[r, "\text{add } 1"] & x - 1 + 1 = 5 + 1 \arrow[d, "\text{simplify}"'] \\
& & x = 6
\end{tikzcd}$$

These transformations are written as follows:

$$2x - 1 = 5 + x \iff 2x - 1 - x = 5 + x - x \iff$$
$$x - 1 = 5 \iff x - 1 + 1 = 5 + 1 \iff \boxed{x = 6}$$

Fast track

There is a trick that may help you to operate **more efficiently** with equations.

The subtraction of x from both sides of the equation $2x-1=5+x$, namely

$$2x-1=5+x \iff 2x-1-x=5+x-x \iff x-1=5$$

is equivalent to relocation x from the right hand side (RHS) of the equation to the left hand side (LHS) with the **opposite** sign:

$$2x-1=5+x \iff 2x-x-1=5 \iff x-1=5$$

Look how **fast** we can solve the equation:

$$2x-1=5+x \iff x-1=5 \iff x=6\,.$$

Multiply both sides by the same non-zero number

> Any equation is equivalent to the equation obtained from it by **multiplying** both sides by the same **non-zero** number.

Example 1. $\dfrac{x}{2}=3 \iff \dfrac{x}{2}\cdot 2=3\cdot 2 \iff \boxed{x=6}$

Example 2. $3x=5 \iff 3x\cdot\dfrac{1}{3}=5\cdot\dfrac{1}{3} \iff \boxed{x=\dfrac{5}{3}}$

Similarly, **dividing** both sides of an equation by the same **non-zero** number gives rise to an equivalent equation:

$$2x=8 \iff \frac{2x}{2}=\frac{8}{2} \iff \boxed{x=4}$$

Adding the same expression to both sides of an equation and **multiplying** both sides by the the same non-zero number are called **elementary transformations** of the equation.

Example of elementary transformations

See how a sequence of **elementary transformations** brings an equation to a simple equivalent equation, which is the solution.

Example. Solve the equation $7x - 5 = 2x + 1$.

Solution. $7x - 5 = 2x + 1$

Move $2x$ to the LHS: $7x - 2x - 5 = 1$

Simplify: $5x - 5 = 1$

Move -5 to the RHS: $5x = 1 + 5$

Simplify: $5x = 6$

Divide by 5: $\boxed{x = \frac{6}{5}}$

Linear equations

An equation is called **linear**, if both its sides are polynomials of degree ≤ 1.

For example, $3(x-2) + 4 = \frac{2}{3}(5x+1) + x$ is a linear equation,

$x^2 + 2 = x$ is not.

A polynomial of degree ≤ 1 is called a **linear expression**.

Both sides of a linear equation are linear expressions.

By a sequence of elementary transformations, any linear equation can be transformed to an equation of the form $ax = b$ where a and b are some numbers and x is an unknown.

To do this, that is, to bring the equation to the form $ax = b$,

- simplify (if needed) both sides of the equation,
- collect all terms involving the unknown on one side of the equation, and all numbers on the other side,
- simplify the equation again.

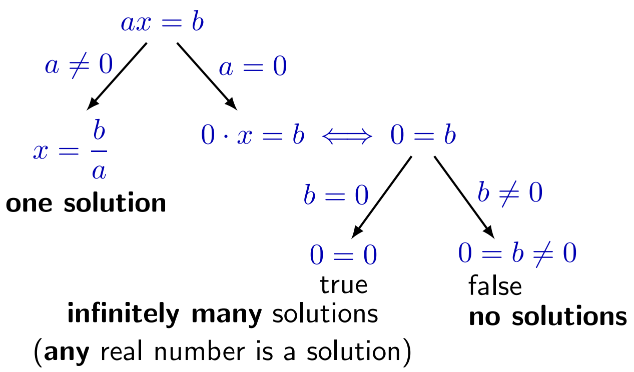
Example

Solve the equation $\dfrac{3}{2}(x-1) = \dfrac{x}{3} + 1$.

$$
\begin{aligned}
\text{Multiply by } 6\text{:} \quad & 6 \cdot \frac{3}{2}(x-1) = 6\left(\frac{x}{3}+1\right) \quad \text{to get rid of fractions} \\
\text{Simplify LHS:} \quad & 9(x-1) = 6\left(\frac{x}{3}+1\right) \\
\text{Distribute:} \quad & 9x - 9 = 2x + 6 \\
\text{Move } 2x \text{ to LHS:} \quad & 7x - 9 = 6 \\
\text{Move } -9 \text{ to RHS:} \quad & 7x = 15 \longleftarrow \text{ equation in the form } ax = b \\
\text{Divide by } 7\text{:} \quad & x = \frac{15}{7} \longleftarrow \text{ solution}
\end{aligned}
$$

Number of solutions of a linear equation

How many solutions may a linear equation $ax = b$ have? It depends on the numbers a and b .



A linear equation with one unknown may have either **one** solution, or **no solutions**, or **infinitely many** solutions.

Examples of linear equations

Example 1. Solve the equation $2(x-3) = 2x+1$.

Solution. Distribute: $2x - 6 = 2x + 1$

Move $2x$ from RHS to LHS: $2x - 2x - 6 = 1$

Simplify: $-6 = 1 \longleftarrow$ **false** numerical equality

Answer. The equation has **no solutions**.

Example 2. Solve the equation $2 - x = \frac{1}{3}(6-3x)$.

Solution. Multiply both sides by 3: $3 \cdot (2-x) = 3 \cdot \frac{1}{3}(6-3x)$

Simplify: $6 - 3x = 6 - 3x$

Add $3x$ to both sides: $6 = 6 \longleftarrow$ **true** numerical equality

Answer. The equation is an **identity**.

Any number is a solution.

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How to check if solution is correct

While solving an equation, we can make mistakes.

There is a opportunity **to check** if the number obtained is a solution.

For this, **plug in** the number into the equation and check if the obtained numerical equality is **true**.

Example. Solve the equation $2x - 1 = 3(2x+1)$ and check your solution by substitution.

Solution. $2x - 1 = 3(2x+1) \iff 2x - 1 = 6x + 3 \iff -1 = 4x + 3 \iff$

$$-4 = 4x \iff -1 = x \iff \boxed{x = -1}$$

Check (substitute $x = -1$ into the original equation):

$$2 \cdot (-1) - 1 \stackrel{?}{=} 3(2 \cdot (-1) + 1)$$
$$-2 - 1 \stackrel{?}{=} 3(-2+1)$$
$$-3 \stackrel{?}{=} 3(-1)$$
$$-3 = -3 \ \checkmark$$

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Summary

In this lecture, we have learned

- ☑ which equations are called **equivalent**
- ☑ that there are elementary **transformations** of equations:
  - adding the same expression to both sides
  - multiplying both sides by the same non-zero number
- ☑ how to solve equations **efficiently**
- ☑ what a **linear** equation is
- ☑ how to solve a linear equation
- ☑ how many solutions a linear equation may have:
  - one
  - infinitely many
  - no solutions
- ☑ how **to check** a solution by substitution into the original equation

Lecture 16

Applications of Linear Equations

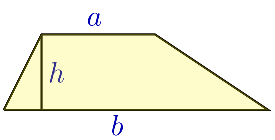
Linear equations in mathematics, physics, and beyond

In this lecture, we will show how

- to solve linear equations originated in **mathematics** and **physics**
- how to use linear equations for solving **word problems**.

Area of trapezoid

Problem 1. The area A of a trapezoid with bases a, b and the height h is given by the formula



$$A = \frac{a+b}{2} h .$$

Using this formula, express b in terms of A, a, and h.

Solution. We have to solve out b from the equation $A = \dfrac{a+b}{2} h$.

Multiply the equation by 2: $2A = (a+b)h$,

divide both sides by h: $\dfrac{2A}{h} = a + b$,

and move a to LHS: $\dfrac{2A}{h} - a = b$.

Answer: $b = \dfrac{2A}{h} - a$.

Motion with constant acceleration

Problem. A car moving at a constant speed of v_0 starts to accelerate with a constant acceleration of a. How long will it take for the car to increase the speed up to v, if the initial speed v_0, the terminal speed v, the acceleration a, and the time t are related by the formula $v = v_0 + at$?

Solution. We have to solve out t from the equation $v = v_0 + at$.

For this, we subtract v_0 from both sides: $v - v_0 = at$,

and divide both sides by a: $\dfrac{v - v_0}{a} = t$.

Answer: $t = \dfrac{v - v_0}{a}$.

Newton's law

Example. According to Newton's law of universal gravitation,

$$F = G\frac{m_1 m_2}{R^2},$$

where F is the gravitational force between masses m_1 and m_2, G is the gravitational constant, and R is the distance between the centers of the masses.
Use this equation to find m_1 in terms of F, G, m_2, and R.

Solution. To solve out m_1 from the equation $F = G\dfrac{m_1 m_2}{R^2}$,

multiply both sides by R^2: $FR^2 = G\,m_1 m_2$,

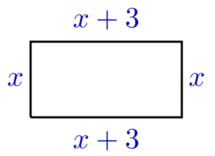
and divide by Gm_2: $\dfrac{FR^2}{Gm_2} = m_1$.

Answer: $m_1 = \dfrac{FR^2}{Gm_2}$

Perimeter of a rectangle

Problem. In a rectangle, one side is 3 feet longer than the other side. Find the lengths of the sides, if the perimeter of the rectangle is 34 feet.

Solution.



Let x be the length of the short side.

Then the length of the long side is $x+3$.

The **perimeter** is the sum of the lengths of all sides: $x+(x+3)+x+(x+3)$.

Simplify this expression: $x+(x+3)+x+(x+3)=4x+6$.

Since the perimeter is 34 feet, $4x+6=34$.

Solve this equation: $4x+6=34 \iff 4x=28 \iff \boxed{x=7}$ feet.

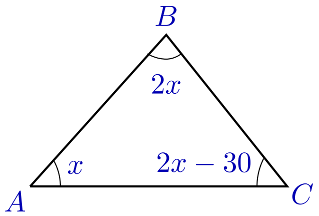
The short side is 7 feet, the long side is $7+3=10$ feet.

Answer. The lengths of the sides are 7 and 10 feet.

Angles in a triangle

Problem. In a triangle ABC, the angle B is twice as large as the angle A, and the angle C is $30°$ less than the angle B. Find the angles.

Solution.



Let x be the measure of A.

Then the measure of B is $2x$,

and the measure of C is $2x-30$.

The **sum** of the angles in a triangle is $180°$. In our case,

$$x+2x+(2x-30)=180.$$

This is a linear equation to solve:

$x+2x+(2x-30)=180 \iff 5x-30=180 \iff 5x=210 \iff x=42.$

The measure of A is $\boxed{42°}$, the measure of B is $2\cdot 42=\boxed{84°}$,

the measure of C is $84-30=\boxed{54°}$.

Uniform motion

Problem. A car traveled for 3 hours at a constant speed. Then it increased the speed by 8 mi/h and traveled for another 2 hours. During this trip, the car traveled for 271 miles. Find the speed of the car on both intervals of driving.

Solution. Let x mi/h be the speed of the car on the first interval of driving. Then the speed on the second interval of driving is $x+8$ mi/h.

3 h | 2 h

x mi/h | $(x+8)$ mi/h

$\underbrace{\qquad\qquad}_{3x \text{ miles}}$ $\underbrace{\qquad\qquad}_{2(x+8) \text{ miles}}$

The **total** distance is $3x+2(x+8)$ miles, which is equal to 271 miles.
Therefore, $3x+2(x+8)=271$. Let us solve this equation to find x.
$3x+2(x+8)=271 \iff 3x+2x+16=271 \iff 5x=255 \iff x=51$

So the speed on the first interval is 51 mi/h, and the speed on the second interval is $51+8=59$ mi/h.

Answer. 51 mi/h and 59 mi/h.

Summary

In this lecture, we have learned

- ☑ how to solve linear equations "with letters" arising from **mathematics and physics**
- ☑ how to solve **word problems** leading to linear equations

Lecture 17

Linear Inequalities

What a linear inequality is

There are four **inequality signs**: $<$, $\leq$, $>$, $\geq$.

$$\begin{array}{ll} a < b & a \text{ is less than } b \\ a \leq b & a \text{ is less than or equal to } b \\ a > b & a \text{ is greater than } b \\ a \geq b & a \text{ is greater than or equal to } b \end{array}$$

A **linear inequality** consists of two linear expressions connected by one of the inequality signs.

For example, $3(x-1) \leq 4+5x$ is a linear inequality in one variable.

Evaluation of both sides of an inequality at a **number**
gives rise to a numerical inequality, which may be either true or false.

For example, at $x=0$ the inequality above holds true:

$$3(0-1) \leq 4+5\cdot 0 \iff -3 \leq 4 \quad \checkmark$$

Solution

To solve an inequality means to find **all** values of the variable, for which the inequality holds true.

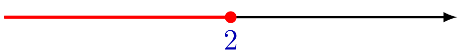
These values form a **solution set**.

A linear **inequality** is very similar to a linear **equation**.

As we remember, the solution set of a linear **equation**
• either consists of a single number (when the equation has one solution),
• or is **empty** (when the equation has no solutions),
• or is the entire number line (when the equation has infinitely many solutions).

The solution set of a linear inequality is quite different.

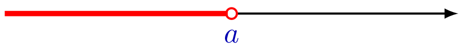
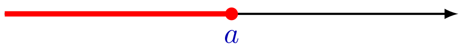
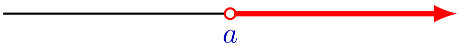
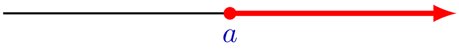
Consider a simple inequality $x \leq 2$. Its solution set consists of all numbers ≤ 2
and is denoted by $\{x \mid x \leq 2\}$. One can **graph** the solutions on the number line:



The solution set is an **interval**. It is denoted by $(-\infty, 2]$.

Intervals

Let us review intervals that we may encounter solving linear inequalities.

inequality	solution	graph	interval
$x < a$	$\{x \mid x < a\}$		$(-\infty, a)$
$x \leq a$	$\{x \mid x \leq a\}$		$(-\infty, a]$
$x > a$	$\{x \mid x > a\}$		(a, ∞)
$x \geq a$	$\{x \mid x \geq a\}$		$[a, \infty)$

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Equivalent inequalities

Two inequalities are called **equivalent** if they have the same solution sets.

It means that each solution of the first inequality is a solution of the second one, and vice versa: each solution of the second inequality is a solution of the first one.

If two inequalities are equivalent, we write the equivalence sign " $\iff$ " between them, like this

$$x + 1 > 3 \iff x > 2 .$$

How to **transform** an inequality into an equivalent inequality?

To this end, we will use three **elementary** transformations.

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Add the same to both sides

> Any inequality is equivalent to the inequality obtained from it by **adding** the same expression to **both** sides.

Example 1. Consider the inequality $x-1>2$. If we add 1 to both sides, then we get an equavalent inequality:

$$x-1>2 \iff x-1+1>2+1 \iff \boxed{x>3}$$

Example 2. $5-x\le 0 \iff 5-x+x\le 0+x \iff 5\le x \iff \boxed{x\ge 5}$

Example 3.

$$5-x<2 \iff 5-x+(x-2)<2+(x-2) \iff$$
$$5-x+x-2<2+x-2 \iff 3<x \iff \boxed{x>3}$$

Similarly, **subtracting** the same expression from both sides of an inequality gives rise to an equivalent inequality:

$$x+2\ge 6 \iff x+2-2\ge 6-2 \iff \boxed{x\ge 4}$$

Fast track

There is a trick that may help you to operate **more efficiently** with inequalities.

The subtraction of x from both sides of the inequality $2x-1\le 5+x$, namely

$$2x-1\le 5+x \iff 2x-1-x\le 5+x-x \iff x-1\le 5$$

is equivalent to relocation x from the right hand side (RHS) of the inequality to the left hand side (LHS) with the **opposite** sign:

$$2x-1\le 5+x \iff 2x-x-1\le 5 \iff x-1\le 5$$

Look how **fast** we can solve the inequality:

$$2x-1\le 5+x \iff x-1\le 5 \iff x\le 6\,.$$

Multiply both sides by the same positive number

Any inequality is equivalent to the inequality obtained from it by **multiplying** both sides by the same **positive** number.

Example 1. $\frac{x}{2} > 3 \iff \frac{x}{2} \cdot 2 > 3 \cdot 2 \iff \boxed{x > 6}$

Example 2. $3x \le 5 \iff 3x \cdot \frac{1}{3} \le 5 \cdot \frac{1}{3} \iff \boxed{x \le \frac{5}{3}}$

Similarly, **dividing** both sides of an inequality by the same **positive** number gives rise to an equivalent inequality:

$$2x \ge 8 \iff \frac{2x}{2} \ge \frac{8}{2} \iff \boxed{x \ge 4}$$

Multiply by negative number and reverse the sign

What happens if we multiply an inequality by a **negative** number?

Consider the inequality $x > 2$. Move x to RHS, and move 2 to LHS (don't forget to change the signs):

$$x > 2 \iff -2 > -x\,.$$

This inequality says that -2 is greater than $-x$. This is the same as $-x$ is less than -2:

$$-2 > -x \iff -x < -2\,.$$

Therefore, $x > 2 \iff -x < -2\,.$

In general, if we multiply both sides of an inequality by a **negative** number, we have to **reverse the sign** of the inequality.

Example 1. $-\frac{x}{3} < 2 \iff (-3) \cdot \left(-\frac{x}{3}\right) > (-3) \cdot 2 \iff x > -6.$

The same rule is valid if we **divide** an inequality by a negative number.

Example 2. $-2x \le 6 \iff \frac{-2x}{-2} \ge \frac{6}{-2} \iff x \ge -3\,.$

Elementary transformations

Elementary transformations of an inequality are

- **adding** the same expression to both sides of an inequality,
- **multiplying** both sides by the the same **positive** number, and
- **multiplying** both sides by the the same **negative** number and **reversing** the sign of the inequality.

See how a sequence of **elementary transformations** brings an inequality
to a simple equivalent inequality.

Examples

Example 1. Solve the inequality $7x - 5 \leq 2x + 1$. Give the answer in interval notation. Show the solution on the number line.

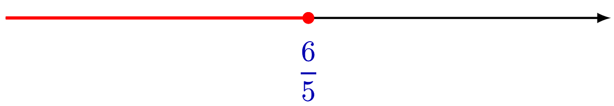
Solution. Move $2x$ to the LHS: $7x - 2x - 5 \leq 1$

Simplify: $5x - 5 \leq 1$

Move -5 to the RHS: $5x \leq 1 + 5$

Simplify: $5x \leq 6$

Divide by 5: $\boxed{x \leq \frac{6}{5}}$



Answer. $\left(-\infty, \frac{6}{5}\right]$

Examples

Example 2. Solve the inequality $-\frac{x}{2}+3<x+4$. Give the answer in interval notation. Show the solution on the number line.

Solution.

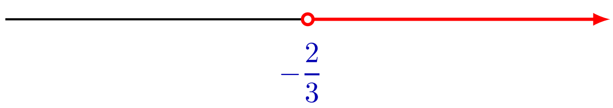
$$
\begin{aligned}
\text{Move } 3 \text{ to the RHS:}\quad & -\frac{x}{2}<x+4-3\\
\text{Simplify:}\quad & -\frac{x}{2}<x+1\\
\text{Multiply by } (-2)\text{:}\quad & (-2)\left(-\frac{x}{2}\right)>(-2)(x+1)\\
\text{Simplify:}\quad & x>-2x-2\\
\text{Move } -2x \text{ to the LHS:}\quad & x+2x>-2\\
\text{Simplify:}\quad & 3x>-2\\
\text{Divide by } 3\text{:}\quad & x>-\frac{2}{3}
\end{aligned}
$$

Writing down the answer

The answer can be written as an inequality $x>-\frac{2}{3}$,

or as a set $\left\{x \mid x>-\frac{2}{3}\right\}$,

or as an interval $\left(-\frac{2}{3},\infty\right)$ on a number line:



Systems of linear inequalities

Two inequalities with the same single variable may form a **system**.

To **solve** a system means

to find all the values of the variable that satisfy **both** inequalities.

Example. Solve the system $\begin{cases} 3x - 2 \le 2x - 1 \\ -2x + 3 < 4. \end{cases}$

Write the answer in interval notation. Show the solution on the number line.

Solution.

$$\begin{cases} 3x - 2 \le 2x - 1 \\ -2x + 3 < 4 \end{cases} \iff \begin{cases} 3x - 2x \le -1 + 2 \\ -2x < 1 \end{cases} \iff \begin{cases} x \le 1 \\ x > -\dfrac{1}{2} \end{cases} \iff -\frac{1}{2} < x \le 1$$

Answer: $\left(-\frac{1}{2}, 1\right]$

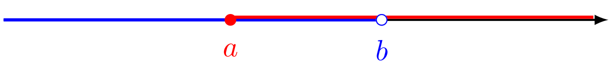
Solution of a system

Geometrically, the solution of a system of two linear inequalities in one variable

is the **intersection** of two intervals.

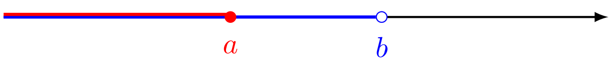
The intersection consists of all points belonging to **both** intervals.

As the intersection, we may get a **finite interval**, for example, $a \le x < b$:



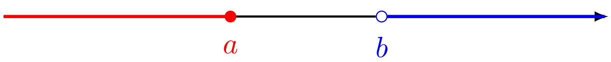
$[a, b)$

an **infinite interval**, for example $x \le a$:



$(-\infty, a]$

or the **empty set** (when the system has no solutions):



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Summary

In this lecture, we have learned

☑ what a **linear inequality** is
☑ what the **solution** of an inequality is
☑ which **intervals** on a real line may appear as solutions of inequalities
☑ which inequalities are called **equivalent**
☑ what **elementary transformations** of inequalities are
- adding the same expression to both sides
- multiplying both sides by the same **positive** number
- multiplying both sides by the same **negative** number and **reversing the sign** of the inequality

☑ how to solve inequalities **efficiently**
☑ how to **write down** the solution of an inequality
☑ how to show the solution on a **number line**
☑ how to solve a **system** of inequalities

Lecture 18

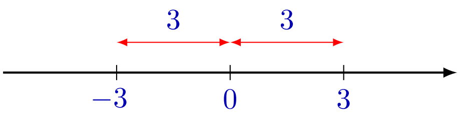
Absolute Value

Absolute value of a number

The **absolute value** of a number is the **distance** between this number and 0 on the number line.

The absolute value of a number a is denoted by $|a|$.

For example, $|3| = 3$, $|-3| = 3$, $|0| = 0$.



In general,

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Observe that if a is negative, then $-a$ is **positive**.

For example, if $a = -5$, then the formula above gives $|-5| = -(-5) = 5$.

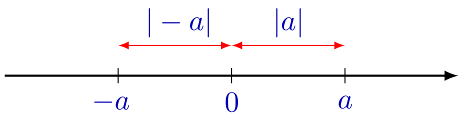
Properties of absolute value

- The absolute value of a number is **non-negative** (positive or zero):

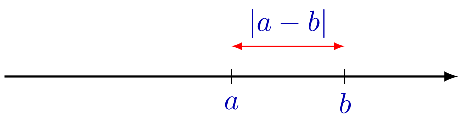
$$|a| \geq 0$$

- A number and its opposite have the same absolute values:

$$|a| = |-a|$$



- The distance between numbers a and b on the number line is given by $|a - b|$



Examples

Example 1. Calculate $|-6+|-2-3||$.

Solution. $|-6+|-2-3|| = |-6+|-5|| = |-6+5| = |-1| = 1$.

Example 2. Which number is greater, $|-2|$ or -3?

Solution. Since $|-2| = 2$, and $2 > -3$, we get $|-2| > -3$.

Example 3. Find the distance between the numbers -7 and -3 on the number line.

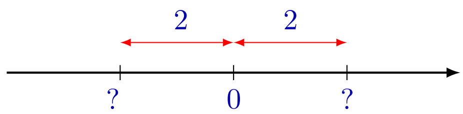
Solution. The distance between two numbers is given by the **absolute value** of the difference between them:

$$|-7-(-3)| = |-7+3| = |-4| = 4.$$

Linear equations involving absolute value

Example 1. Solve the equation $|x| = 2$.

Solution. We have to find **all** values of the unknown x for which $|x| = 2$, that is, all numbers which are located at the distance of 2 from 0.



These numbers are 2 and -2.

It is convenient to write down our solution as follows:

$$|x| = 2$$

$$x = 2 \qquad x = -2$$

Answer. $x = 2$ or $x = -2$.

Linear equations involving absolute value

Example 2. Solve the equation $|3x-1|=2$. Check your answer by substitution.

Solution.

$$|3x-1| = 2$$

$$\begin{array}{ccc} 3x-1=2 & \textbf{or} & 3x-1=-2 \\ 3x=3 & & 3x=-1 \\ x=1 & \textbf{or} & x=-1/3 \end{array}$$

Check now that both $x=1$ and $x=-1/3$ satisfy the original equation.

Plug in $x=1$:

$|3\cdot 1-1| \stackrel{?}{=} 2$

$|2| \stackrel{?}{=} 2$

$2 \stackrel{\checkmark}{=} 2$

Plug in $x=-1/3$:

$|3\cdot\left(-\frac{1}{3}\right)-1| \stackrel{?}{=} 2$

$|-1-1| \stackrel{?}{=} 2$

$|-2| \stackrel{?}{=} 2$

$2 \stackrel{\checkmark}{=} 2$

Answer. $x=1$ or $x=-1/3$.

Linear inequalities involving absolute value

Example 1. Solve the inequality $|3x-1|<2$.
Give your answer in interval notation. Show the solution on the number line.

Solution. The inequality means that
the number $3x-1$ is on the distance **less** than 2 units from 0.

Therefore, this number should be in between -2 and 2:

$$-2 < 3x-1 < 2\,.$$

This double inequality is nothing but a system of inequalities:

$$-2<3x-1<2 \iff \begin{cases} -2<3x-1 \\ 3x-1<2 \end{cases} \iff \begin{cases} -2+1<3x \\ 3x<2+1 \end{cases} \iff$$

$$\begin{cases} -1<3x \\ 3x<3 \end{cases} \iff \begin{cases} -1/3<x \\ x<1 \end{cases} \iff -1/3<x<1$$

Answer. $(-1/3, 1)$

Linear inequalities involving absolute value

Example 2. Solve the inequality $|1-x| \geq 3$.

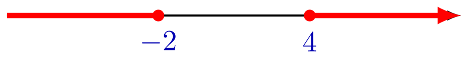
Give your answer in interval notation. Show the solution on the number line.

Solution. The inequality means that

the number $1-x$ is on the distance **more** than or equal to 3 units from 0.

Therefore, the number $1-x$ should be ≥ 3 **or** ≤ -3:

$$\begin{array}{rcl} 1-x \geq 3 & \text{or} & 1-x \leq -3 \\ -x \geq 2 & \text{or} & -x \leq -4 \\ x \leq -2 & \text{or} & x \geq 4 \end{array}$$



The solution is the **union** of two intervals: $(-\infty,-2) \cup (4,\infty)$.

Answer. $(-\infty,-2) \cup (4,\infty)$

Summary

In this lecture, we have learned

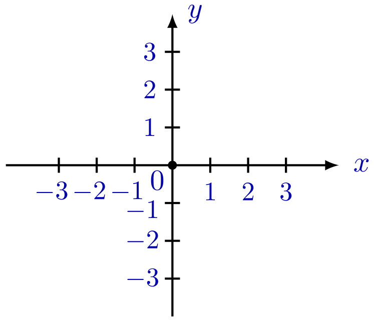
- ☑ what **absolute value** of a number is
- ☑ what the **properties** of absolute value are
- ☑ how to solve **linear equations** involving absolute value
- ☑ how to solve **linear inequalities** involving absolute value

Lecture 19

Lines on a Plane. Part 1

Cartesian coordinate system on a plane

Cartesian (or **rectangular**) coordinate system is defined by
• a point, called the **origin**,
• two perpendicular **number lines** drawn through the origin.

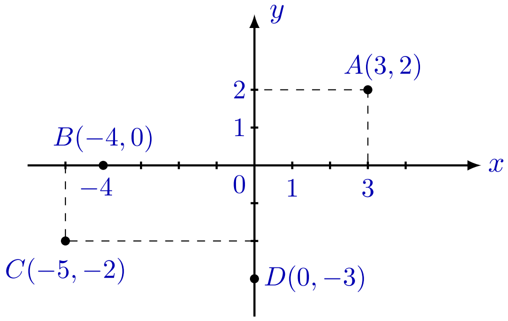


Usually, one line is drawn horizontally, and the other one vertically.

The horizontal line is called $\boldsymbol{x}$-**axis**, the vertical line is called the $\boldsymbol{y}$-**axis**.

Points and their coordinates

Given the coordinate system, each point on the plane gets its **coordinates** –
two numbers which determine the location of the point on the plane.

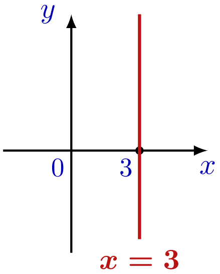


The first number is called $\boldsymbol{x}$-**coordinate**, the second number is called the $\boldsymbol{y}$-**coordinate**.
For example, the coordinates of point A are $(3, 2)$,
where 3 is the x-coordinate and 2 is the y-coordinate.

Vertical lines

Example. Describe geometrically the set of all points on the coordinate plane whose x-coordinate is 3.

Solution. These are the points with coordinates $(3, y)$, where y is an arbitrary number.



All such points form a **vertical** line passing through the point 3 on the x-axis.

This vertical line is the **graph** of the equation $\boldsymbol{x = 3}$.

The graph of the equation $x = a$, where a is a number, is the **vertical** line passing through the point a on the x-axis.

The y-axis, which is a vertical line, has the equation $x = 0$.

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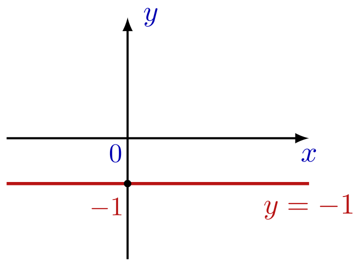
Horizontal lines

Example. Draw the graph of the equation $y = -1$.

Solution. The graph of $y = -1$ is the set of all points on the plane whose coordinates are $(x, -1)$, where x is an arbitrary number.
It is a **horizontal** line passing through the point -1 on the y-axis.



The graph of the equation $y = b$, where b is a number, is the **horizontal** line passing through the point b on the y-axis.

The x-axis, which is a horizontal line, has the equation $y = 0$.

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General linear equation in two variables

The equation $Ax + By = C$, where A, B, C are given numbers and x, y are variables, is called a **linear equation** in two variables.

The numbers A, B, C are called the **coefficients**.

Examples of linear equations in two variables:

$-2x + y = 4 \quad (A = -2,\ B = 1,\ C = 4)$,

$x = 1 \iff x + 0 \cdot y = 1 \quad (A = 1,\ B = 0,\ C = 1)$,

$y = 0 \iff 0 \cdot x + y = 0 \quad (A = 0,\ B = 1,\ C = 0)$,

$0 = 3 \iff 0 \cdot x + 0 \cdot y = 3 \quad (A = 0,\ B = 0,\ C = 3)$,

$0 = 0 \iff 0 \cdot x + 0 \cdot y = 0 \quad (A = 0,\ B = 0,\ C = 0)$.

The **graph** of an equation is the set of **all** points on the plane whose coordinates satisfy the equation.

The graph of a linear equation in two variables

What is the **graph** of the equation $Ax + By = C$? It depends on the coefficients A, B, C.

• If all the coefficients are zeros, that is $A = B = C = 0$, then the equation is

$$0 \cdot x + 0 \cdot y = 0 \iff 0 = 0,$$

and it is satisfied by **any** pair of numbers (x, y). Therefore, its graph is the **entire plane**.

• If $A = B = 0$ and $C \neq 0$ then the equation is

$$0 \cdot x + 0 \cdot y = C \iff 0 = C,$$

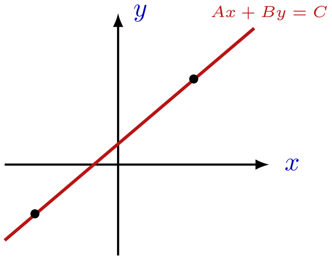
and there are **no** (x, y) satisfying it. Its graph is the **empty set**.

• If $A,\ B$ are **not** both zero, that is either $A \neq 0$ or $B \neq 0$, then the graph is a **straight line**.

Line as the graph of a linear equation

If A, B are **not** both zero, then there are **infinitely many** points (x, y) satisfying the equation $Ax + By = C$.

They are located on a **straight line**. This line is the **graph** of the equation $Ax + By = C$.



A line is determined by any two of its points. Therefore, to draw the line, it is enough to specify the location of **two** points on it.

How to draw a line by its equation

Example. Draw the line $3x - 4y = 12$ on the coordinate plane.

Solution. Let us pick up two points on the line. A point on the line is defined by a pair of numbers (x, y), satisfying the equation $3x - 4y = 12$.

For simplicity, let us choose $x = 0$. Then

$$3 \cdot 0 - 4y = 12 \iff -4y = 12 \iff y = -3.$$

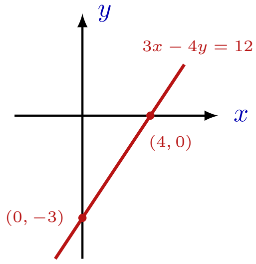
Therefore, $(0, -3)$ is a point on the line.

Now put $y = 0$. Then

$$3x - 4 \cdot 0 = 12 \iff 3x = 12 \iff x = 4.$$

Therefore, $(4, 0)$ is a point on the line.

Draw a line through $(0, -3)$ and $(4, 0)$:



A line through two points

Remark. When we search for two points belonging to the line $3x - 4y = 12$, it is convenient to put the coordinates in the table:

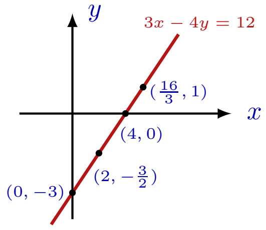
x	y
0	-3
4	0

$$x = 0 \implies y = -3$$
$$y = 0 \implies x = 4$$

One may choose any two other points on the line, for example,

x	y
2	$-\frac{3}{2}$
$\frac{16}{3}$	1

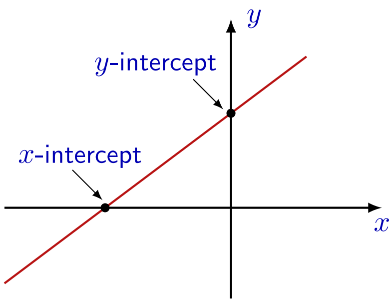
$$x = 2 \implies 3 \cdot 2 - 4y = 12 \implies 6 - 4y = 12 \implies y = -\frac{3}{2}$$
$$y = 1 \implies 3x - 4 \cdot 1 = 12 \implies 3x = 16 \implies x = \frac{16}{3}$$



Intercepts

The point where the line intersects the x-axis is called the x-**intercept**.
The x-intercept has coordinates $(x, 0)$, its y-coordinate equals 0.

The point where the line intersects the y-axis is called the y-**intercept**.
The y-intercept has coordinates $(0, y)$, its x-coordinate equals 0.

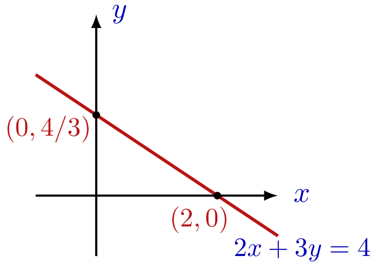


How to find intercepts

Example. Determine the intercepts of the line $2x + 3y = 4$.
Draw the line on the coordinate system.

Solution.
The x-intercept is the point where $y = 0$. Plug in $y = 0$ into the equation:
$2x + 3 \cdot 0 = 4 \iff 2x = 4 \iff x = 2$. So the x-intercept is $(2, 0)$.

The y-intercept is the point where $x = 0$. Plug in $x = 0$ into the equation:
$2 \cdot 0 + 3y = 4 \iff 3y = 4 \iff y = 4/3$. So the y-intercept is $(0, 4/3)$.



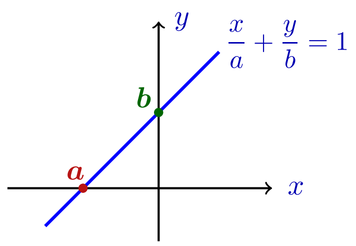
Two-intercept form of a linear equation

The equation $\boxed{\dfrac{x}{a} + \dfrac{y}{b} = 1}$ where x, y are variables and a, b are non-zero numbers,
is called the **two-intercept** equation of a line.

The coefficients $\boldsymbol{a}$ and $\boldsymbol{b}$ represent the x- and y-intercepts respectively.

Indeed, $(\boldsymbol{a}, \mathbf{0})$ and $(\mathbf{0}, \boldsymbol{b})$ satisfy the equation:

$$\frac{\boldsymbol{a}}{a} + \frac{\mathbf{0}}{b} = 1 \quad \text{and} \quad \frac{\mathbf{0}}{a} + \frac{\boldsymbol{b}}{b} = 1 .$$



Quick drawing

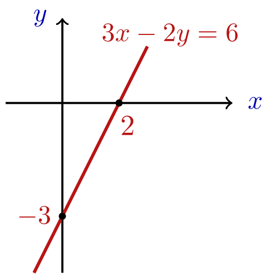
The two intercept form of the equation helps to draw a line in no time.

Example. Draw the line $3x - 2y = 6$.

Solution. Rewrite the equation in the **two-intercept** form:

$$3x - 2y = 6 \iff \frac{3x}{6} - \frac{2y}{6} = 1 \iff \frac{x}{2} + \frac{y}{-3} = 1.$$

The x-intercept is $(2, 0)$, the y-intercept is $(0, -3)$.



Summary

In this lecture, we have learned

- ☑ what a **Cartesian** coordinate system is
- ☑ what the equation of a **vertical** line is ($x = a$)
- ☑ what the equation of a **horizontal** line is ($y = b$)
- ☑ what the general **linear equation in two variables** is ($Ax + By = C$)
- ☑ what the **graph** of a linear equation is
- ☑ how to draw a line by its equation
- ☑ what the **intercepts** are
- ☑ what the **two-intercept** equation is $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

Lecture 20

Lines on a Plane. Part 2

Linear equation $y = mx + b$

Consider a general linear equation $Ax + By = C$ whose graph is a line.

If $B \neq 0$, then the equation can be rewritten as follows:

$$Ax + By = C \iff By = -Ax + C \iff y = -\frac{A}{B}x + \frac{C}{B} \iff y = mx + b\,,$$

where $m = -\dfrac{A}{B}$ and $b = \dfrac{C}{B}$.

If $B = 0$, then $Ax + By = C \iff Ax = C \iff x = \dfrac{C}{A}$,
and the graph is a **vertical** line.

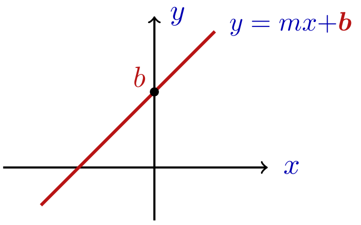
Any non-vertical line can be described by the equation $y = mx + b$.

The y-intercept

Consider a linear equation $y = mx + b$. What do the coefficients m and b represent?

The coefficient $\boldsymbol{b}$ represents the y-**intercept** of the line $y = mx + \boldsymbol{b}$.

Indeed, if $x = 0$ then $y = m \cdot 0 + \boldsymbol{b} \iff y = \boldsymbol{b}$ and $(0, \boldsymbol{b})$ is the y-**intercept**.



The coefficient $\boldsymbol{b}$ in the equation $y = mx + \boldsymbol{b}$ shows where the line meets the y-**axis**.

The coefficient $\boldsymbol{b}$ is called the $\boldsymbol{y}$-**intercept**.

Slope-intercept equation of a line

The coefficient m in the equation $y = mx + b$ is called the **slope** of the line.

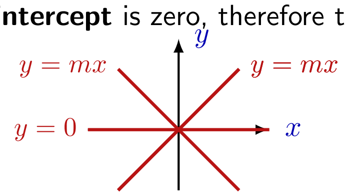
$$y = \underbrace{m}_{\text{slope}}x + \overbrace{b}^{y\text{-intercept}}$$

The equation $\boxed{y = mx + b}$ is called the **slope-intercept** equation of a line.

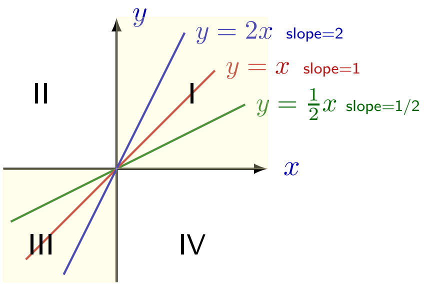
What does the slope of the line represent?

Slope measures the inclination of a line

Let us study a line $y = mx$. The y-**intercept** is zero, therefore the line passes through the origin.



Here are several lines with **positive** slopes:



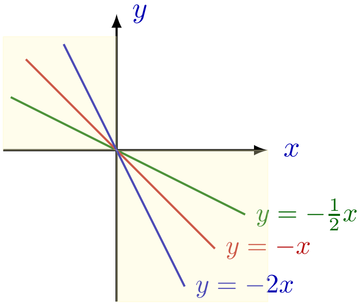
The larger the slope, the steeper the line.

A line with positive slope **rises** as we move from left to right.

Lines $y = mx$ with **positive** m are located in the **first** and **third** quadrants of the plane.

Negative slope. Zero slope

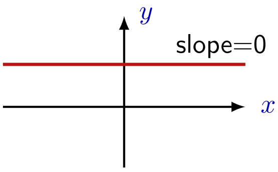
Here are several lines with **negative** slopes:



A line with negative slope **falls** as we move from left to right.

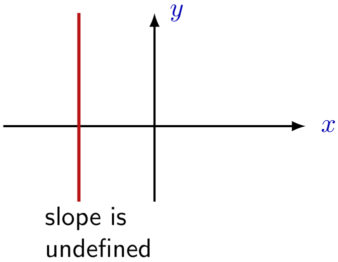
A line $y = mx$ with **negative** m is located in the **second** and **fourth** quadrants.

If the slope $m = 0$, then
$y = mx + b \iff y = 0 \cdot x + b \iff y = b,$
and the line is **horizontal**.



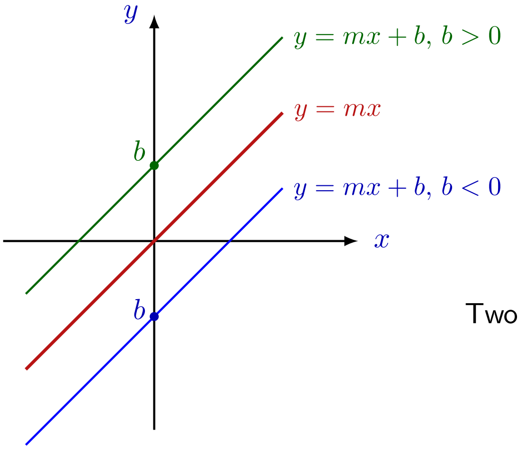
Slope of vertical line

The slope of a vertical line is **undefined**.



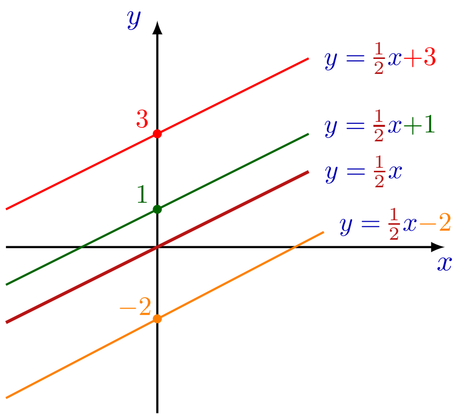
Parallel lines have the same slope

A line $y = mx + b$ is obtained from the line $y = mx$ by a **vertical shift** along the y-axis.



Two non-vertical lines are **parallel**
if and only if
they have **the same** slope.

Example of parallel lines



Parallel or not?

Example 1. Are the lines $3x - 2y = 1$ and $-6x + 4y = 5$ parallel?

Solution. To answer the question, we have to determine the **slopes** of the lines. For this, we rewrite the equations in the **slope-intercept** form $y = mx + b$.

$$3x - 2y = 1 \iff 2y = 3x - 1 \iff y = \frac{3}{2}x - \frac{1}{2}$$

$$-6x + 4y = 5 \iff 4y = 6x + 5 \iff y = \frac{6}{4}x + \frac{5}{4} \iff y = \frac{3}{2}x + \frac{5}{4}.$$

Since the lines have the same slope of $\frac{3}{2}$, they are **parallel**.

Example 2. Are the lines $y = 2$ and $y = 2x$ parallel?

Solution. The slope of the line $y = 2$ is 0, since $y = 2 \iff y = 0 \cdot x + 2$.
The slope of line $y = 2x$ is 2. Since the lines have different slopes, they are **not** parallel.
Remark. $y = 2$ is a **horizontal** line, while $y = 2x$ is not. So the lines are not parallel.

Slope of a line through two given points

Theorem. A line passing though two points (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ has the slope $\dfrac{y_2 - y_1}{x_2 - x_1}$.

Proof. Let $y = mx + b$ be an equation of the line. We have to prove that the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
Since the points (x_1, y_1) and (x_2, y_2) are on the line $y = mx + b$,
their coordinates satisfy the equation $y = mx + b$:

$$y_1 = mx_1 + b \text{ and } y_2 = mx_2 + b.$$

Subtracting the first equality from the second one, we get

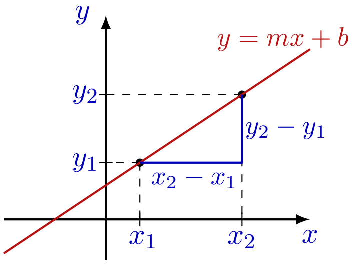
$y_2 - y_1 = (mx_2 + b) - (mx_1 + b) \iff y_2 - y_1 = m(x_2 - x_1) \iff m = \dfrac{y_2 - y_1}{x_2 - x_1}$. as required.

Notice that $x_2 - x_1 \neq 0$ since $x_1 \neq x_2$.

Slope as a ratio

Let us give a **geometric** interpretation of this result:

A line $y = mx + b$ passing through the points (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ has the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



When we move along the line from a point (x_1, y_1) to another point (x_2, y_2), the difference $x_2 - x_1$ shows the change in x-coordinate, and the difference $y_2 - y_1$ shows the change in y-coordinate.

The slope is the ratio of the change: $\text{slope} = \dfrac{\text{change in } y}{\text{change in } x}$

Examples

Example 1. Find the equation of the line passing through the points $(1, -1)$ and $(-3, 7)$.
Solution. Let $y = mx + b$ be the equation of the line.
We have to determine the coefficients m and b.

The slope m of the line passing through the points $(x_1, y_1) = (1, -1)$ and $(x_2, y_2) = (-3, 7)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{-3 - 1} = \frac{8}{-4} = -2.$$

Our line has the equation $y = -2x + b$.
To determine b, we plug in any of two given points into this equation.
Plugging in $(x_1, y_1) = (1, -1)$, we get

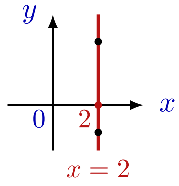
$$\underbrace{-1}_{y_1} = \underbrace{-2}_{m} \cdot \underbrace{1}_{x_1} + b \iff -1 = -2 + b \iff b = 1.$$

Therefore, the line has equation $\boxed{y = -2x + 1}$

Examples

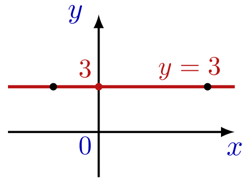
Example 2. Find the equation of the line passing through the points $(2,-1)$ and $(2,3)$.

Solution. The given points have the **same** x-coordinate.
Therefore, they belong to a **vertical** line.
The equation of the line is $x=2$.



Example 3. Find the equation of the line passing through the points $(-1,3)$ and $(4,3)$.

Solution. The given points have the **same** y-coordinate.
Therefore, they belong to a **horizontal** line.
The equation of the line is $y=3$.



Point-slope equation

Theorem. A line that has a slope of m and passes through the point (x_1,y_1)
has the equation $\boxed{y-y_1=m(x-x_1)}$

Proof. Let us show that the equation above describes
a line that has a slope of m and passes through (x_1,y_1).
Rewrite the equation in a slope-intercept form:

$$y-y_1=m(x-x_1) \iff y=\underbrace{m}_{slope}x+(-mx_1+y_1).$$

The coefficient in front of x is the slope m.
Moreover, the point (x_1,y_1) satisfies the equation $y-y_1=m(x-x_1)$:
$y_1-y_1=m(x_1-x_1) \iff 0=0$, so it belongs to the line.

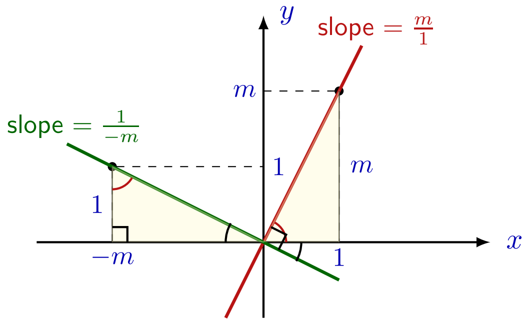
Example. Find a slope-intercept equation of a line that has a slope of 3 and passes through the point $(-1,2)$.

Solution. Using the point-slope equation $y-y_1=m(x-x_1)$, we get

$$y-2=3(x-(-1)) \iff y-2=3(x+1) \iff \boxed{y=3x+5}$$

Perpendicular lines

Theorem. Two non-vertical lines are perpendicular if the product of their slopes is -1.
Proof.



The triangles are congruent.
The lines are perpendicular.

Example. Prove that the lines $x - 2y = 1$ and $6x + 3y = 2$ are perpendicular.

Solution. $x - 2y = 1 \iff 2y = x - 1 \iff y = \frac{1}{2}x - 1/2$

$6x + 3y = 2 \iff 3y = -6x + 2 \iff y = -2x + 2/3$

The slopes $1/2$ and -2 are negative reciprocals of each other.

Therefore, the lines are perpendicular.

Summary

In this lecture, we have learned

- ☑ the **slope-intercept equation** of a line $y = mx + b$
- ☑ what the **slope** of a line represents
- ☑ that **parallel lines** have the same slope
- ☑ how to find equation of a line passing through two points
- ☑ what the **point-slope equation** of a line is $y - y_1 = m(x - x_1)$
- ☑ that **perpendicular lines** have negative reciprocals slopes

Lecture 21

Linear Systems. Part 1

What is a linear system?

We will study systems consisting of two linear equations in two unknowns,

like this: $\begin{cases} -2x+3y=-8 \\ 5x+2y=1 \end{cases}$ $\quad x,\ y$ are called unknowns.

To **solve a system** means to find all values of x and y which satisfy **both** equations.

The brace $\Big\{$ means that **both** equations should be satisfied by the same values of x and y.

The values $x=1$ and $y=-2$ satisfy $\begin{cases} -2x+3y=-8 \\ 5x+2y=1, \end{cases}$

because $\begin{cases} -2\cdot 1+3(-2)=-2+(-6)=-8 \\ 5\cdot 1+2(-2)= \quad 5+(-4)=1. \end{cases}$ Therefore,

$\begin{cases} x=1 \\ y=-2 \end{cases}$ (or just the pair $(1,-2)$) is a **solution** of $\begin{cases} -2x+3y=-8 \\ 5x+2y=1. \end{cases}$

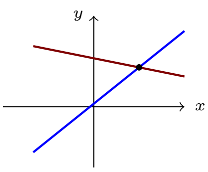
Are there **other solutions?** To solve a system means to find **all** its solutions!

How many solutions may a system have?

The graph of each equation of the system is a **line**.

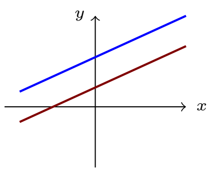
A solution of the system is a point which belongs to **both** lines.

How can two lines on a plane be **positioned** with respect to each other?



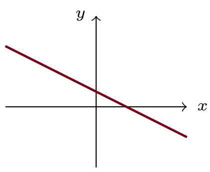
lines intersect at one point

system has **one** solution



lines are parallel

system has **no** solution



lines coincide

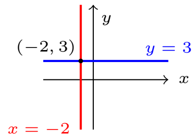
system has **infinitely many** solutions

How to solve a system?

Some systems are easy.

$$\begin{cases} x = -2 \\ y = 3 \end{cases}$$

is a linear system,
but it looks like a solution,
and it is a **solution** for itself.



To solve a more complicated system, we propose to turn it into an easy one by a sequence of elementary **transformations**.

The transformations must preserve the set of all solutions.

If two systems have the same solutions, we call them equivalent .
and write $\iff$ between the systems,

like this:

$$\begin{cases} x + 3 = 1 \\ 2y = 6 \end{cases} \iff \begin{cases} x = -2 \\ y = 3 \end{cases}$$

Elementary transformations

There are **three** elementary transformations.

1. Adding equations, that is replacing one equation by its sum with the other equation.

$$\begin{cases} -x + 2y = 3 \\ x - y = 0 \end{cases} \begin{matrix} \xrightarrow{\text{sum up}} \\ \xrightarrow{\text{keep}} \end{matrix} \iff \begin{cases} -x + 2y + (x - y) = 3 + 0 \\ x - y \ = \ 0 \end{cases}$$

$$\iff \begin{cases} -x + 2y + (x - y) = 3 + 0 \\ x - y \ = \ 0 \end{cases} \iff \begin{cases} y = 3 \\ x - y = 0 \end{cases}$$

Adding the first equation to the second one completes the solution:

$$\begin{cases} y = 3 \\ x - y = 0 \end{cases} \iff \begin{cases} y = \ 3 \\ x - y + y = 0 + 3 \end{cases} \iff \begin{cases} y = 3 \\ x = 3 \end{cases}$$

Elementary transformations

2. Subtracting equations.

$$\begin{cases} \quad y = -1 \\ x + y = 1 \end{cases} \begin{array}{c} \xrightarrow{\text{keep}} \\ \xrightarrow{\text{subtract}} \end{array} \iff \begin{cases} \qquad y &= \quad -1 \\ x + y - y &= 1 - (-1) \end{cases} \iff \begin{cases} y = -1 \\ x = 2 \end{cases}$$

3. Multiplying an equation by a non-zero number.

$$\begin{cases} -\dfrac{1}{2}x = 1 \\ \quad 3y = -5 \end{cases} \begin{array}{c} \xrightarrow{\text{multiply by } (-2)} \\ \xrightarrow{\text{divide by } 3} \end{array} \begin{cases} x = -2 \\ y = -\dfrac{5}{3} \end{cases}$$

Division by 3 is multiplication by $\dfrac{1}{3}$.

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Summary

In this lecture, we have learned

- ☑ what a **linear system** is
- ☑ what **solutions** of a linear system are
- ☑ what it means to **solve** a system
- ☑ **how many solutions** a linear system may have
- ☑ which systems are called **equivalent**
- ☑ what **elementary transformations** are

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Lecture 22

Linear Systems. Part 2

Preface

In Lecture 21, we learned

- what a linear system is
- what its solution is
- how many solutions a system may have
- how to solve a system by **elementary transformations:**
  adding/subtracting equations and
  multiplying an equation by a non-zero number.

We continue our journey through the theory shifting the attention to examples.

We will solve one by one specific systems,
gradually learning new practical tricks and fragments of theory.

Substitution

Example 1. Solve the system $\begin{cases} x - 3y = 1 \\ \quad\;\; y = 2. \end{cases}$

It's a nice system: the second equation says the unknown y is actually **known**!
Solution: Plug $y = 2$ into the first equation:

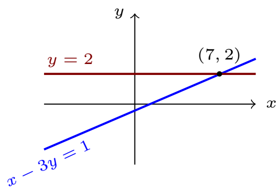
$$\begin{cases} x - 3y = 1 \\ \quad\;\; y = 2 \end{cases} \iff \begin{cases} x - 3(2) \quad = 1 \\ \qquad y \quad = 2 \end{cases} \iff \begin{cases} x = 1 + 6 \\ y = 2 \end{cases} \iff \begin{cases} x = 7 \\ y = 2 \end{cases}$$

This method is called substitution.

This system could be solved also by elementary transformations:

$$\begin{cases} x - 3y = 1 \\ \quad\;\; y = 2 \end{cases} \iff \begin{cases} x - 3y = 1 \\ \quad\; 3y = 6 \end{cases} \iff \begin{cases} x \quad\; = 7 \\ \; 3y = 6 \end{cases} \iff \begin{cases} x = 7 \\ y = 2 \end{cases}$$

Geometric interpretation:



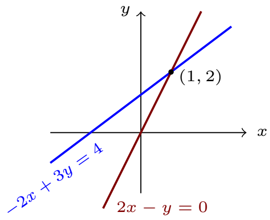
Elimination by addition

Example 2. Solve the system $\begin{cases} -2x + 3y = 4 \\ 2x - y = 0 \end{cases}$

Solution.
The coefficients for x are -2 and 2, so adding the equations will **eliminate** x:

$$\begin{cases} -2x + 3y = 4 \\ 2x - y = 0 \end{cases} \iff \begin{cases} 2y = 4 \\ 2x - y = 0 \end{cases} \iff \begin{cases} y = 2 \\ 2x - y = 0 \end{cases} \iff \begin{cases} y = 2 \\ 2x = 2 \end{cases}$$

$$\iff \begin{cases} y = 2 \\ x = 1 \end{cases} \iff (x, y) = (1, 2)$$



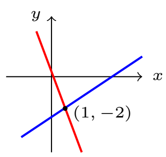
All methods together

Example 3. Solve the system $\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1 \end{cases}$

Solution.
Let us **eliminate** one of the unknowns, say x:

$$\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1 \end{cases} \begin{matrix} \xrightarrow{\text{multiply by } 5} \\ \xrightarrow{\text{multiply by } 2} \end{matrix} \begin{cases} -10x + 15y = -40 \\ 10x + 4y = 2 \end{cases} \iff \begin{cases} 19y = -38 \\ 10x + 4y = 2 \end{cases}$$

$$\iff \begin{cases} y = -2 \\ 5x + 2y = 1 \end{cases} \iff \begin{cases} y = -2 \\ 5x + 2(-2) = 1 \end{cases} \iff (x, y) = (1, -2)$$



How to check a solution?

It is easy to **check** if a solution of a linear system is correct.

Let us check if $(x, y) = (1, -2)$ is indeed a correct solution of the system

$$\begin{cases} -2x + 3y = -8 \\ 5x + 2y = 1 \end{cases}$$

Plug in $x = 1$, $y = -2$ into the system:

$$\begin{cases} -2(1) + 3(-2) \overset{?}{=} -8 \\ 5(1) + 2(-2) \overset{?}{=} 1 \end{cases} \iff \begin{cases} -8 \overset{\checkmark}{=} -8 \\ 1 \overset{\checkmark}{=} 1 \end{cases}$$

Systems with no solutions

Solve the system $\begin{cases} x + 2y = -1 \\ -2x - 4y = 3. \end{cases}$

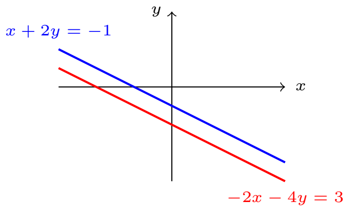
Solution.

$$\begin{cases} x + 2y = -1 \\ -2x - 4y = 3 \end{cases} \iff \begin{cases} 2x + 4y = -2 \\ -2x - 4y = 3 \end{cases} \iff \begin{cases} 2x + 4y = -2 \\ 0 = 1 \end{cases}$$

The statement $0 = 1$ is **false**.

It is false **no matter** what values x and y take.

A system, which includes an equation $0 = 1$, has **no** solution.



Systems with infinitely many solutions

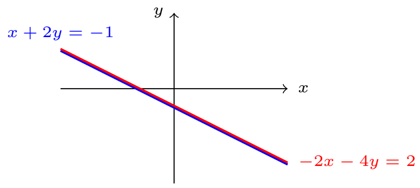
Solve the system $\begin{cases} x + 2y = -1 \\ -2x - 4y = 2. \end{cases}$

Solution.

$$\begin{cases} x + 2y = -1 \\ -2x - 4y = 2 \end{cases} \iff \begin{cases} 2x + 4y = -2 \\ -2x - 4y = 2 \end{cases} \iff \begin{cases} 2x + 4y = -2 \\ 0 = 0 \end{cases}$$

The statement $0 = 0$ is **true**. It is true, **no matter** what values x and y take. Removing the equation $0 = 0$ from a system does not change the set of solutions. Our system is equivalent to a single equation:

$$2x + 4y = -2 \iff x + 2y = -1 \iff x = -1 - 2y$$



Answer: $(x, y) = (-1 - 2y, y)$, where y is an arbitrary number.

Summary

In this lecture, we have learned

- ☑ how to solve a system by a **substitution**
- ☑ how to **eliminate** an unknown
- ☑ how to **check** a solution
- ☑ how to handle systems with **no solutions**
- ☑ how to handle systems with **infinitely many solutions**

Lecture 23

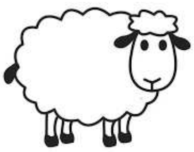
Linear Systems. Part 3

Applications of linear systems

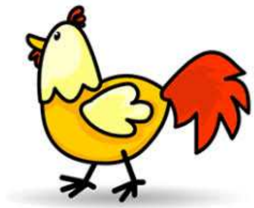
In this lecture, we will learn how to solve **word problems** using systems of linear equations.

On a farm

Problem. On a farm, there are sheep and chicken.



All together,
they have 44 feet and 17 heads.
How many sheep and
how many chicken
are on the farm?



Solution. Let x be the number of sheep, and y be the number of chicken.

How many feet do all sheep have? $4x$

How many feet do all chicken have? $2y$

How many feet do sheep and chicken have all together? $4x + 2y$

How many heads do they have all together? $x + y$

What is given in the problem?

- all together they have 44 feet, so $4x + 2y = 44$.
- all together they have 17 heads, so $x + y = 17$.

Solve a system

How to find x, the number of sheep, and y, the number of chicken?

Solve the system

$$\begin{cases} 4x + 2y = 44 \\ x + \ \ y = 17 \end{cases} \iff \begin{cases} 2x + y = 22 \\ x + y = 17 \end{cases} \iff \begin{cases} x \quad\ \ = 5 \\ x + y = 17 \end{cases} \iff \begin{cases} x = 5 \\ y = 12 \end{cases}$$

Therefore, the number of sheep is 5, the number of chicken is 12.

Let us check if our answer is correct.

How many feet do 5 sheep and 12 chicken have?

$$4 \cdot 5 + 2 \cdot 12 = 20 + 24 = 44 \ \checkmark$$

How many heads do 5 sheep and 12 chicken have?

$$5 + 12 = 17 \ \checkmark$$

The problem is solved correctly!

Answer. There are 5 sheep and 12 chicken on the farm.

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In a movie theater

Problem. A family of two adults and five children pays $\$61$ for tickets in a movie theater.
A family of three adults and two children pays $\$53$.
Find a ticket price for an adult and a ticket price for a child.

Solution. Let $\$x$ be the price for an adult ticket, and $\$y$ be the price for a child ticket.

How much a family of **two** adults and **five** children will pay then? $\$(2x + 5y)$

How much a family of **three** adults and **two** children will pay? $\$(3x + 2y)$

What is given in the problem?

A family of **two** adults and **five** children pays $\$61$. So $2x + 5y = 61.$

A family of **three** adults and **two** children pays $\$53$. So $3x + 2y = 53.$

How to find x and y?

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Solve a system

$$\begin{cases} 2x+5y=61 \\ 3x+2y=53 \end{cases} \iff \begin{cases} 6x+15y=183 \\ 6x+\ \ 4y=106 \end{cases} \iff \begin{cases} \qquad 11y=77 \\ 3x+\ \ 2y=53 \end{cases} \iff$$

$$\begin{cases} \qquad y=7 \\ 3x+2\cdot 7=53 \end{cases} \iff \begin{cases} y=7 \\ 3x=53-14 \end{cases} \iff \begin{cases} y=7 \\ 3x\ \ =39 \end{cases} \iff \begin{cases} x=13 \\ y=7 \end{cases}$$

Therefore, the price for an adult ticket is $\$13$, and the price for a children ticket is $\$7$.

Let us check if our answer is correct.

How much a family of **two** adults and **five** children will pay, in dollars?

$$2\cdot 13+5\cdot 7=26+35=61 \ \checkmark$$

How much a family of **thee** adults and **two** children will pay, in dollars?

$$3\cdot 13+2\cdot 7=39+14=53 \ \checkmark$$

Answer. The ticket price for an adult is $\$13$, the ticket price for a child is $\$7$.

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In a winery

Problem. A winemaker has in his cellar
1620 liters of wine
aging in **three** small and **five** large barrels.
Find the volumes of the barrels
if a large barrel
contains 20 liters more than a small one.

Solution. Let x be the volume (in liters) of a small barrel,
and y be the volume (in liters) of a large barrel.

What is the **total** volume of **three** small and **five** large barrels?
$3x+5y$ (liters)

What is the difference in volumes between a large and a small barrel? $y-x$

What is given in the problem?

- total volume: $3x+5y=1620$
- the difference in volumes: $y-x=20$

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Solve a system

$$\begin{cases} 3x+5y=1620 \\ -x+\ \ y=20 \end{cases} \iff \begin{cases} 3x+5y=1620 \\ y=x+20 \end{cases} \iff \begin{cases} 3x+5(x+20)=1620 \\ y=x+20 \end{cases}$$

$$\begin{cases} 3x+5x+100=1620 \\ y=x+20 \end{cases} \iff \begin{cases} 8x=1520 \\ y=x+20 \end{cases} \iff \begin{cases} x=190 \\ y=190+20 \end{cases} \iff$$

$$\begin{cases} x=190 \\ y=210 \end{cases}$$

Therefore, a small barrel contains 190 liters,

and a large one contains 210 liters.

Let us check if our answer is correct.
What is the total volume of **three** small barrels and **five** large ones?

$3\cdot 190+5\cdot 210=570+1050=1620$ ✓

How many liters more does a large barrel contain than a small one?

$210-190=20$ ✓

Answer. 190 and 210 liters.

Summary

In this lecture, we have learned

- ☑ how to solve word problems using linear systems
- ☑ how to check if the answer is correct

Lecture 24

Radicals

Squares and square roots

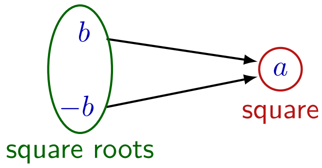
A number and its opposite have the same square:

for example, $3^2 = 9$ and $(-3)^2 = 9$.

Number 9 is called the **square** of 3 (or -3).

Numbers 3 and -3 are called the **square roots** of 9.

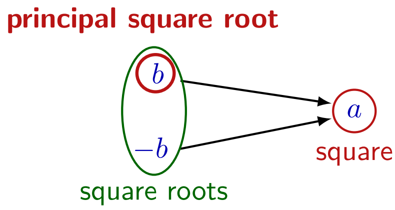
Let a be a non-negative number. A **square root of** a is a number b such that $b^2 = a$.
If a is positive, then there are two numbers, b and $-b$, whose square is a:



If $a = 0$, then there is only one number, 0, whose square is 0: $0 = 0^2$.

Definition of radical

Let a be a non-negative number.
The **principal square root** of a is a non-negative number b such that $b^2 = a$.



Notation for the principal square root: $\sqrt{a} = b$

The symbol $\sqrt{}$ is called a **radical sign**.

The formula $\sqrt{a} = b$ reads "the square root of a is equal to b".

By definition, $\sqrt{a} = b \iff b^2 = a$ for non-negative a and b.

Radicals and perfect squares

Examples. $\sqrt{0} = 0$ since $0^2 = 0$,

$\sqrt{1} = 1$ since $1^2 = 1$,

$\sqrt{4} = 2$ since $2^2 = 4$,

$\sqrt{9} = 3$ since $3^2 = 9$,

$\sqrt{16} = 4$ since $4^2 = 16$.

A number a is called a **perfect square** if $\sqrt{a}$ is an integer.

Here are some perfect squares: $0, \ 1, \ 4, \ 9, \ 16, \ 25, \ 36, \ 49, \ 64, \ 81, \ 100.$

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Precautions

• When we work with **real numbers**, the number under the radical sign should be **non-negative**: $\sqrt{a}$ is defined only for $a \geq 0$.

For example, $\sqrt{-9}$ is **not** defined.

• A square root is always **non-negative**: $\sqrt{a} \geq 0$.

For example, it is **incorrect** to write $\sqrt{9} = -3$, since $\sqrt{9}$, by definition, should be non-negative.

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Taking principal square root is opposite to squaring

$$\begin{tikzcd} 3 \arrow[r, bend left, "\text{squaring}"] & 9 \arrow[l, bend left, "\text{taking principal root}"] \end{tikzcd}$$

It means that $\sqrt{3^2} = 3$ and $(\sqrt{9})^2 = 9.$

For any **non-negative** a, $\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$.

Example. Find the value of the following expressions:

$$\sqrt{5^2}, \ \sqrt{(-5)^2}, \ \sqrt{-5^2}, \ (\sqrt{5})^2, \ (-\sqrt{5})^2, \ (\sqrt{-5})^2.$$

Solution. $\sqrt{5^2} = 5,$ $\sqrt{(-5)^2} = \sqrt{5^2} = 5,$ $\sqrt{-5^2} = \sqrt{-25}$ is undefined

$(\sqrt{5})^2 = 5,$ $(-\sqrt{5})^2 = (\sqrt{5})^2 = 5,$ $(\sqrt{-5})^2$ is undefined

Properties of radicals

Let a, b be non-negative numbers. Then $\sqrt{a}\sqrt{b} = \sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.

Indeed, $(\sqrt{a}\sqrt{b})^2 = (\sqrt{a})^2(\sqrt{b})^2 = ab$. Therefore, $\sqrt{a}\sqrt{b} = \sqrt{ab}$.

$\left(\dfrac{\sqrt{a}}{\sqrt{b}}\right)^2 = \dfrac{(\sqrt{a})^2}{(\sqrt{b})^2} = \dfrac{a}{b}$. Therefore, $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.

Example. Simplify the following expressions: $\sqrt{3}\,\sqrt{12}$, $\sqrt{75}$, $\dfrac{\sqrt{27}}{\sqrt{12}}$.

Solution. $\sqrt{3}\,\sqrt{12} = \sqrt{3}\,\sqrt{3 \cdot 4} = \sqrt{3}\,\sqrt{3}\,\sqrt{4} = (\sqrt{3})^2\sqrt{2^2} = 3 \cdot 2 = 6$.

Another way to calculate: $\sqrt{3}\,\sqrt{12} = \sqrt{3 \cdot 12} = \sqrt{36} = \sqrt{6^2} = 6$.

$\sqrt{75} = \sqrt{3 \cdot 25} = \sqrt{3 \cdot 5^2} = \sqrt{3}\sqrt{5^2} = \sqrt{3} \cdot 5 = 5\sqrt{3}.$

$\dfrac{\sqrt{27}}{\sqrt{12}} = \dfrac{\sqrt{3 \cdot 9}}{\sqrt{3 \cdot 4}} = \dfrac{\sqrt{3} \cdot \sqrt{9}}{\sqrt{3} \cdot \sqrt{4}} = \dfrac{\sqrt{3^2}}{\sqrt{2^2}} = \dfrac{3}{2}$.

What is $\sqrt{x^2}$?

We know that x^2 is non-negative for any value of x. So $\sqrt{x^2}$ is defined.

Is it true that $\sqrt{x^2} = x$ for **all** x? No!

For **non-negative** x, $\sqrt{x^2} = x$ by definition of the radical.

For **negative** x, $\sqrt{x^2} = -x$, since $-x > 0$ and $(-x)^2 = x^2$.

Therefore, $\sqrt{x^2} = |x|$. Reminder: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Example 1. $\sqrt{(-5)^2} = |-5| = 5$.

Example 2. Simplify the following expressions: $\sqrt{x^4}$, $\sqrt{x^6}$.

Solution. $\sqrt{x^4} = \sqrt{(x^2)^2} = |x^2| = x^2$

$\sqrt{x^6} = \sqrt{(x^3)^2} = |x^3| = |x^2 \cdot x| = |x^2| \cdot |x| = x^2 \cdot |x|$

Why $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$?

It is **not** true that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ for arbitrary x, y.

Indeed, if $x = 9$ and $y = 16$, then

$\sqrt{x+y}\Big|_{x=9,\, y=16} = \sqrt{9+16} = \sqrt{25} = 5$, while

$(\sqrt{x} + \sqrt{y})\Big|_{x=9,\, y=16} = \sqrt{9} + \sqrt{16} = 3 + 4 = 7$ and $5 \neq 7$.

Are there any x, y for which $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$? Yes!

For example, $x = y = 0$: $\sqrt{0+0} = \sqrt{0} + \sqrt{0}$

or $x = 1$ and $y = 0$: $\sqrt{1+0} = \sqrt{1} + \sqrt{0}$.

Actually, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ only if at least one of x, y is zero.

Simplest radical form

An expression involving radicals can be written in many different forms. For example,

$$\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

It is a custom to write radical expressions in a special form, which is called **simplest radical form**.

In simplest radical form, the expression

- doesn't contain perfect square factors:

  $\sqrt{12}$ is not in the simplest form, but $2\sqrt{3}$ is. ($\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$)

- doesn't contain fractions under the radical:

  $\sqrt{\frac{3}{4}}$ is not in the simplest form, but $\frac{\sqrt{3}}{2}$ is. $\left(\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}\right)$

- doesn't contain radicals in denominators:

  $\frac{1}{\sqrt{2}}$ is not in the simplest form, but $\frac{\sqrt{2}}{2}$ is. $\left(\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}\right)$

Simplest radical form

Example. Bring the following expressions in simplest radical form:

$$\frac{1}{\sqrt{3}},\quad \sqrt{\frac{2}{5}},\quad \frac{1}{3-\sqrt{2}}$$

Solution. $\frac{1}{\sqrt{3}} = \frac{1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{3}}{(\sqrt{3})^2} = \frac{\sqrt{3}}{3}$

$$\sqrt{\frac{2}{5}} = \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{\sqrt{10}}{(\sqrt{5})^2} = \frac{\sqrt{10}}{5}$$

$$\frac{1}{3-\sqrt{2}} = \frac{1\cdot(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3+\sqrt{2}}{3^2-(\sqrt{2})^2} = \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$$

Remember: $(a-b)(a+b) = a^2 - b^2$, so

$$(3-\sqrt{2})(3+\sqrt{2}) = 3^2 - (\sqrt{2})^2$$

Operating with radical expressions

Example 1. Simplify the expression: $\sqrt{6}(\sqrt{18}-\sqrt{24})$

Solution. $\sqrt{6}(\sqrt{18}-\sqrt{24}) = \sqrt{6}\sqrt{18}-\sqrt{6}\sqrt{24} = \sqrt{6\cdot 18}-\sqrt{6\cdot 24} =$

$\sqrt{6\cdot 6\cdot 3}-\sqrt{6\cdot 6\cdot 4} = \sqrt{6^2\cdot 3}-\sqrt{6^2\cdot 2^2} = \sqrt{6^2}\sqrt{3}-\sqrt{6^2}\sqrt{2^2} =$

$6\sqrt{3}-6\cdot 2 = 6\sqrt{3}-12.$

Example 2. Bring the expression in simplest radical form: $\dfrac{\sqrt{6}-3}{\sqrt{3}-\sqrt{2}}$.

Solution.

$$\frac{\sqrt{6}-3}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3\cdot 2}-(\sqrt{3})^2}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}\sqrt{2}-(\sqrt{3})^2}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}(\sqrt{2}-\sqrt{3})}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\sqrt{3}(-1)(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}(-1)(\cancel{\sqrt{3}-\sqrt{2}})}{\cancel{\sqrt{3}-\sqrt{2}}} = -\sqrt{3}.$$

Summary

In this lecture, we have learned

- ☑ what the **square roots** of a non-negative number are
- ☑ what the **principal square root** is
- ☑ what the perfect squares are
- ☑ the defining identities for radical: $\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$
- ☑ the properties of radicals: $\sqrt{a}\sqrt{b} = \sqrt{ab}$, $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$
- ☑ $\sqrt{x^2} = |x|$ that for all x
- ☑ $\sqrt{x+y} \neq \sqrt{x}+\sqrt{y}$ for arbitrary x, y
- ☑ what the simplest radical form is
- ☑ how to operate with radical expressions

Lecture 25

Radicals as Powers with Rational Exponents

Roots

Let a be a real number. The **n-th root** of a is a number b such that $b^n = a$.

If $n = 2$ then the n-th root is the square root which we studied in the preceding lecture.

Examples. The **2nd** root of 49 is 7, since $7^2 = 49$

The **4th** root of 81 is 3, since $3^4 = 81$.

The **5th** root of -32 is -2, since $(-2)^5 = -32$.

The **4th** root of -81 does **not** exist,

since there is no real number which **4th** power is negative.

Cube root

The **3rd** root has a special name: it is called a **cube root**.

Notation for the cube root: $\sqrt[3]{\ }$. By definition, $b = \sqrt[3]{a} \iff b^3 = a$.

For any number a, there exists a unique cube root of a,

since the equation $x^3 = a$ has a unique solution.

Examples. $\sqrt[3]{1} = 1$ since $1^3 = 1$,

$\sqrt[3]{8} = 2$ since $2^3 = 8$,

$\sqrt[3]{27} = 3$ since $3^3 = 27$,

$\sqrt[3]{64} = 4$ since $4^3 = 64$,

$\sqrt[3]{0} = 0$ since $0^3 = 0$,

$\sqrt[3]{-1} = -1$ since $(-1)^3 = -1$,

$\sqrt[3]{-8} = -2$ since $(-2)^3 = -8$.

Odd-order roots

Let n be a positive **odd** integer, and a be a real number.

Then the equation $x^n = a$ has a unique solution. So there exists a unique n-th root of a.

Notation for the n-th root: $\sqrt[n]{\ }$. By definition, $b = \sqrt[n]{a} \iff b^n = a$.

The number n is called the **index** of the n-th root.

Examples. $\sqrt[5]{1} = 1$ since $1^5 = 1$,

$\sqrt[9]{-1} = -1$ since $(-1)^9 = -1$,

$\sqrt[3]{-125} = -5$ since $(-5)^3 = -125$,

$\sqrt[5]{243} = 3$ since $3^5 = 243$,

$\sqrt[7]{128} = 2$ since $2^7 = 128$,

$\sqrt[7]{-128} = -2$ since $(-2)^7 = -128$.

Even-order roots

Let n be a positive **even** integer, and a be a **non-negative** real number.

Then the equation $x^n = a$ has **two** solutions, which differ by their signs.

So there exist two n-th roots of a.

The positive root is called the **principal** n-th root and denoted by $\sqrt[n]{\ }$.

By definition, $b = \sqrt[n]{a} \iff b^n = a$.
The number n is called the **index** of the n-th root.

It's a custom to omit the index of 2: the second root $\sqrt[2]{a}$ is written as $\sqrt{a}$.

Examples. $\sqrt[4]{1} = 1$ since $1^4 = 1$,

$\sqrt[4]{16} = 2$ since $2^4 = 16$,

$\sqrt[4]{-16}$ is undefined since 4 is even and $-16 < 0$,

$\sqrt[6]{64} = 2$ since $2^6 = 64$,

$\sqrt[4]{81} = 3$ since $3^4 = 81$,

$\sqrt[6]{-81}$ is undefined since 6 is even and $-81 < 0$.

Precautions

Dealing with n-th roots, we have to distinguish two cases: when n is odd and when n is even.

- For **odd** n, $\sqrt[n]{a}$ is defined for **all** a.
  In this case, $\sqrt[n]{a}$ may be positive, negative, or zero (depending on a).
- For **even** n, $\sqrt[n]{a}$ is defined only for **non-negative** a. In this case, $\sqrt[n]{a} \geq 0$.

Operations of taking the n-th power and n-th root are **inverse** to each other:

$$\begin{tikzcd} \sqrt[n]{a} = b \arrow[rr, bend left, "n\text{-th power}"] & & a = b^n \arrow[ll, bend left, "n\text{-th root}"] \end{tikzcd}$$

For even n, we have to restrict ourselves to non-negative a and b.
Then $\sqrt[n]{b^n} = b$ and $(\sqrt[n]{b})^n = b$.

Examples

Example 1. Find the value of the following expressions:

$$\sqrt[3]{5^3},\ \sqrt[3]{(-5)^3},\ \sqrt[3]{-5^3},\ (\sqrt[3]{5})^3,\ (-\sqrt[3]{5})^3,\ (\sqrt[3]{-5})^3.$$

Solution. $\sqrt[3]{5^3} = 5, \quad \sqrt[3]{(-5)^3} = -5, \quad \sqrt[3]{-5^3} = \sqrt[3]{-125} = -5,$

$(\sqrt[3]{5})^3 = 5, \quad (-\sqrt[3]{5})^3 = -(\sqrt[3]{5})^3 = -5, \quad (\sqrt[3]{-5})^3 = -5.$

Example 2. Find the value of the following expressions:

$$\sqrt[4]{5^4},\ \sqrt[4]{(-5)^4},\ \sqrt[4]{-5^4},\ (\sqrt[4]{5})^4,\ (-\sqrt[4]{5})^4,\ (\sqrt[4]{-5})^4.$$

Solution. Caution! 4 is even and $\sqrt[4]{}$ may be not defined.

$\sqrt[4]{5^4} = 5, \quad \sqrt[4]{(-5)^4} = \sqrt[4]{5^4} = 5, \quad \sqrt[4]{-5^4} = \sqrt[4]{-625}$ is undefined.

$(\sqrt[4]{5})^4 = 5, \quad (-\sqrt[4]{5})^4 = (\sqrt[4]{5})^4 = 5, \quad (\sqrt[4]{-5})^4$ is undefined.

Properties of n-th roots

Let a, b be numbers for which n-th roots are defined. Then $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$ and $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$.

Indeed, $(\sqrt[n]{a}\,\sqrt[n]{b})^n = (\sqrt[n]{a})^n(\sqrt[n]{b})^n = ab$. Therefore, $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$.

$\left(\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \dfrac{(\sqrt[n]{a})^n}{(\sqrt[n]{b})^n} = \dfrac{a}{b}$. Therefore, $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$.

Radicals as powers with rational exponents

Reminder:

If n is a **positive** integer, then $x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{n\text{ times}}$, and $x^{-n} = \dfrac{1}{x^n}$.

If $n = 0$, then $x^0 = 1$.

What is $x^{\frac{1}{n}}$? Calculate the n-th power of $x^{\frac{1}{n}}$:

$\left(x^{\frac{1}{n}}\right)^n = \underbrace{x^{\frac{1}{n}} \cdot x^{\frac{1}{n}} \cdot \cdots \cdot x^{\frac{1}{n}}}_{n\text{ times}} = x^{\frac{1}{n}+\frac{1}{n}+\cdots+\frac{1}{n}} = x^{n\cdot\frac{1}{n}} = x^1 = x.$

This means that n-th power of $x^{\frac{1}{n}}$ is x, therefore, $x^{\frac{1}{n}} = \sqrt[n]{x}$.

For positive integers m and n, define a power with **fractional** exponent as follows:

$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$.

One can prove that all power rules are valid for fractional exponents.

Operating with fractional exponents

Example. Simplify the following expressions:

$25^{\frac{3}{2}}$, $27^{-\frac{5}{3}}$, $(64)^{\frac{2}{3}}$, $(-64)^{\frac{2}{3}}$, $(64)^{\frac{3}{2}}$, $(-64)^{\frac{3}{2}}$.

Solution.

$25^{\frac{3}{2}} = 25^{\frac{1}{2}\cdot 3} = (25^{\frac{1}{2}})^3 = (\sqrt{25})^3 = 5^3 = 125$

$27^{-\frac{5}{3}} = \dfrac{1}{27^{\frac{5}{3}}} = \dfrac{1}{(\sqrt[3]{27})^5} = \dfrac{1}{3^5} = \dfrac{1}{243}$

$(64)^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

$(-64)^{\frac{2}{3}} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$

$(64)^{\frac{3}{2}} = (\sqrt{64})^3 = 8^3 = 512$

$(-64)^{\frac{3}{2}} = (\sqrt{-64})^3$ is undefined since $-64 < 0$.

Summary

In this lecture, we have learned

- ☑ what the n-th root is
- ☑ what $\sqrt[n]{a}$ is
- ☑ the difference between cases when n is **odd** and **even**
- ☑ defining identities for n-th root: $(\sqrt[n]{x})^n = x$, $\sqrt[n]{x^n} = x$ for $x \geq 0$
- ☑ properties of n-th root
- ☑ that radicals may be written as **powers** with rational exponents:
  $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
- ☑ how to operate with rational exponents

Lecture 26

Quadratic Equations

Quadratic polynomials

A **quadratic polynomial** is a polynomial of degree two.

It can be written in the standard form $ax^2 + bx + c$,
where x is a variable, a, b, c are constants (numbers) and $a \neq 0$.

The constants a, b, c are called the **coefficients** of the polynomial.

Example 1 (quadratic polynomials).

$-3x^2 + x - \frac{4}{5}$ ($a = -3,\ b = 1,\ c = -\frac{4}{5}$)

x^2 ($a = 1,\ b = c = 0$)

$\frac{x^2}{7} - 5x + \sqrt{2}$ ($a = \frac{1}{7},\ b = -5,\ c = \sqrt{2}$)

$4x(x+1) - x$ (this is a quadratic polynomial which is not written in the standard form.
Its standard form is $4x^2 + 3x$, where $a = 4, b = 3, c = 0$)

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Quadratic polynomials

Example 2 (polynomials, but not quadratic)
$x^3 - 2x + 1$ (this is a polynomial of degree 3, not 2)
$3x - 2$ (this is a polynomial of degree 1, not 2)
Example 3 (not polynomials)
$x^2 + x^{\frac{1}{2}} + 1$, $x - \frac{1}{x}$ are not polynomials

A quadratic polynomial $ax^2 + bx + c$ is called sometimes a **quadratic trinomial**.
A trinomial consists of three terms.

Quadratic polynomials of type $ax^2 + bx$ or $ax^2 + c$
are called **quadratic binomials**. A binomial consists of two terms.

Quadratic polynomials of type ax^2 are called **quadratic monomials**.
A monomial consists of one term.

Quadratic polynomials (together with polynomials of degree 1 and 0) are the **simplest** polynomials.

Due to their simplicity, they are among the most important algebraic objects.

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Quadratic equations and their roots

A **quadratic equation** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where x is an unknown, $a,\ b,\ c$ are constants and $a \neq 0$.

Examples. $-x^2 + 3x + 5 = 0$ is quadratic equation in standard form with
$a = -1,\ b = 3,\ c = 5$.

$x + 1 = 2x(3 - 4x)$ is a quadratic equation, but not in standard form.
We obtain its standard form as follows:

$$x + 1 = 2x(3 - 4x) \iff x + 1 = 6x - 8x^2 \iff 8x^2 - 5x + 1 = 0.$$

To **solve** an equation means to find **all** values of the unknown
which turn the equation into a numerical identity.

The values of x that turn the equation $ax^2 + bx + c = 0$ into a numerical identity

are called the **roots** or **solutions** of the equation.

Also, they are called the **roots** of the polynomial $ax^2 + bx + c$.

How to solve a binomial quadratic equation

Example 1. Solve the equation $x^2 - 3 = 0$.

Solution. Alternative 1.

$$x^2 - 3 = 0 \iff x^2 = 3 \iff \sqrt{x^2} = \sqrt{3} \iff |x| = \sqrt{3}$$
$$\iff x = \sqrt{3} \text{ or } x = -\sqrt{3}.$$

One can shorten the answer: $x = \pm\sqrt{3}$.

Alternative 2. Let us write 3 as $(\sqrt{3})^2$ and use the **difference of squares** formula:

$$x^2 - 3 = 0 \iff x^2 - (\sqrt{3})^2 = 0 \iff (x - \sqrt{3})(x + \sqrt{3}) = 0.$$

The product of two terms, $(x - \sqrt{3})$ and $(x + \sqrt{3})$, equals 0
if and only if either one term equals 0, or the other term equals 0:

$$(x - \sqrt{3})(x + \sqrt{3}) = 0 \iff x - \sqrt{3} = 0 \text{ or } x + \sqrt{3} = 0$$
$$\iff x = \sqrt{3} \text{ or } x = -\sqrt{3}$$

Answer. $x = \pm\sqrt{3}$.

Solution in simplest radical form

Example 2. Solve the equation $3x^2 - 5 = 0$. Give the answer in simplest radical form.

Solution.

$$3x^2 - 5 = 0 \iff 3x^2 = 5 \iff x^2 = \frac{5}{3} \iff x = \pm\sqrt{\frac{5}{3}}.$$

To write the number $\sqrt{\frac{5}{3}}$ in the **simplest radical form**,

we have to get rid of the radical in the denominator:

$$\sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5}\,\sqrt{3}}{\sqrt{3}\,\sqrt{3}} = \frac{\sqrt{15}}{3}.$$

Therefore, the solution is $x = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$.

Answer. $x = \pm\frac{\sqrt{15}}{3}$.

Quadratic equations with no roots

Example 3. Solve the equation $x^2 + 4 = 0$.

Solution. $x^2 + 4 = 0 \iff x^2 = -4$.

We know that the square of any real numbers is **non-negative** (positive or zero).

Therefore, the equation has **no** real solutions.

Example 4. Solve the equation $x(2 - 3x) = (x + 1)^2$.

Solution. The equation is **not** in the standard form. Let us bring it to this form.

$$x(2 - 3x) = (x + 1)^2 \iff 2x - 3x^2 = x^2 + 2x + 1$$
$$\iff -4x^2 = 1 \iff x^2 = -\frac{1}{4}.$$

The square of a real number can't be negative, therefore, the equation has **no** real solutions.

Solving binomial equations by factoring

Example 5. Solve the equation $-3x^2+4x=0$.

Solution. By **factoring**, we get

$-3x^2+4x=0 \iff x(-3x+4)=0$.

The product of two unknown numbers, x and $-3x+4$ equals zero.

This may happen **if and only if** either one number equals 0, or the other number equals 0:

$x(-3x+4)=0 \iff x=0$ or $-3x+4=0 \iff x=0$ or $x=\dfrac{4}{3}$.

Answer. $x=0$ or $x=\dfrac{4}{3}$

Don't lose roots!

Example 6. Solve the equation $x(x-1)=x$.

Solution. Rewrite the equation to bring it to the standard form:

$x(x-1)=x \iff x^2-x=x \iff x^2-2x=0$.

Solve this binomial equation by factoring:

$x^2-2x=0 \iff x(x-2)=0 \iff x=0$ or $x=2$.

Warning. Let us have a look on an "alternative solution":

$\cancel{x}\,x\,(x-1)=\cancel{x}\,x \iff x-1=1 \iff x=2$.

We have got only one solution, the other solution, $x=0$, has been **lost**.
The reason for this is an **illegal** cancellation of x.
A cancellation of x is the division by x, which makes sense only if $x\neq 0$.
But $x=0$ is in fact a solution,
and cancellation of it leads to the loss of this solution.

⚠ Don't cancel anything unknown while solving an equation!

Summary

In this lecture, we have learned

- ☑ what a quadratic polynomial is
- ☑ what the standard form of a quadratic polynomial is $ax^2 + bx + c$
- ☑ why quadratic polynomials are important
- ☑ what a quadratic equation is
- ☑ what it means to solve an equation
- ☑ what the **roots** (or solutions) of a quadratic equation are
- ☑ how to solve a **binomial** quadratic equation

Lecture 27

Quadratic Formula

Goal: to solve any quadratic equation

In previous lecture, we learned how to solve some special quadratic equations, namely, **binomial** equations, that is, equations of types $ax^2 + c = 0$ or $ax^2 + bx = 0$.

In this lecture, we will learn how to solve a general quadratic equation $ax^2 + bx + c = 0$ for **arbitrary** coefficients $a \neq 0, b$ and c.

This will take some time and efforts,

but we'll get a formula which allows to solve **any** quadratic equation!

Quadratic formula

Theorem. Let $ax^2 + bx + c = 0$ be a quadratic equation

with **arbitrary** coefficients $a \neq 0, b$ and c.

Its solution is given by the **quadratic formula**

$$\boxed{x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\,, \quad \text{provided } b^2 - 4ac \geq 0}$$

If $b^2 - 4ac < 0$, then the equation has **no** solutions.

Remarks. We are going to **prove** and discuss the quadratic formula,

and master it by various numerical examples.

The deduction of the quadratic formula is the most **difficult** part of our course.

It's normal to go over this proof several times until complete understanding.

Important. Quadratic formula will be used throughout all your math studies.

It makes sense to **memorize** it.

Plan

Let $ax^2 + bx + c = 0$ be a quadratic equation,
where x is unknown, a, b, c are given numbers (coefficients) and $a \neq 0$.

We have to **solve** this equation, that is to find the unknown x in terms of the coefficients a, b, c.

For this, we perform a standard trick which turns any quadratic **tri**nomial into a quadratic **bi**nomial.

This trick is called **completing the square**.

Once the quadratic trinomial is converted to a quadratic binomial,
the equation becomes a **binomial equation**, which we know how to solve.

Completing the square

Let $ax^2 + bx + c$ be a quadratic trinomial.

The expression $ax^2 + bx$ may be considered as a "sprout" of a square, an **incomplete** square:

$$ax^2 + bx = a\left(x^2 + \frac{b}{a}x\right) = a\underbrace{\left(x^2 + 2 \cdot x \cdot \frac{b}{2a}\right)}_{\text{incomplete square}}$$

To complete this incomplete square,
we add (and then subtract to keep the balance) the missing term, namely, $\left(\frac{b}{2a}\right)^2$:

$$a\underbrace{\left(x^2 + 2 \cdot x \cdot \frac{b}{2a}\right)}_{\text{incomplete square}} = a\left(\underbrace{x^2 + 2 \cdot x \cdot \frac{b}{2a} + \left(\frac{b}{2a}\right)^2}_{\text{complete square}} - \left(\frac{b}{2a}\right)^2\right) = a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right)$$

Completing the square

We have got that

$$ax^2 + bx = a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a}.$$

The **tri**nomial may be rewritten as

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c.$$

Note that the resulting expression is a quadratic **binomial**.

Indeed, x is a variable, so is $x + \frac{b}{2a}$. Since a, b, c are constants, so is $-\frac{b^2}{4a} + c$.

If we denote $x + \frac{b}{2a}$ by y and $-\frac{b^2}{4a} + c$ by d,

then the expression $a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$ turns to $ay^2 + d$, which is a **bi**nomial.

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Proving quadratic formula

By completing the square,

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

Therefore

$$ax^2 + bx + c = 0 \iff a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0.$$

Let us solve the latter **binomial** equation:

$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$ Move $-\frac{b^2}{4a} + c$ to RHS

$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$ Divide both sides by a

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$ Combine terms in RHS: $\frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

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Proving quadratic formula

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Notice that the equation has a solution **only if** $\dfrac{b^2 - 4ac}{4a^2} \geq 0$.

$\dfrac{b^2 - 4ac}{4a^2} \geq 0 \iff \boxed{b^2 - 4ac \geq 0}$ since $4a^2 > 0$.

Take the square roots from both sides of the equation:

$\left|x + \dfrac{b}{2a}\right| = \sqrt{\dfrac{b^2 - 4ac}{4a^2}}$ Take down the absolute value sign

$x + \dfrac{b}{2a} = \pm\sqrt{\dfrac{b^2 - 4ac}{4a^2}}$ Simplify the radical

$x + \dfrac{b}{2a} = \pm\dfrac{\sqrt{b^2 - 4ac}}{2a}$ Move $\dfrac{b}{2a}$ to RHS

Proving quadratic formula

$x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$ Combine terms on RHS

$\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Done!

This is the **quadratic formula** for finding roots (solutions) of a quadratic equation.

It is applicable **only** if $\boldsymbol{b^2 - 4ac \geq 0}$.

The expression $\boldsymbol{b^2 - 4ac}$ is of special importance,
it is called the **discriminant** of the quadratic equation.

A quadratic equation has solutions if and only if its discriminant is **non-negative**.

Discriminant

What does the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ give us?

Case 1. If the discriminant is positive, that is $b^2 - 4ac > 0$,
then the quadratic formula gives **two** solutions (roots):

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Case 2. If the discriminant equals zero, that is $b^2 - 4ac = 0$,
then the quadratic formula gives **one** solution (root):

$$x = -\frac{b}{2a}.$$

Case 3. If the discriminant is negative, that is $b^2 - 4ac < 0$,
then the quadratic equation has **no** solutions (roots).

How to apply the quadratic formula

Example 1. Solve the equation $x^2 + 2x - 3 = 0$.

Solution. The quadratic equation is written in the standard form

$ax^2 + bx + c = 0$ with $a = 1,\ b = 2$ and $c = -3$.

The solution is given by the quadratic formula $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In our case,

$$x_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}.$$

From this, $x_1 = \dfrac{-2 + 4}{2} = 1$ and $x_2 = \dfrac{-2 - 4}{2} = -3$.

Answer. $x = 1$ or $x = -3$.

How to apply the quadratic formula

Example 2. Solve the equation $2x^2 - 3x - 1 = 0$.

Solution. In this case, $a = 2,\ b = -3,\ c = -1$. The solution is

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$$
$$= \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$$

Answer. $x_{1,2} = \dfrac{3 \pm \sqrt{17}}{4}$

Example 3. Solve the equation $x^2 - x + 1 = 0$.

Solution. In this case, $a = 1,\ b = -1,\ c = 1$. The solution is

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{1 \pm \sqrt{-3}}{2}$$

This equation has **no** real solutions.

How to apply the quadratic formula

Example 4. Solve the equation $-x^2 + 6x - 9 = 0$.

Solution. In this case, $a = -1,\ b = 6,\ c = -9$. The solution is

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot (-1) \cdot (-9)}}{2 \cdot (-1)}$$
$$= \frac{-6 \pm \sqrt{36 - 36}}{-2} = \frac{-6}{-2} = 3.$$

Answer. $x = 3$.

Remark. Let us have another look on the equation:

$-x^2 + 6x - 9 = 0 \iff x^2 - 6x + 9 = 0$. The left hand side on the latter equation is, actually, a perfect square trinomial: $x^2 - 6x + 9 = (x-3)^2$.

Therefore, $x^2 - 6x + 9 = 0 \iff (x-3)^2 = 0 \iff x - 3 = 0 \iff x = 3$.

When an equation is not in the standard form

Example 5. Solve the equation $5 + x(2 - x) = 4 + x^2$.

Solution. To use the quadratic formula, we have to bring the equation into the standard form:

$5 + x(2 - x) = 4 + x^2 \iff 5 + 2x - x^2 = 4 + x^2 \iff 0 = 2x^2 - 2x - 1$.

The equation is in the standard form with $a = 2,\ b = -2,\ c = -1$.

The solution is

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{2 \pm \sqrt{4 + 8}}{4}$$
$$= \frac{2 \pm \sqrt{12}}{4}.$$

Let us bring the answer to simplest radical form:

$$\frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{2(1 \pm \sqrt{3})}{4} = \frac{1 \pm \sqrt{3}}{2}.$$

Answer. $x_{1,2} = \dfrac{1 \pm \sqrt{3}}{2}$

When the quadratic formula is not the best choice

If a quadratic equation is **not** a trinomial, but a binomial,
then the quadratic formula is valid, but is **not** the most efficient tool for solving the equation.

Example. Solve the equation $4x^2 - x = 0$.

Solution. Alternative 1 (using the quadratic formula) $a = 4, b = -1, c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 4 \cdot 0}}{2 \cdot 4} = \frac{1 \pm \sqrt{1}}{8} = \frac{1 \pm 1}{8}.$$

By this, $x_1 = \dfrac{1+1}{8} = \dfrac{1}{4}$ and $x_2 = \dfrac{1-1}{8} = 0$.

Alternative 2 (by factoring):

$4x^2 - x = 0 \iff x(4x - 1) = 0 \iff x = 0$ or $4x - 1 = 0$

$\iff x = 0$ or $x = \dfrac{1}{4}$.

Answer. $x = 0$ or $x = \dfrac{1}{4}$

Summary

In this lecture, we have learned

- ☑ the **quadratic formula** $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- ☑ how to complete the square
- ☑ how to prove the quadratic formula
- ☑ when the quadratic formula is valid
- ☑ what the **discriminant** of a quadratic equation is
- ☑ how many solutions a quadratic equation has depending on its determinant
- ☑ how to apply the quadratic formula to solving quadratic equations
- ☑ when the quadratic formula is not the best tool to solve a quadratic equation

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Lecture 28

Factoring Quadratic Polynomials

Factoring polynomials

To **factor** a polynomial means to present this polynomial as a product of polynomials of degree **less** than the original polynomial.

For example, $x^2 - 1 = (x-1)(x+1)$ is a factoring, but

$3x^2 + 3 = 3(x^2+1)$ is **not** a polynomial factoring, since the degree of x^2+1 is **not** less than the degree of $3x^2+3$.

Factoring is an important algebraic tool that helps to solve various problems.

The same polynomial can be factored in different ways.

For example, $x^3 - x$ can factored as $x(x^2-1)$ or as $x(x-1)(x+1)$ or as $2x(x-1)\left(\frac{1}{2}x+\frac{1}{2}\right)$

Monomials, that is, polynomials of type ax^n, are easy to factor.

For example, $4x^3 = 4x^2 \cdot x$ or $4x^3 = 4x \cdot x \cdot x$.

In this lecture we will learn how to factor quadratic **binomials** and **trinomials**.

Irreducible polynomials

If a polynomial can't be factored, it is called **irreducible**.

Polynomials of degree one are **irreducible**, they can't be factored: we can't present a polynomial of degree one as a product of polynomials of degrees **less** than one.

Some polynomials are easy to factor: $x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$.
The factors, $x-2$ and $x+2$, contain only **integer** coefficients. Such factoring is called factoring over the integers.

Consider another factoring: $x^2 - 3 = x^2 - (\sqrt{3})^2 = (x-\sqrt{3})(x+\sqrt{3})$.

Here the factors, $x-\sqrt{3}$ and $x+\sqrt{3}$, have **real coefficients**, Such factoring is called factoring over the reals.

The polynomial $x^2 - 3$ can't be factored over the integers. It is **irreducible** over the integers.

The polynomial x^2+1 is **irreducible** over the reals, but can be factored over the complex numbers: $x^2 - 1 = (x-i)(x+i)$.

Factoring quadratic binomials

Quadratic binomials are expressions of type $ax^2 + bx$ or $ax^2 + c$,
they are special types of quadratic polynomials.

It's easy to factor the binomial $ax^2 + bx$: $ax^2 + bx = x(ax + b)$

The binomial $ax^2 + c$ can be factored over the reals
only if the coefficients a and c have opposite signs.

If a and c are of the same sign (both positive or both negative) then $ax^2 + c$ is **irreducible**.

Example. Factor the following polynomials: $9x^2 - 4$, $9x^2 + 4$.

Solution. $9x^2 - 4 = (3x)^2 - 2^2 = (3x - 2)(3x + 2)$.

The polynomial $9x^2 + 4$ is **irreducible**.

For the rest of the course, we will say that a polynomial is irreducible,
if it is irreducible over the reals.

Factorization theorem for quadratic trinomials

Theorem. Let $ax^2 + bx + c$ be a quadratic polynomial
with non-negative discriminant, that is, $b^2 - 4ac \geq 0$.

Then

$$\boxed{ax^2 + bx + c = a(x - x_1)(x - x_2)}$$

where x_1, x_2 are the **roots** of the polynomial,
that is, the solutions of the equation $ax^2 + bx + c = 0$.

Remarks.

1. If the discriminant is 0, then $x_1 = x_2$ is the only root of the equation, and the factoring is

$$ax^2 + bx + c = a(x - x_1)(x - x_1) = a(x - x_1)^2.$$

2. Factoring is simple when $a = 1$:

$$x^2 + bx + c = (x - x_1)(x - x_2).$$

3. If the discriminant is negative, then the polynomial is **irreducible**.

Proving factorization formula

By **completing the square**,

$$ax^2 + bx + c = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right) =$$

$$a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2\right) =$$

$$a\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right) =$$

$$a\left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) = a(x - x_1)(x - x_2)\,,$$

as required.

Factoring by finding roots

Example 1. Factor $2x^2 - x - 1$.

Solution. By the factoring theorem,

$2x^2 - x - 1 = 2(x - x_1)(x - x_2)$, where

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$$

$$= \frac{1 \pm \sqrt{1 + 8}}{4} = \frac{1 \pm \sqrt{9}}{4} = \frac{1 \pm 3}{4}$$

So $x_1 = \dfrac{1+3}{4} = 1$ and $x_2 = \dfrac{1-3}{4} = -\dfrac{1}{2}$.

The factoring is

$$2x^2 - x - 1 = 2(x - 1)\left(x - \left(-\frac{1}{2}\right)\right) = 2(x - 1)\left(x + \frac{1}{2}\right) = (x - 1)(2x + 1)\,.$$

Factoring by finding roots

Example 2. Factor $x^2 - x - 1$.

Solution. By the factoring theorem,

$x^2 - x - 1 = (x - x_1)(x - x_2)$, where

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} =$$

$$= \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

So $x_1 = \dfrac{1+\sqrt{5}}{2}$ and $x_2 = \dfrac{1-\sqrt{5}}{2}$.

The factoring is

$$x^2 - x - 1 = \left(x - \frac{1+\sqrt{5}}{2}\right)\left(x - \frac{1-\sqrt{5}}{2}\right).$$

Factoring by finding roots

Example 3. Factor $x^2 - 4x + 4$.

Solution. By the factoring theorem,

$x^2 - 4x + 4 = (x - x_1)(x - x_2)$, where

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} =$$

$$= \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4 \pm 0}{2} = 2$$

So $x_1 = x_2 = 2$.

The factoring is $x^2 - 4x + 4 = (x-2)(x-2) = (x-2)^2$.

Remark. If you recognize a perfect square trinomial formula in the expression $x^2 - 4x + 4$, then the factoring can be achieved faster:

$$x^2 - 4x + 4 = x^2 - 2 \cdot x \cdot 2 + (2)^2 = (x-2)^2.$$

Factoring by finding roots

Example 4. Factor $3x^2 - x + 1$.

Solution. The **discriminant** is

$$b^2 - 4ac = (-1)^2 - 4 \cdot 3 \cdot 1 = 1 - 12 = -11 < 0\,,$$

therefore, the polynomial has **no** roots and is **irreducible**.

Vieta's theorem

Theorem. If x_1 , x_2 are the roots of the equation $ax^2 + bx + c = 0$,

then $x_1 + x_2 = -\dfrac{b}{a}$ and $x_1 \cdot x_2 = \dfrac{c}{a}$.

Proof. By the Factorization theorem,

$ax^2 + bx + c = a(x - x_1)(x - x_2)$.

Let us expand RHS of this identity:

$a(x - x_1)(x - x_2) = a(x^2 - x_1 x - x_2 x + x_1 x_2) = ax^2 - a(x_1 + x_2)x + ax_1 x_2$.

Therefore, $ax^2 + \boldsymbol{b}\, x + \boldsymbol{c} = ax^2\, \boldsymbol{-a(x_1 + x_2)}\, x + \boldsymbol{ax_1 x_2}$.

By comparison of the coefficients of these two polynomials, we get

$\boldsymbol{b = -a(x_1 + x_2)}$ and $\boldsymbol{c = ax_1 x_2}$. From this,

$x_1 + x_2 = -\dfrac{b}{a}$ and $x_1\, x_2 = \dfrac{c}{a}$, as required.

Vieta's theorem relates the **roots** and the **coefficients** of a quadratic equation.

Vieta's theorem for finding roots

Vieta's theorem is especially simple if $a = 1$. In this case,

- the roots x_1, x_2 of the equation $x^2 + bx + c = 0$ satisfy

$$x_1 + x_2 = -b \text{ and } x_1x_2 = c.$$

Vieta's theorem may be used for finding the roots of a quadratic equation, provided that the coefficients of the equation and the roots are **integers**.

Example. Solve the equation $x^2 + x - 6 = 0$.
Solution. If x_1 and x_2 are the roots of $x^2 + x - 6 = 0$, then
$x_1 + x_2 = -b = -1$ and $x_1x_2 = c = -6$.
Let us guess two numbers, whose sum equals -1 and the product equals -6. The numbers are 2 and -3. **Answer.** $x = 2$ or $x = -3$.
⚠ **Warning.** Guessing out the roots may be **not** a good idea.
It may happen that the equation has **irrational** roots or **no roots** at all.
Although Vieta's theorem is valid, it can't be used to find the roots in these cases.

Don't waste your time guessing!

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Solving quadratic equations by factoring

Example. Solve the equation $x^2 - 2x - 15 = 0$.

Solution. For this equation, $a = 1,\ b = -2,\ c = -15$.

The discriminant of the equation is $b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-15) = 64$, which a perfect square.

It means that the roots are **rational** numbers, and we may guess them out.

The factoring is

$x^2 - 2x - 15 = (x - \ ?\ )(x - \ ?\ )$

By guessing, we get

$x^2 - 2x - 15 = (x - 5)(x + 3)$.

So $x^2 - 2x - 15 = 0 \iff (x - 5)(x + 3) = 0$

$$\iff x - 5 = 0 \text{ or } x + 3 = 0 \iff x = 5 \text{ or } x = -3.$$

Answer. $x = 5$ or $x = -3$

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Summary

In this lecture, we have learned

- ☑ what it means to factor a polynomial
- ☑ what an irreducible polynomial is
- ☑ how to factor quadratic binomials
- ☑ how to factor quadratic trinomials $ax^2 + bx + c = a(x - x_1)(x - x_2)$
- ☑ how to prove the factorization formula
- ☑ Vieta's theorem
- ☑ how to use Vieta's theorem for solving quadratic equations
- ☑ how to solve quadratic equations by factoring

Lecture 29

Equations Reducible to Quadratic

Applications of quadratic equations

In this lecture we will learn how to apply our knowledge about quadratic equations to other problems.

We will discuss

- Polynomial equations
- Biquadratic equations
- Rational equations
- Word problems leading to quadratic equations

Polynomial Equations

Example 1. Solve the equation $x^3 - 3x^2 - 4x = 0$.

Solution. This is a **polynomial equation**, since $x^3 - 4x^2 - 3x$ is a polynomial.

To solve the equation, we factor LHS:

$x^3 - 4x^2 - 3x = 0 \iff x(x^2 - 4x - 3) = 0$.

The product of two factors, x and $x^2 - 4x - 3$, equals 0 if and only if $x = 0$ or $x^2 - 4x - 3 = 0$.

By this, the first root is $x_1 = 0$. To find other roots, we have to solve the quadratic equation $x^2 - 4x - 3 = 0$.

$$x^2 - 4x - 3 = 0 \iff x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 + 12}}{2}$$
$$= \frac{4 \pm \sqrt{28}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}.$$

Therefore, the equation has three roots: $x_1 = 0$, $x_2 = 2 + \sqrt{7}$, $x_3 = 2 - \sqrt{7}$.

Biquadratic equations

Example 2. Solve the equation $x^4 + 2x^2 - 3 = 0$.

Solution. This equation is called **biquadratic**.

It is solved by the **substitution** $t = x^2$. Observe that $t \geq 0$.

$$x^4 + 2x^2 - 3 = 0 \iff t^2 + 2t - 3 = 0 \iff (t-1)(t+3) = 0$$
$$\iff t = 1 \text{ or } t = -3.$$

Since $t \geq 0$, we reject the negative root $t = -3$.

By this, the only solution is given by $t = 1$, that is $x^2 = 1$. So $x = \pm 1$.

Answer. $x = \pm 1$

Rational equations

Example 3. Solve the equation $\dfrac{1}{x} + \dfrac{2}{x+1} = 1$.

Solution. This equation is called **rational**, since it contains rational expressions.

To solve the equation, we bring RHS to 0:

$$\frac{1}{x} + \frac{2}{x+1} = 1 \iff \frac{1}{x} + \frac{2}{x+1} - 1 = 0.$$

Bring all terms to the **common denominator**:

$$\frac{x+1}{x(x+1)} + \frac{2x}{x(x+1)} - \frac{x(x+1)}{x(x+1)} = 0$$

Combine the terms in a single fraction:

$$\frac{x+1+2x-x(x+1)}{x(x+1)} = 0$$ and simplify

$$\frac{-x^2+2x+1}{x(x+1)} = 0$$

Rational equations

We have got that the original equation is equivalent to the following equation:

$$\frac{-x^2+2x+1}{x(x+1)} = 0\,.$$

When is a fraction equal to 0?
Only if its **numerator** equals 0 and the denominator is **not** equal to 0
(since one can't divide by 0).

Therefore,

$$\frac{-x^2+2x+1}{x(x+1)} = 0 \iff -x^2+2x+1=0 \text{ and } x \neq 0\,,\ x \neq -1\,.$$

Let us solve the quadratic equation:

$$-x^2+2x+1=0 \iff x^2-2x-1=0$$

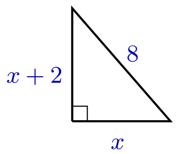
$$\iff x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 1\cdot(-1)}}{2\cdot 1} = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm 2\sqrt{2}}{2} = \boxed{1\pm\sqrt{2}}$$

We accept both roots, since none of them is 0 or -1.

Word problems

Problem 1. The hypotenuse of a right triangle is 8 cm long.
One leg is 2 cm shorter than the other. Find the lengths of the legs of the triangle.

Solution.



Let x cm be the length of the shorter leg.
Then the other leg has the length of $x+2$ cm.
The hypotenuse is 8 cm.

By the **Pythagorean** theorem, $x^2+(x+2)^2=8^2$.

To find x, we have to solve this quadratic equation.

Word problems

To solve the equation, we have bring it to the **standard** form.

$$x^2 + (x+2)^2 = 8^2 \iff x^2 + x^2 + 4x + 4 = 64 \iff 2x^2 + 4x - 60 = 0$$
$$\iff x^2 + 2x - 30 = 0\,.$$

The equation is in the standard form now, and we can use the **quadratic formula**:

$$x_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-30)}}{2} = \frac{-2 \pm \sqrt{124}}{2} = \frac{-2 \pm 2\sqrt{31}}{2} = -1 \pm \sqrt{31}\,.$$

We have got two solutions, $x_1 = -1 + \sqrt{31}$ and $x_2 = -1 - \sqrt{31}$.

One of the solutions, $x_2 = -1 - \sqrt{31}$, is **negative**, and should be rejected, since x, being the length of a side in a triangle, is positive.

Therefore, one leg is $-1 + \sqrt{31}$ cm long, the other leg is $-1 + \sqrt{31} + 2 = 1 + \sqrt{31}$ cm long.

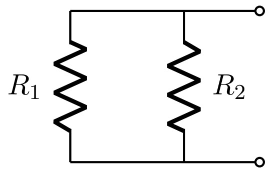
Answer. The lengths of the legs are $-1 + \sqrt{31}$ cm and $1 + \sqrt{31}$ cm.

Word problems

Problem 2. Two parallel resistors provide the total resistance of 2 Ohms.
Find the value of each resistor if one of them is 3 Ohms more than the other.
Use the law for parallel resistors:

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Solution.



Given: $R_{\text{total}} = 2$, $R_2 = R_1 + 3$.
Plug these into the given equation:

$$\frac{1}{2} = \frac{1}{R_1} + \frac{1}{R_1 + 3}.$$

To find R_1, we have to solve this **rational** equation.

Word problems

$\frac{1}{2} = \frac{1}{R_1} + \frac{1}{R_1+3} \iff \frac{1}{R_1} + \frac{1}{R_1+3} - \frac{1}{2} = 0$

Bring all terms to the common denominator:

$\frac{2(R_1+3)}{2R_1(R_1+3)} + \frac{2R_1}{2R_1(R_1+3)} - \frac{R_1(R_1+3)}{2R_1(R_1+3)} = 0$ Combine the terms in a single fraction:

$\frac{2(R_1+3)+2R_1-R_1(R_1+3)}{2R_1(R_1+3)} = 0$ Simplify:

$\frac{-R_1^2+R_1+6}{2R_1(R_1+3)} = 0 \iff -R_1^2+R_1+6=0 \iff R_1^2-R_1-6=0$

$\iff (R_1-3)(R_1+2)=0 \iff R_1=3$ or $R_1=-2$.

We reject the negative root $R_1=-2$ since a negative resistance makes no sense.

So $R_1=3$ Ohms and $R_2=R_1+3=3+3=6$ Ohms.

Summary

In this lecture, we have learned

- ☑ how to solve **polynomial** equations reducible to quadratic ones
- ☑ how to solve **biquadratic** equations
- ☑ how to solve **rational** equations
- ☑ how to solve word problems leading to quadratic equations

Lecture 30

Parabolas

Quadratic functions

A **quadratic function** is a function $y = ax^2 + bx + c$, where a, b, c are given numbers and $a \neq 0$.

Examples of quadratic functions:

$$y = x^2$$
$$y = x^2 + x$$
$$y = -3x^2 + 2x - 5$$
$$y = \frac{1}{3}x^2 - \sqrt{2}\,x + 1$$

Functions and, in particular, quadratic functions, are studied in the precalculus and calculus courses.

In this lecture we will learn how to draw the **graph** of a quadratic function.

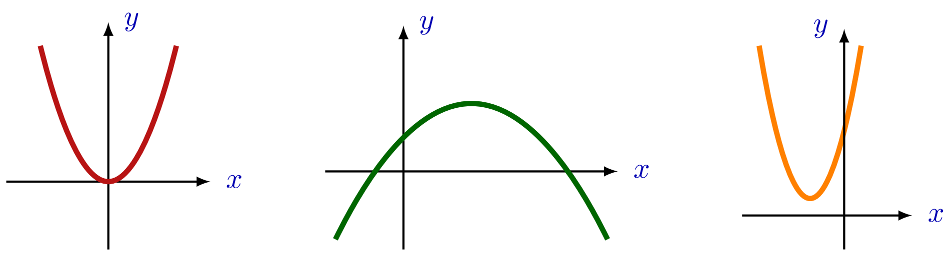
The graph of a function provides a **visualization** of various properties of the function, and helps to understand these properties.

What is the graph

The **graph** of a quadratic function $y = ax^2 + bx + c$ is the set of all points on the plane whose coordinates (x, y) satisfy the equation $y = ax^2 + bx + c$.

The graph of a quadratic function is a plane **curve**, it is called a **parabola**.

Here are a few examples of parabolas:

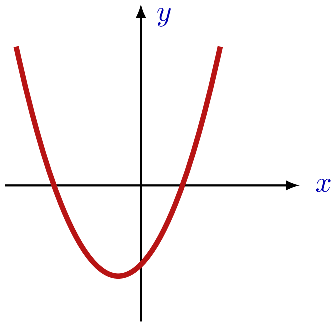


In this lecture, we will learn how to draw a parabola by its equation.

Geometry of a parabola

Any parabola has certain geometric elements which are common for all parabolas.

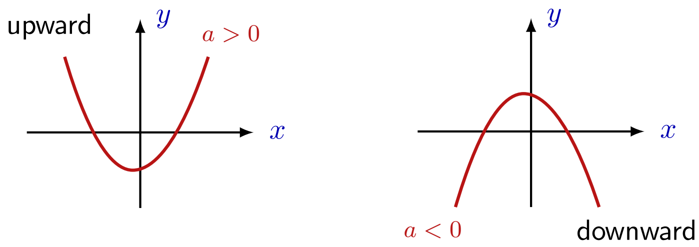
Let us have a look on a typical parabola:



Which geometric elements do we observe on this parabola?

Horns: upward or downward

A parabola has its "horns" turned **upward** or **downward.** (A parabola opens upward or downward.)

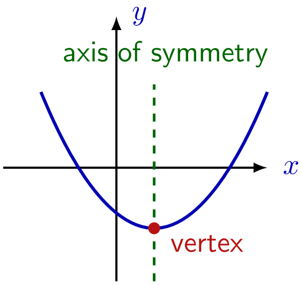


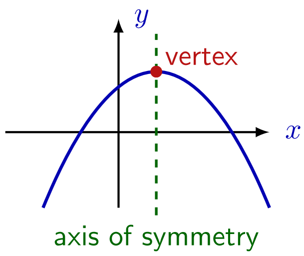
It is the coefficient a (called the **leading coefficient**) which is responsible for this.

- If $a > 0$, then the parabola opens **upward**
- If $a < 0$, then the parabola opens **downward**

Vertex and axis of symmetry

There is a characteristic point on a parabola, where the parabola makes a **turn**.



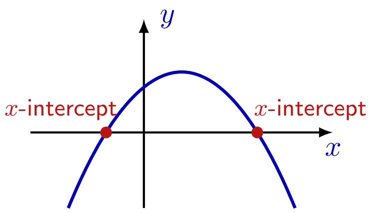


This point is called the **vertex**.
The vertex is the **lowest** point on the parabola if $a > 0$, and the **highest** point if $a < 0$.

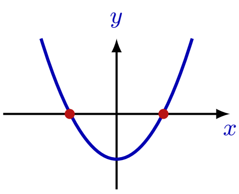
A vertical line passing through the vertex is called the **axis of symmetry**,
because a parabola is symmetric about its axis of symmetry.

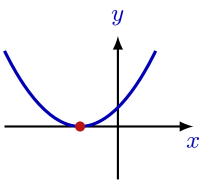
The x-intercepts

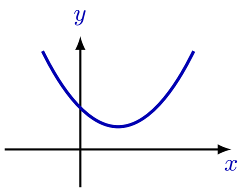
The points where the parabola intersects the x-axis, are called the **x-intercepts**.



A parabola may have **two**, **one**, or **no** **x-intercepts**.

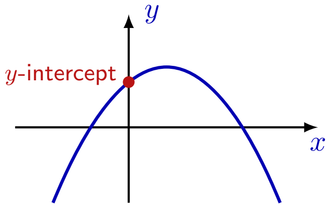






The y-intercept

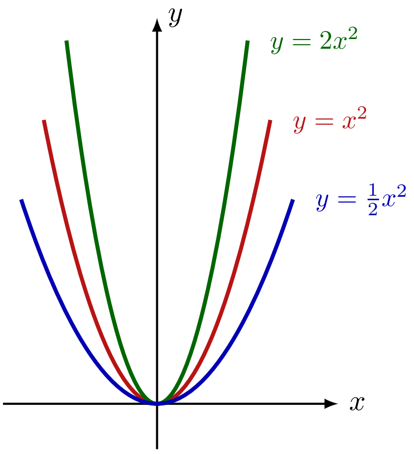
A point where the parabola intersects the y-axis is called the **y-intercept**.



Each parabola has exactly **one** y-intercept.

Wide or narrow?

Some parabolas are wider than others:



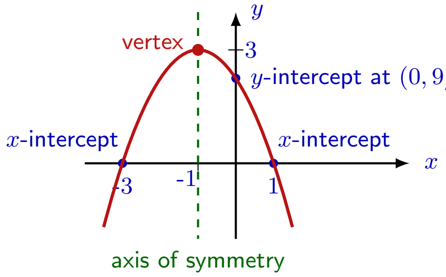
$|a|$ is responsible
for the **width** of the parabola
.
The smaller $|a|$,
the wider the parabola.

What do we need to sketch a parabola?

- the vertex
- the axis of symmetry
- the sign of a (upward or downward)
- the y-intercept
- the x-intercepts (if any)

Example. Sketch a parabola which opens downward, has the vertex at $(-1, 3)$, the y-intercept at $(0, 9/4)$, and the x-intercepts at $(-3, 0)$ and $(1, 0)$.

Solution.



How to find the vertex

The vertex of the parabola $y = ax^2 + bx + c$ is located

at the point with coordinates $\left(-\frac{b}{2a}, -\frac{b^2}{4a} + c\right)$.

Why is this so? Rewrite the equation of the parabola using **completing the square**:

$$y = ax^2 + bx + c \iff y = a\left(x + \frac{b}{2a}\right)^2 + \left(-\frac{b^2}{4a} + c\right)$$

If $a > 0$, then the vertex is located at the **lowest** point on the parabola,

that is at the point, where y takes the **minimal** value.

Since $a\left(x + \frac{b}{2a}\right)^2 \geq 0$ for all x, the minimal value of $y = a\left(x + \frac{b}{2a}\right)^2 + \left(-\frac{b^2}{4a} + c\right)$ occurs exactly when $\left(x + \frac{b}{2a}\right)^2 = 0$, that is when $x = -\frac{b}{2a}$.

Therefore, the vertex is located at $\left(-\frac{b}{2a}, -\frac{b^2}{4a} + c\right)$.

How to find the vertex and the axis of symmetry

If $a < 0$, then the vertex is located at the **highest** point on the parabola,
that is at the point, where y takes the **maximal** value.

Since $a\left(x+\frac{b}{2a}\right)^2 \le 0$ for all x, the maximal value of $y = a\left(x+\frac{b}{2a}\right)^2 + \left(-\frac{b^2}{4a}+c\right)$ occurs exactly when $\left(x+\frac{b}{2a}\right)^2 = 0$, that is when $x = -\frac{b}{2a}$.

Therefore, the vertex is located at $\left(-\frac{b}{2a}, -\frac{b^2}{4a}+c\right)$.
Remember that

> The **vertex** of the parabola $y = ax^2+bx+c$
> is located at the point where $x = \frac{-b}{2a}$.

The **axis of symmetry** is the vertical line passing through the vertex.

Its equation is $\boxed{x = -\frac{b}{2a}}$

How to find the vertex and the axis of symmetry

Example. Find the vertex and the axis of symmetry of the parabola $y = x^2 - 4x + 1$.

Solution. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-4)}{2 \cdot 1} = \frac{4}{2} = 2.$$

To find the y-coordinate of the vertex, we plug in $x = 2$ into the equation of the parabola:

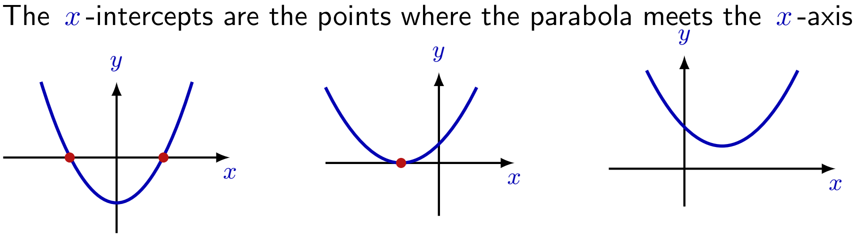
$$y = 2^2 - 4 \cdot 2 + 1 = 4 - 8 + 1 = -3.$$

Therefore, the **vertex** of the parabola is at the point with coordinates $(2, -3)$.

The **axis of symmetry** is the vertical line $x = 2$.

How to find the x-intercepts

The x-intercepts are the points where the parabola meets the x-axis.



A parabola may have two, one or no x-intercepts.
At x-intercept, the y-value is equal to 0. Therefore,

> To find the x-intercepts of the parabola $y = ax^2 + bx + c$,
> solve the equation $ax^2 + bx + c = 0$.

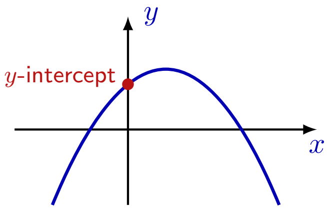
If the quadratic equation $ax^2 + bx + c = 0$ has **two** roots,
then the parabola intersects the x-axis at **two** points.

If the equation has **one** root, then the parabola touches the x-axis at **one** point.

If the equation has **no** roots, then the parabola does **not** intersect the x-axis.

How to find the y-intercept

The y-intercept is easy to find.



This is the point where the parabola intersects the y axis.

At this point, the x-coordinate equals 0.

When we plug $x = 0$ into the equation of the parabola $ax^2 + bx + c$, we get

$y = a \cdot 0^2 + b \cdot 0 + c = c$.

Therefore,

> The y-intercept of the parabola $y = ax^2 + bx + c$
> is located at the point $(0, c)$

Step-by-step instruction for drawing a parabola

To draw the parabola $y = ax^2 + bx + c$,

- Determine the **vertex**. It's located at the point where $x = \dfrac{-b}{2a}$.
- Draw the **axis of symmetry**. It's the vertical line $x = \dfrac{-b}{2a}$.
- Determine if the parabola opens **upward** ($a > 0$) or **downward** ($a < 0$).
- Determine the **y-intercept**. It's located at the point $(0, c)$.
- Determine the **x-intercepts** (if any). They are located at the points $(x_{1,2}, 0)$,

  where $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Draw the parabola, using the information above.
  Make sure that your parabola is smooth and symmetric.

Example 1

Example 1. For the parabola $y = x^2 - x - 2$, determine the vertex, the axis of symmetry, the intercepts, and draw the graph.

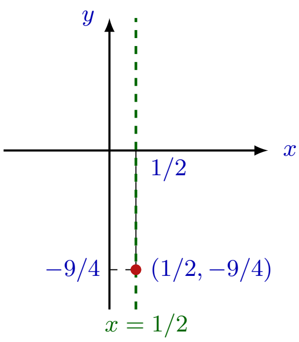
Solution.
- The **vertex** is at $x = \dfrac{-b}{2a} = \dfrac{-(-1)}{2} = \dfrac{1}{2}$. The y-coordinate of the vertex is

$y = \left(\dfrac{1}{2}\right)^2 - \dfrac{1}{2} - 2 = \dfrac{1}{4} - \dfrac{1}{2} - 2 = -9/4$. So the vertex is located at $(1/2, -9/4)$.

Draw the vertex.

- The **axis of symmetry** is the vertical line $x = 1/2$.

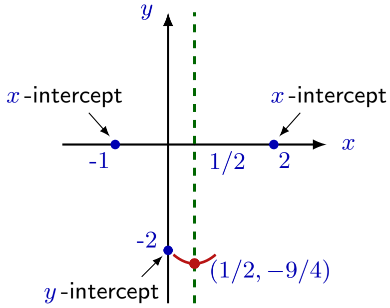
Draw the axis of symmetry.



Example 1

- $a = 1 > 0$, therefore, the parabola opens **upward**.

Draw a small **sprout** of a parabola at the vertex.



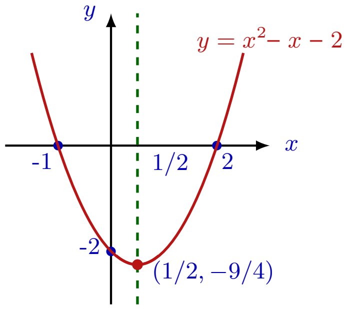
- The $\boldsymbol{y}$**-intercept** is at $(0, c) = (0, -2)$.
- The $\boldsymbol{x}$**-intercepts** are the roots of $x^2 - x - 2 = 0$.

$x^2 - x - 2 = 0 \iff (x+1)(x-2) = 0 \iff x = -1,\ x = 2$.

So the x-intercepts are $(-1, 0)$ and $(2, 0)$.

Example 1

Now we are ready to draw the parabola:



Be neat: the parabola should be smooth and symmetric.

Example 2.

Example 2. For the parabola $y = -x^2 - 2x - 2$, determine the vertex, the axis of symmetry, the intercepts, and draw the graph.

Solution. The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2 \cdot (-1)} = -1$.

The y-coordinate of the vertex is $y = -(-1)^2 - 2 \cdot (-1) - 2 = -1 + 2 - 2 = -1$. By this, the vertex is $\boxed{(-1, -1)}$.

The axis of symmetry is $\boxed{x = -1}$.

$a = -1 < 0$, so the parabola opens $\boxed{\text{downward}}$.

The y-intercept is $(0, c) = (0, -2)$.

For the x-intercepts, solve the equation $-x^2 - 2x - 2 = 0$:

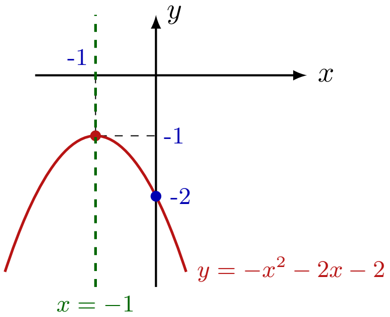
$$-x^2 - 2x - 2 = 0 \iff x^2 + 2x + 2 = 0.$$

The discriminant is $b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 2 = -4 < 0$.

Therefore, there are no solutions, and the parabola doesn't meet the x-axis.

Example 2.

Now put all the information on the graph.



The graph of a quadratic monomial

What do we know about the graph of the parabola $y = ax^2$?

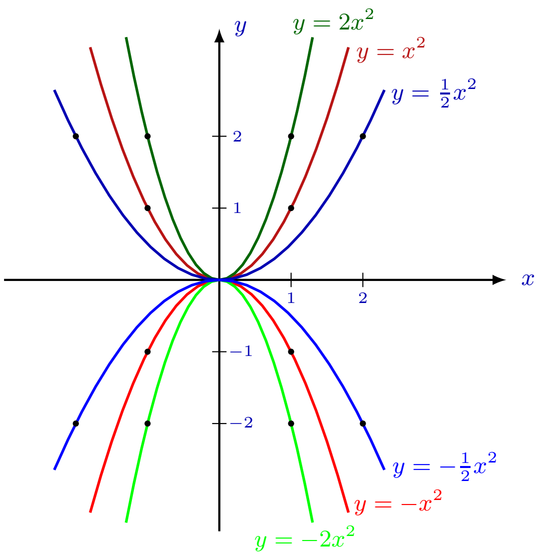
- The vertex at the origin $(0,0)$, since $\dfrac{-b}{2a} = 0$.
- The axis of symmetry is the line $x = 0$, that is, the y -axis.
- The parabola opens upward if $a > 0$, and downward if $a < 0$.
- The y -intercept is $(0,0)$.
- The only x -intercept is $(0,0)$.

This information is not sufficient for a drawing.
We may need to plot a support point, say, $(x,y) = (1,a)$ belonging to the parabola.

By symmetry, we get another point $(x,y) = (-1,a)$ on the parabola.

The graph of a quadratic monomial

Let us draw several parabolas $y = ax^2$ with different coefficients a .



Summary

In this lecture, we have learned

- ☑ what the **graph** of a quadratic function is
- ☑ what a **parabola** looks like
- ☑ what the essential **geometric elements** of the parabola are (vertex, axis of symmetry, intercepts)
- ☑ when a parabola opens **upward** ($a > 0$) or **downward** ($a < 0$)
- ☑ how to find the **vertex** and the **axis of symmetry** of a parabola
- ☑ how to find the x-**intercepts** (if any) and the y-**intercept** of a parabola
- ☑ how to **draw** the parabola from its equation
- ☑ how to draw the graph of a **quadratic monomial**

Lecture 31

Quadratic Inequalities

Quadratic inequalities

We will solve inequalities of the following types:

$ax^2 + bx + c \geq 0$, $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c < 0$,

where $a \neq 0$, b , c are given coefficients, and x is unknown.

For example, $x^2 + 5x - 6 \leq 0$ is a quadratic inequality.
Here $a = 1$, $b = 5$, $c = -6$.

The coefficient a is **not** zero, otherwise the inequality would be not quadratic , but rather linear .

What does it mean to solve inequality ?

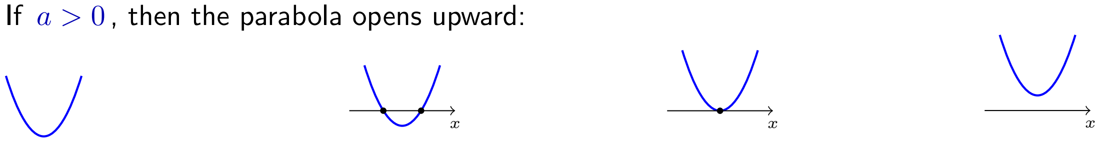
It means to find **all** the values of unknown x for which the inequality holds true.

Visualization

Let us draw a picture illustrating a quadratic inequality.

We know that the equation $y = ax^2 + bx + c$ defines a parabola ,
and know how to draw this parabola.

If $a > 0$, then the parabola opens upward:



two x-intercepts | one x-intercept | no x-intercepts

If $a < 0$, then the parabola opens downward:

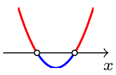
two x-intercepts | one x-intercept | no x-intercepts

Geometric solution

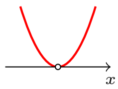
Let us solve the inequality $ax^2 + bx + c > 0$ in the case when $a > 0$.

Let $y = ax^2 + bx + c$. Then $ax^2 + bx + c > 0 \iff y > 0$.

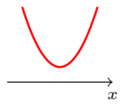
Thus, to solve the inequality $ax^2 + bx + c > 0$, we need to find where the parabola $y = ax^2 + bx + c$ is **above** the x-axis.



two x-intercepts

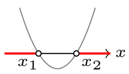


one x-intercept

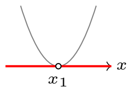


no x-intercepts

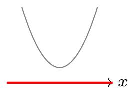
For which x is the parabola **above** the x-axis?



$x \in (-\infty, x_1) \cup (x_2, \infty)$



$x \in (-\infty, x_1) \cup (x_1, \infty)$



$x \in (-\infty, \infty)$

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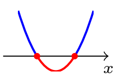
Geometric solution

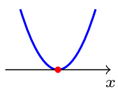
Now let us solve the inequality $ax^2 + bx + c \leq 0$ again in the case when $a > 0$.

Let $y = ax^2 + bx + c$. Then $ax^2 + bx + c \leq 0 \iff y \leq 0$.

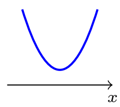
Thus, to solve the inequality $ax^2 + bx + c \leq 0$, we need to find where the parabola $y = ax^2 + bx + c$ is **below or on** the x-axis.



two x-intercepts

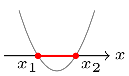


one x-intercept

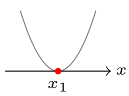


no x-intercepts

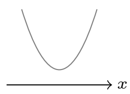
For which x is the parabola **below or on** the x-axis?



$x \in [x_1, x_2]$



$x = x_1$



no solution

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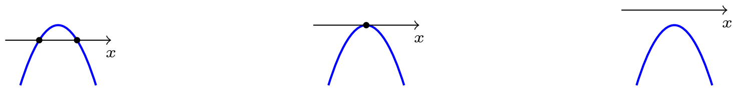
What if $a < 0$?

We have a choice:

• **either** to solve the inequality using a parabola, as we did in the case $a > 0$,

Don't forget that the parabola $y = ax^2 + bx + c$ with $a < 0$ opens down:



• **or** multiply both sides of the inequality by -1 , like

$$-3x^2 + x - 2 \geq 0 \iff 3x^2 - x + 2 \leq 0,$$

in order to make a-coefficient positive.

Don't forget to reverse the sign of inequality!

Example 1

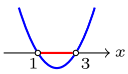
Solve the inequality $x^2 - 4x + 3 < 0$.

Solution. The parabola $y = x^2 - 4x + 3$ opens **upward**, since $a = 1 > 0$.

Determine the x-intercepts. They are the **roots** of the equation $x^2 - 4x + 3 = 0$.

$x^2 - 4x + 3 = 0 \iff (x-1)(x-3) = 0 \iff x_1 = 1, \ x_2 = 3$.

Therefore, the parabola looks as follows:



To solve the inequality $x^2 - 4x + 3 < 0$, we have to find **all** x for which the parabola is **below** the x-axis.

As we see, those x fill the interval $(1, 3)$.

The **answer** can be written in several ways:

$1 < x < 3$, or $x \in (1, 3)$, or simply $(1, 3)$.

Example 2

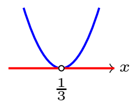
Solve the inequality $9x^2 - 6x + 1 > 0$.

Solution. The parabola $y = 9x^2 - 6x + 1$ opens **upward**, since $a = 9 > 0$.

Determine the x-intercepts. They are the **roots** of the equation $9x^2 - 6x + 1 = 0$.

$$9x^2 - 6x + 1 = 0 \iff (3x-1)^2 = 0 \iff x_1 = \frac{1}{3}.$$

Therefore, the parabola looks as follows:



To solve the inequality $9x^2 - 6x + 1 > 0$, we have to find **all** x for which the parabola is **above** the x-axis.

As we see, those x fill the whole line except the point $\frac{1}{3}$.

The **answer** can be written as $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$ or $\mathbb{R} \setminus \left\{\frac{1}{3}\right\}$.

Example 3

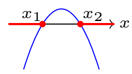
Solve the inequality $-x^2 + 3x - 1 \le 0$.

Solution. The parabola $y = -x^2 + 3x - 1$ opens **downward**, since $a = -1 < 0$.

Determine the x-intercepts. They are the **roots** of the equation $-x^2 + 3x - 1 = 0$. Solve the equation:

$$-x^2 + 3x - 1 = 0 \iff x^2 - 3x + 1 = 0 \iff x_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}.$$

Therefore, the parabola looks as follows:



To solve the inequality $-x^2 + 3x - 1 \le 0$, we have to find **all** x for which the parabola is **below or on** the x-axis.

Answer: $x \in \left(-\infty, \frac{3-\sqrt{5}}{2}\right] \cup \left[\frac{3+\sqrt{5}}{2}, \infty\right)$.

Example 4

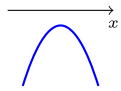
Solve the inequality $-x^2 - x - 1 > 0$.
Solution. Alternative 1. The parabola $y = -x^2 - x - 1$ opens **downward**, since $a = -1 < 0$.

Determine the x-intercepts. They are the **roots** of the equation $-x^2 - x - 1 = 0$.

$-x^2 - x - 1 = 0 \iff x^2 + x + 1 = 0 \iff$

$x_{1,2} = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2}$. No real roots!

Therefore, the parabola looks as follows:



To solve the inequality $-x^2 - x - 1 > 0$, we have to find **all** x
for which the parabola is **above** the x-axis.

As we see, there are no such x. **Answer:** no solutions.

Example 4

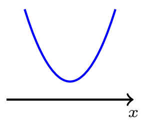
Let us solve the inequality $-x^2 - x - 1 > 0$ in a different way.

Alternative 2. $-x^2 - x - 1 > 0 \iff x^2 + x + 1 < 0$.

Instead of solving $-x^2 - x - 1 > 0$, we will solve an equivalent inequality $x^2 + x + 1 < 0$.

The parabola $y = x^2 + x + 1$ opens **upward** since $a = 1 > 0$,
and has **no** x-intercepts, since the discriminant $b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot 1 = -3$ is negative.

Therefore, the parabola is situated above the x-axis:



To solve the inequality $x^2 + x + 1 < 0$
means to find all values of x for which the parabola is below the x-axis.

But there are **no** such x. **Answer:** the inequality has **no** solutions.

Summary

In this lecture, we have learned

- ☑ what a **quadratic inequality** is
- ☑ what it means to **solve** a quadratic inequality
- ☑ how to **visualize** a quadratic inequality by a **parabola**
- ☑ how to solve a quadratic inequality
  in terms of the **leading coefficient** and the **roots**
- ☑ how to write down the **answer**

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Student: ____________________ **Instructor:** Deb Wertz **Assignment:** Homework #1
Date: ____________________ **Course:** MAP102 MASTER

1. Select the answer that best completes the given statement.

The (1) ____________ are {..., −3, −2, −1, 0, 1, 2, 3, ...}.

(1) ◯ rational numbers
◯ integers
◯ natural numbers
◯ irrational numbers

2. Select the correct choice to complete the following sentence.

The number $\sqrt{5}$ is a(n) (1) ____________

(1) ◯ natural number.
◯ rational number.
◯ irrational number.
◯ whole number.

3. Select the answer that best completes the given statement.

The number $\frac{5}{7}$ is a(n) (1) ____________

(1) ◯ natural numbers.
◯ rational number.
◯ irrational numbers.
◯ whole number.

4. List the elements in the set described.

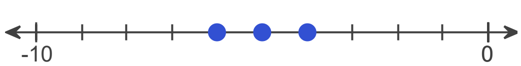
{x|x is a natural number less than 2}

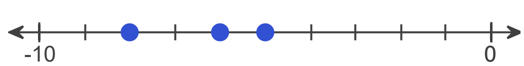
{____________}
(Use a comma to separate answers as needed. Use ascending order.)

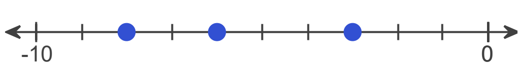
5. Graph the set on a number line.

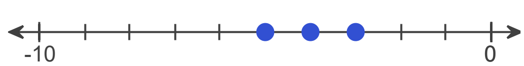
{−5, −6, −8}

Choose the correct graph below.

◯ **A.** 

◯ **B.** 

◯ **C.** 

◯ **D.** 

6. List the elements of the set $\left\{2, 0, \sqrt{13}, \sqrt{25}, \frac{3}{5}, -129\right\}$ that are also the elements of the set of whole numbers.

The elements of the given set that are also elements of the set of whole numbers are $\{\underline{\qquad\qquad}\}$.
(Use a comma to separate answers as needed.)

Student: ____________________	**Instructor:** Deb Wertz	**Assignment:** Homework #2
Date: ____________________	**Course:** MAP102 MASTER	

1. Add.

$-5+15$

$-5+15=$ __________

2. Subtract.

$11-13$

$11-13=$ __________

3. Subtract as indicated.

$\frac{7}{6}-\left(-\frac{1}{3}\right)$

$\frac{7}{6}-\left(-\frac{1}{3}\right)=$ __________

(Simplify your answer.)

4. Subtract $20-8-16$.

$20-8-16=$ __________

5. Subtract as indicated.

$-\frac{4}{5}-\left(-\frac{7}{15}\right)$

$-\frac{4}{5}-\left(-\frac{7}{15}\right)=$ __________

(Simplify your answer. Type an integer or a simplified fraction.)

6. Divide.

$\frac{-8}{-4}$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** $\frac{-8}{-4}=$ __________ (Simplify your answer.)

○ **B.** The expression is undefined.

7. Multiply as indicated.

$$2\left(-\frac{1}{18}\right)$$

Select the correct choice and, if necessary, fill in the answer box to complete your choice.

- A. $2\left(-\frac{1}{18}\right) =$ ______
  (Type an integer or a simplified fraction.)
- B. The expression is undefined.

---

8. Simplify the expression.

$$-14\left(-\frac{2}{7}\right)-14$$

$-14\left(-\frac{2}{7}\right)-14 =$ ______

---

9. Simplify the expression.

$$4-[(7-6)+(9-19)]$$

$4-[(7-6)+(9-19)] =$ ______

---

10. Divide.

$$\frac{-16}{8}$$

Select the correct choice and, if necessary, fill in the answer box to complete your choice.

- A. $\frac{-16}{8} =$ ______ (Simplify your answer. Type an integer or a fraction.)
- B. The expression is undefined.

---

11. Find the product.

$$(-6)(-8)(-1)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. $(-6)(-8)(-1) =$ ______
- B. The expression is undefined

---

12. Simplify.

$$4\{-5+3[3-5(-3+1)]\}$$

$4\{-5+3[3-5(-3+1)]\} =$ ______

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #3

1. Simplify the expression.

$$\frac{0.5-(-1.5)}{-0.5}$$

$\frac{0.5-(-1.5)}{-0.5} =$ ____________

2. Simplify the expression.

$$\frac{\frac{1}{2}\cdot 4-7}{5+\frac{1}{3}\cdot 9}$$

$\frac{\frac{1}{2}\cdot 4-7}{5+\frac{1}{3}\cdot 9} =$ ____________ (Type an integer or a simplified fraction.)

3. Evaluate the expression when $x = 5$ and $y = -6$.

$$5x-3y$$

$5x-3y =$ ____________

4. Evaluate the expression when $y = -3$.

$$-9y^2$$

$-9y^2 =$ ____________

5. Evaluate the expression when $x = 25$ and $y = -6$.

$$\frac{\sqrt{x}}{y}-\frac{y}{x}$$

$\frac{\sqrt{x}}{y}-\frac{y}{x} =$ ____________
(Type an integer or a simplified fraction.)

6. Find the value of the expression when $x_1 = 4$, $x_2 = 6$, $y_1 = -3$, $y_2 = 8$.

$$\frac{y_2-y_1}{x_2-x_1}$$

$\frac{y_2-y_1}{x_2-x_1} =$ ____________

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #4

1. Use the commutative property of addition to write an expression equivalent to the following.

   $13x + y$

   The answer is ____________.

2. Use the commutative property of multiplication to write an expression equivalent to the following.

   $g \cdot h$

   The answer is ____________.

3. Use the commutative property of multiplication to write an expression equivalent to the following.

   $\frac{1}{6} \cdot \frac{x}{8}$

   The answer is ____________.
   (Do not multiply.)

4. Use the associative property of multiplication to write an expression equivalent to the following.

   $10 \cdot (2x)$

   $10 \cdot (2x) =$ ____________ (Do not simplify.)

5. Use the associative property of addition to write an expression equivalent to the following.

   $(x + 9.7) + y$

   The answer is ____________.

6. Use an associative property to write an equivalent expression.

   $(22x) \cdot y$

   $(22x) \cdot y =$ ____________
   (Type the terms of your expression in the same order as they appear in the original expression.)

7. Write an expression for the amount of money (in cents) in n quarters.

   ____________ cents
   (Use integers or decimals for any numbers in the expression.)

8. Use a commutative property to complete the statement.

   $3x + 13 =$ ________

   $3x + 13 =$ ____________

9. Complete the following statement to illustrate the additive inverse property.

$$\frac{2}{4} + \left(-\frac{2}{4}\right) = \underline{?}$$

$\frac{2}{4} + \left(-\frac{2}{4}\right) =$ ____________

10. Complete the following statement to illustrate the multiplicative identity property.

$$3 \cdot 1 = \underline{?}$$

$3 \cdot 1 =$ ____________

11. Complete the statement to illustrate the associative property.

$$12(4y) = \_\_\_\_\_\_$$

$12(4y) =$ ____________
(Type the terms of your expression in the same order as they appear in the original expression. Do not perform the calculation.)

12. In the statement, a property of real numbers has been incorrectly applied. Correct the right side of the statement.

$$3(6y) = (3 \cdot 6)(3y)$$

$3(6y) =$ ____________
(Do not perform the calculation. Type the terms of your expression in the same order as they appear in the original expression.)

13. Name the only real number that is its own opposite, and explain why this is so.

Select the correct choice below and fill in the answer box to complete your choice.

- **A.** If a real number a satisfies the given condition, then $a = a$. The only real number that satisfies this equation is ____________.
- **B.** If a real number a satisfies the given condition, then $a^2 = -a$. The only real number that satisfies this equation is ____________.
- **C.** If a real number a satisfies the given condition, then $a = \frac{1}{a}$. The only real number that satisfies this equation is ____________.
- **D.** If a real number a satisfies the given condition, then $a = -a$. The only real number that satisfies this equation is ____________.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #5

1. Select the answer that best completes the given statement.

$0 \cdot a =$ (1) ____________

(1)
- ○ 0
- ○ $\frac{1}{a}$
- ○ 1
- ○ a

2. Select the answer that best completes the given statement.

The (1) ____________ of the nonzero number b is $\frac{1}{b}$.

(1)
- ○ opposite
- ○ square root
- ○ reciprocal
- ○ absolute value
- ○ exponent

3. Select the correct choices that complete the sentence below.

$\frac{0}{4}$ is (1) ____________ while $\frac{4}{0}$ is (2) ____________

(1)
- ○ undefined
- ○ 0

(2)
- ○ 4.
- ○ 0.
- ○ undefined.

4. Select the correct choices that complete the sentence below.

The fraction $-\frac{a}{b} =$ (1) ____________ = (2) ____________

(1)
- ○ $\frac{a}{b}$
- ○ $\frac{-a}{b}$

(2)
- ○ $\frac{a}{-b}$.
- ○ $\frac{a}{b}$.

5. Select the answer that best completes the given statement.

The opposite of nonzero number a is (1) ____________

(1)
- ○ $\frac{1}{a}$.
- ○ $-\frac{1}{a}$.
- ○ $-a$.
- ○ a.

6. Select the correct choice that completes the sentence below.

The reciprocal of nonzero number a is (1) __________

(1)
- ○ $\frac{1}{a}$.
- ○ $-a$.

---

7. Select the answer that best completes the given statement.

The (1) __________ property has to do with "order."

(1)
- ○ commutative
- ○ distributive
- ○ associative

---

8. Select the correct choice that completes the sentence below.

The (1) __________ property has to do with "grouping."

(1)
- ○ commutative
- ○ associative
- ○ distributive

---

9. Evaluate.

$$-3^2$$

$-3^2 =$ __________

---

10. Find the value of the expression.

$$\left(-\frac{1}{10}\right)^3$$

$\left(-\frac{1}{10}\right)^3 =$ __________

(Simplify your answer.)

11. Choose the fraction(s) equivalent to the given fraction.

$$-\frac{1}{5}$$

Select all that apply.

- ☐ **A.** $\frac{1}{-5}$
- ☐ **B.** $\frac{1}{5}$
- ☐ **C.** $\frac{-1}{5}$
- ☐ **D.** $\frac{-1}{-5}$

---

12. Choose the fraction(s) equivalent to the given fraction.

$$\frac{8}{-(p+r)}$$

Select all that apply.

- ☐ **A.** $-\frac{8}{(p+r)}$
- ☐ **B.** $\frac{8}{(p+r)}$
- ☐ **C.** $\frac{-8}{(p+r)}$
- ☐ **D.** $\frac{-8}{-(p+r)}$

---

13. Choose the fraction(s) equivalent to the given fraction.

$$\frac{-8r}{-9s}$$

Select all that apply.

- ☐ **A.** $-\frac{8r}{9s}$
- ☐ **B.** $\frac{-8r}{9s}$
- ☐ **C.** $\frac{8r}{-9s}$
- ☐ **D.** $\frac{8r}{9s}$

---

14. Evaluate $40 \div (8 \div 4)$ and $(40 \div 8) \div 4$. Use these two expressions and discuss whether division is associative.

$40 \div (8 \div 4) =$ ______ (Type an integer or a simplified fraction.)

$(40 \div 8) \div 4 =$ ______ (Type an integer or a simplified fraction.)

Therefore, division (1) ______ associative.

(1) ◯ is
◯ is not

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #6

1. Select the answer that best completes the given statement.

   a(b + c) = ab + ac illustrates the (1) ____________ property.

   (1) ◯ commutative
   ◯ associative
   ◯ distributive

2. Watch the section lecture video and answer the question listed below. Note: The counter in the lower right corner of the screen displays the Example number.

   From Examples 12-14, how are algebraic expressions simplified? If the expression contains parentheses, what property might be applied first?

   From Examples 12-14, how are algebraic expressions simplified?

   ◯ **A.** They are simplified by combining like terms.
   ◯ **B.** Algebraic expressions in those examples cannot be simplified.
   ◯ **C.** They are simplified by substitution.
   ◯ **D.** They are simplified by solving.

   If the expression contains parentheses, what property might be applied first?

   ◯ identity property
   ◯ commutative property
   ◯ associative property
   ◯ distributive property

3. Select the correct choice that completes the sentence below.

   The (1) ____________ of an expression are the addends of the expression.

   (1) ◯ degree
   ◯ terms
   ◯ grouping symbols

4. In the statement, a property of real numbers has been incorrectly applied. Correct the right side of the statement.

   6(x + 3) = 6x + 3

   The correct statement is 6(x + 3) = ____________.

5. Fill in the chart.

Number	Opposite	Reciprocal
5	?	?

Number	Opposite	Reciprocal
5	(1) ________	(2) ________

(1) ◯ -5 ◯ 0 ◯ $\frac{1}{5}$ ◯ $-\frac{1}{5}$ ◯ undefined

(2) ◯ -5 ◯ $-\frac{1}{5}$ ◯ 0 ◯ $\frac{1}{5}$ ◯ undefined

6. Fill in the chart.

Number	Opposite	Reciprocal
?	6	?

Number	Opposite	Reciprocal
(1) ________	6	(2) ________

(1) ◯ $-\frac{1}{6}$ ◯ $\frac{1}{6}$ ◯ 6 ◯ Undefined ◯ -6

(2) ◯ -6 ◯ Undefined ◯ 6 ◯ $-\frac{1}{6}$ ◯ $\frac{1}{6}$

7. Use the commutative property of addition to write an expression equivalent to the following.

$10a + b$

The answer is ________.

8. Use the distributive property to find the product of the following.

$8(x + 1)$

$8(x + 1) =$ ________ (Simplify your answer.)

9. Use the distributive property to find the product of the following.

$-(3x + y)$

The answer is ________.

10. Use the distributive property to multiply.

$3(4x + 5y + 3z)$

$3(4x + 5y + 3z) =$ ________

11. Use the distributive property to find the product.

$$-6(x-2y+9)$$

$-6(x-2y+9) =$ ____________
(Simplify your answer.)

12. Simplify.

$$-6+7x+14-12x$$

$-6+7x+14-12x =$ ____________

13. Simplify the following expression.

$$7y-6+19y-17y$$

$7y-6+19y-17y =$ ____________

14. Simplify.

$$8k-(4k-18)$$

$8k-(4k-18) =$ ____________

15. Simplify the expression.

$$-9c-(4-2c)$$

$-9c-(4-2c) =$ ____________

16. Simplify the following expression.

$$(12-11y)-(12+17y)$$

$(12-11y)-(12+17y) =$ ____________

17. Simplify.

$$4(xy-3)+xy+18-x^2$$

$4(xy-3)+xy+18-x^2 =$ ____________

18. Simplify.

$$-(n+1)+(2n-2)$$

$-(n+1)+(2n-2) =$ ____________

19. Simplify the expression.

$$9(10n^2-2)-5(18n^2+6)$$

$9(10n^2-2)-5(18n^2+6) =$ ____________
(Use integers or fractions for any numbers in the expression.)

20. Simplify.

$$\frac{7}{9}b - \frac{1}{5} + \frac{8}{15}b - \frac{1}{3}$$

$\frac{7}{9}b - \frac{1}{5} + \frac{8}{15}b - \frac{1}{3} =$ ____________

(Use integers or fractions for any numbers in the expression.)

---

21. Simplify the following expression.

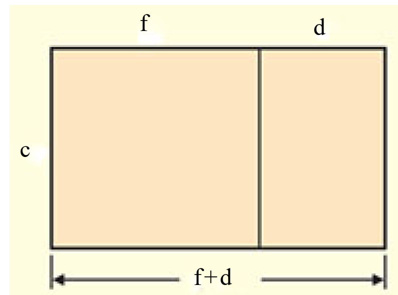
$$\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y)$$

$\frac{1}{3}(27x - 18) - \frac{1}{4}(20x - 3y) =$ ____________

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

---

22. To demonstrate the distributive property geometrically, represent the area of the larger rectangle in two ways, first as width times length and second as the sum of the areas of the smaller rectangles.



The area of the larger rectangle obtained by multiplying width times length is ____________.

(Do not simplify.)

The area of the larger rectangle obtained by finding the sum of the areas of the two smaller rectangles is ____________. (Simplify your answer.)

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #7

1. State the base of the exponent 8 in the expression.

   $(-6)^8$

   The base of the exponent 8 is ____________.

2. State the base of the exponent 4 in the expression.

   -8^4

   The base of the exponent 4 is ____________.

3. State the base of the exponent 7 in the expression.

   cx^7

   The base of the exponent 7 is ____________.

4. Select the answer that best completes the given statement.

   A(n) (1) ____________ is a shorthand notation for repeated multiplication of the same number.

   (1) ○ absolute value
   ○ square root
   ○ base
   ○ exponent

5. Select the correct choices that complete the sentence below.

   In $(-5)^2$, the 2 is the (1) ____________ and the -5 is the (2) ____________

   (1) ○ exponent ○ base
   (2) ○ exponent. ○ base.

6. Evaluate.

   -9^2

   $-9^2 =$ ____________

7. Evaluate.

   $(-2)^2$

   $(-2)^2 =$ ____________

8. Find the value of the expression.

$$\left(-\frac{1}{10}\right)^3$$

$\left(-\frac{1}{10}\right)^3 =$ ____________
(Simplify your answer.)

9. Write the expression with positive exponents.

$$4a^{-1}u^{-3}$$

$4a^{-1}u^{-3} =$ ____________ (Simplify your answer.)

10. Write the expression with positive exponents.

$$a^3b^{-1}c^{-9}$$

$a^3b^{-1}c^{-9} =$ ____________ (Simplify your answer.)

11. Simplify. Use positive exponents for any variables. Assume that all bases are not equal to 0.

$$\frac{p^{-5}}{q^{-7}}$$

$\frac{p^{-5}}{q^{-7}} =$ ____________ (Simplify your answer.)

12. Evaluate the following. Assume that all bases are not equal to 0.

$$(-2x+8)^0$$

$(-2x+8)^0 =$ ____________

13. Evaluate the expression. Assume that all bases are not equal to 0.

$$-5x^0$$

$-5x^0 =$ ____________
(Simplify your answer.)

14. Evaluate the expression. Assume that all bases are not equal to 0.

$$3x^0+5$$

$3x^0+5 =$ ____________
(Simplify your answer.)

15. Simplify. Use positive exponents for any variables.

9^{-2}

$9^{-2} =$ ____________ (Type an integer or a simplified fraction.)

16. Simplify. Use positive exponents for any variables.

$(-3)^{-3}$

$(-3)^{-3} =$ ____________
(Type an integer or a fraction.)

17. Simplify. Use positive exponents for any variables. Assume that all bases are not equal to 0.

$9x^{-2}$

$9x^{-2} =$ ____________ (Simplify your answer.)

18. Simplify. Use positive exponents for any variables. Assume that all bases are not equal to 0.

$4^{0} - 3x^{0}$

$4^{0} - 3x^{0} =$ ____________

19. Simplify. Use positive exponents for any variables.

$3^{-1} + 2^{-2}$

$3^{-1} + 2^{-2} =$ ____________
(Type an integer or a simplified fraction.)

20. Simplify. Use positive exponents for any variables.

$5^{-2} \bullet y$

$5^{-2} \bullet y =$ ____________
(Simplify your answer. Use integers or fractions for any numbers in the expression.)

Student: ____________________	**Instructor:** Deb Wertz	**Assignment:** Homework #8
Date: ____________________	**Course:** MAP102 MASTER	

1. Use the quotient rule for exponents to simplify.

$$\frac{y^{17}}{y^4}$$

$\frac{y^{17}}{y^4} =$ ____________
(Type your answer using exponential notation. Use positive exponents only.)

2. Use the quotient rule to simplify.

$$-\frac{12z^{12}}{6z^9}$$

$-\frac{12z^{12}}{6z^9} =$ ____________
(Type your answer using exponential notation.)

3. Use the quotient rule to simplify.

$$\frac{x^6y^7}{x^2y^7}$$

$\frac{x^6y^7}{x^2y^7} =$ ____________
(Type your answer using exponential notation.)

4. Simplify. Use positive exponents for any variables.

$$\frac{x^9}{x^{13}}$$

$\frac{x^9}{x^{13}} =$ ____________ (Type exponential notation with positive exponents.)

5. Simplify. Use positive exponents for any variables.

$$\frac{10r^6}{2r^{-4}}$$

$\frac{10r^6}{2r^{-4}} =$ ____________ (Type exponential notation with positive exponents.)

6. Simplify. Use positive exponents for any variables.

$$\frac{4x^{-7}x^{3}}{x^{-4}}$$

$\frac{4x^{-7}x^{3}}{x^{-4}} =$ ____________

7. Simplify. Use positive exponents for any variables.

$$\frac{4a^{-6}b^{5}}{20a^{2}b^{-3}}$$

$\frac{4a^{-6}b^{5}}{20a^{2}b^{-3}} =$ ____________

(Use integers or fractions for any numbers in the expression. Type exponential notation with positive exponents.)

8. Simplify. Use positive exponents for any variables. Assume that all bases are not equal to 0.

$$-8x^{-4}$$

$-8x^{-4} =$ ____________ (Simplify your answer.)

9. Simplify. Use positive exponents for any variables.

$$\left(-5x^{2}y\right)\left(4x^{5}\right)\left(-2xy^{4}\right)$$

$\left(-5x^{2}y\right)\left(4x^{5}\right)\left(-2xy^{4}\right) =$ ____________

(Type exponential notation with positive exponents.)

10. Simplify. Use positive exponents for any variables.

$$\frac{6x^{-6}yz^{-7}}{2x^{5}yz}$$

$\frac{6x^{-6}yz^{-7}}{2x^{5}yz} =$ ____________

(Simplify your answer. Type exponential notation with positive exponents.)

11. Simplify. Assume that the variable in the exponent represents a nonzero integer and that x is not 0.

$$x^{6} \cdot x^{6a}$$

$x^{6} \cdot x^{6a} =$ ____________

12. Simplify. Assume that variable in the exponents represents nonzero integer and that x is not 0.

$$\frac{x^{9t-3}}{x^{t}}$$

$\frac{x^{9t-3}}{x^{t}} =$ ____________

13. Use the power rule to simplify the expression.

$$\left(n^{4}\right)^{3}$$

$\left(n^{4}\right)^{3} =$ ____________

14. Simplify.

$$\left(g^{-8}\right)^{-7}$$

$\left(g^{-8}\right)^{-7} =$ ____________

(Simplify your answer. Type exponential notation using positive exponents.)

15. Simplify.

$$\left(3^{-1}\right)^{3}$$

$\left(3^{-1}\right)^{3} =$ ____________ (Type an integer or a simplified fraction. Use positive exponents only.)

16. Simplify. Write the answer using positive exponents only.

$$\left(5x^{8}y^{9}\right)^{3}$$

$\left(5x^{8}y^{9}\right)^{3} =$ ____________

17. Simplify. Write each answer using positive exponents only.

$$\left(4a^{2}bc^{-6}\right)^{-3}$$

$\left(4a^{2}bc^{-6}\right)^{-3} =$ ____________

18. Simplify.

$$\left(\frac{x^{2}y^{-7}}{z^{-1}}\right)^{-2}$$

$\left(\frac{x^{2}y^{-7}}{z^{-1}}\right)^{-2} =$ ____________ (Use positive exponents only.)

19. Simplify.

$$\left(\frac{4}{5}\right)^{-3}$$

$\left(\frac{4}{5}\right)^{-3}$ = ____________ (Type an integer or a fraction.)

20. Simplify.

$$\left(\frac{2x^4}{4x^2}\right)^3$$

$\left(\frac{2x^4}{4x^2}\right)^3$ = ____________

(Type an integer or a simplified fraction. Use positive exponents only.)

21. Simplify.

$$x^7\left(x^7bc\right)^{-5}$$

$x^7\left(x^7bc\right)^{-5}$ = ____________ (Use positive exponents only.)

22. Simplify.

$$\frac{2^{-2}x^2y^{-5}}{5^{-2}x^7y^{-1}}$$

$\frac{2^{-2}x^2y^{-5}}{5^{-2}x^7y^{-1}}$ = ____________

(Type the ratio as a simplified fraction. Use positive exponents only.)

23. Simplify the following. Assume that variables in the exponents represent integers and that all other variables are not 0.

$$\left(x^{3a+7}\right)^2$$

$\left(x^{3a+7}\right)^2$ = ____________ (Simplify your answer.)

24. Simplify the expression.

$$-7x-(6x-3)$$

$-7x-(6x-3)$ = ____________ (Simplify your answer.)

Student: ____________________	**Instructor:** Deb Wertz	**Assignment:** Homework #9
Date: ____________________	**Course:** MAP102 MASTER	

1. Fill in the blank.

   The numerical factor of a term is the _________.

   The numerical factor of a term is the (1) ____________

   (1)
   - ◯ coefficient.
   - ◯ degree.

2. Select the correct choice that completes the sentence below.

   A(n) (1) ____________ is a finite sum of terms in which all variables are raised to nonnegative integer powers and no variables appear in any denominator.

   (1)
   - ◯ polynomial
   - ◯ equation
   - ◯ coefficient

3. Fill in the blank.

   A (1) ____________ is a polynomial with exactly two terms.

   (1)
   - ◯ binomial
   - ◯ monomial
   - ◯ trinomial
   - ◯ constant

4. Select the correct choice that completes the sentence below.

   A (1) ____________ is a polynomial with 1 term.

   (1)
   - ◯ binomial
   - ◯ monomial
   - ◯ trinomial

5. Fill in the blank.

   A (1) ____________ is a polynomial with exactly three terms.

   (1)
   - ◯ binomial
   - ◯ monomial
   - ◯ trinomial
   - ◯ constant

6. Fill in the blank.

The __________ of a polynomial is the largest degree of all its terms.

The (1) __________ of a polynomial is the largest degree of all its terms.

(1) ○ degree
○ coefficient

7. Select the correct choice that completes the sentence below.

(1) __________ terms contain the same variables raised to the same powers.

(1) ○ Like
○ Unlike

8. Find the degree of the given term.

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The degree is __________.

9. Find the degree of the polynomial and indicate whether the polynomial is a monomial, binomial, trinomial, or none of these.

$6x + 0.7$

Classify the given polynomial.

○ monomial
○ binomial
○ trinomial
○ none of these

The degree of the polynomial is __________.

10. Classify the polynomial as a monomial, binomial, trinomial, or none of these. Also, give the degree.

$x^2 - 16x + 64$

Choose the correct type of polynomial.

○ Trinomial
○ Binomial
○ Monomial
○ None of these

What is the degree of the polynomial?

The degree is __________.

11. Simplify by combining like terms.

$$9y + 8y - 7y^2 - 2y^2$$

$9y + 8y - 7y^2 - 2y^2 =$ ____________

12. Simplify by combining like terms.

$$-6x^2y + 8x - 5x^2y - \frac{1}{3} - 4x$$

$-6x^2y + 8x - 5x^2y - \frac{1}{3} - 4x =$ ____________

Student: _______________ **Date:** _______________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #10

1. If $P(x) = x^2 + x + 3$ and $Q(x) = 6x^2 - 3$, find $P(7)$.

$P(7) =$ ____________
(Type an integer or a fraction.)

2. If $P(x) = x^2 + x + 6$ and $Q(x) = 4x^2 - 1$, find $Q(-10)$.

$Q(-10) =$ ____________
(Type an integer or a fraction.)

3. If $P(x) = x^2 + x + 2$ and $Q(x) = 71x^2 - 1$, find $Q\left(\frac{1}{9}\right)$.

$Q\left(\frac{1}{9}\right) =$ ____________
(Type an integer or a fraction.)

4. An object is dropped from the top of a tower with a height of 1130 feet. Neglecting air resistance, the height of the object at time t seconds is given by the polynomial $-16t^2 + 1130$. Find the height of the object at $t = 8$ seconds.

The height of the object at 8 seconds is ____________ feet.

5. Add.

$$\left(9y^2 + y - 8\right) + \left(6y^2 - y - 5\right)$$

$\left(9y^2 + y - 8\right) + \left(6y^2 - y - 5\right) =$ ____________
(Simplify your answer.)

6. Add.

$$\left(8x^3y - 7xy + 3\right) + \left(7x^3y + 7xy + 3x\right)$$

$\left(8x^3y - 7xy + 3\right) + \left(7x^3y + 7xy + 3x\right) =$ ____________ (Simplify your answer.)

7. Subtract.

$$\left(2y^2 - 9y + 4\right) - \left(4y^2 - 9y + 9\right)$$

$\left(2y^2 - 9y + 4\right) - \left(4y^2 - 9y + 9\right) =$ ____________ (Simplify your answer. Do not factor.)

8. Perform the indicated operation.

$$(9x^3 + 9x^2 - 10x + 8) - (-11x^3 - 11x^2 - 3x + 3)$$

$(9x^3 + 9x^2 - 10x + 8) - (-11x^3 - 11x^2 - 3x + 3) =$ ____________
(Simplify your answer. Do not factor.)

9. Perform the subtraction and simplify.

$$\left(7x^2+3x+5\right)-\left(3x^2-5\right)$$

$\left(7x^2+3x+5\right)-\left(3x^2-5\right)=$ ____________

10. Perform the subtraction and simplify.

$$\left(14ab-11a^2b+2b^2\right)-\left(18a^2-19a^2b-2b^2\right)$$

$\left(14ab-11a^2b+2b^2\right)-\left(18a^2-19a^2b-2b^2\right)=$ ____________
(Do not factor.)

11. Perform the indicated operations and simplify.

$$\left(8x^2-7\right)+\left(-4x^2-2\right)-\left(4x^2-9\right)$$

$\left(8x^2-7\right)+\left(-4x^2-2\right)-\left(4x^2-9\right)=$ ____________

12. Subtract.

$$\left(\frac{3}{4}x^2-\frac{6}{7}x+\frac{2}{3}\right)-\left(\frac{1}{4}x^2+\frac{1}{14}x-\frac{1}{6}\right)$$

$\left(\frac{3}{4}x^2-\frac{6}{7}x+\frac{2}{3}\right)-\left(\frac{1}{4}x^2+\frac{1}{14}x-\frac{1}{6}\right)=$ ____________
(Use integers or fractions for any numbers in the expression. Simplify your answer. Do not factor.)

13. For the following pair of functions, find $P(x)+Q(x)$.

$P(x)=3x+5$ and $Q(x)=6x^2-7x+2$

$P(x)+Q(x)=$ ____________ (Simplify your answer. Do not factor.)

14. For the following polynomial, find $P(a)$, $P(-x)$ and $P(x+h)$.

$P(x)=3x-7$

$P(a)=$ ____________ (Simplify your answer. Do not factor.)

$P(-x)=$ ____________ (Simplify your answer. Do not factor.)

$P(x+h)=$ ____________ (Simplify your answer. Do not factor.)

15. For the following polynomial, find $P(a)$, $P(-x)$ and $P(x+h)$.

$P(x)=6x-7$

$P(a)=$ ____________ (Simplify your answer. Do not factor.)

$P(-x)=$ ____________ (Simplify your answer. Do not factor.)

$P(x+h)=$ ____________ (Simplify your answer. Do not factor.)

16. Complete the expression.

$(x + 18)^2 =$ ______

Choose the correct answer below.

- **A.** $(x + 18)^2 = x^2 - 324$
- **B.** $(x + 18)^2 = x^2 + 18x + 324$
- **C.** $(x + 18)^2 = x^2 + 324$
- **D.** $(x + 18)^2 = x^2 + 36x + 324$

---

17. Choose the product of $(x + 3)(x - 3)$ from the following list.

$x^2 + 3x - 9$	$x^2 + 6x - 9$
$x^2 + 9$	$x^2 - 9$

Choose the correct answer below.

- **A.** $x^2 - 9$
- **B.** $x^2 + 9$
- **C.** $x^2 + 3x - 9$
- **D.** $x^2 + 6x - 9$

---

18. Select the correct choice that completes the sentence below.

The product of $(3x - 1)(4x^2 - 2x + 1)$ is a polynomial of degree (1) ______________

(1)
- $12x^3$.
- 12.
- 3.
- 2.

---

19. Fill in the blank.

If $f(x) = x^2 + 9$, then $f(a + 4) =$ _____.

$f(a + 4) =$ (1) ______________

(1)
- $a + 4$
- $(a + 4)^2$
- $(a + 4)^2 + (a + 4)$
- $(a + 4)^2 + 9$

---

20. Select the correct choice that completes the sentence below.

$[x+(2y+1)]^2 = $ (1) ____________

(1) ○ $[x+(2y+1)]\,[x-(2y+1)]$
○ $[x+(2y+1)]\,[x+(2y+1)]$
○ $[x+(2y+1)]\,[x+(2y-1)]$

---

21. Multiply.

$-6xy(3x+y)$

$-6xy(3x+y) = $ ____________ (Simplify your answer.)

---

22. Multiply.

$3ab\left(xa^2+ya^7+5\right)$

$3ab\left(xa^2+ya^7+5\right) = $ ____________

---

23. Multiply.

$(a-3)(2a+5)$

$(a-3)(2a+5) = $ ____________ (Simplify your answer.)

---

24. Multiply.

$(-6x+2)\left(x^3-x-5\right)$

$(-6x+2)\left(x^3-x-5\right) = $ ____________ (Simplify your answer.)

---

25. Multiply.

$(x+3)^2$

$(x+3)^2 = $ ____________ (Simplify your answer.)

---

26. Multiply using the rule for the product of the sum and difference of two terms.

$(6x+7)(6x-7)$

$(6x+7)(6x-7) = $ ____________

---

27. Multiply using special product methods.

$(8x-y)^2$

$(8x-y)^2 = $ ____________
(Simplify your answer.)

28. Use special products to multiply.

$$\left(3x+\frac{1}{2}\right)\left(3x-\frac{1}{2}\right)$$

$\left(3x+\frac{1}{2}\right)\left(3x-\frac{1}{2}\right) =$ ____________

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

---

29. Multiply.

$$\left(5x^3+3\right)\left(7x^2+3x+5\right)$$

$\left(5x^3+3\right)\left(7x^2+3x+5\right) =$ ____________

(Simplify your answer.)

---

30. If $f(x)=x^2-15x$, find the following.

$f(a+h)$

$f(a+h) =$ ____________ (Simplify your answer.)

---

31. If $f(x)=x^2-5x$, find $f(b-9)$.

$f(b-9) =$ ____________

---

32. Find the greatest common factor for the list of terms.
x^3, x^6, x^8

The greatest common factor is ____________.

---

33. Find the greatest common factor for the list of monomials.

$$x^5y^5z^4,\ y^2z^4,\ xy^2z^3$$

The GCF is ____________.

(Simplify your answer.)

---

34. Find the greatest common factor for the list of monomials.

$$42x^4y^3z,\ 21xy^3,\ 84x^3y^4$$

The greatest common factor is ____________.

---

35. Factor out the GCF in the polynomial.

$$12x-18$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $12x-18 =$ ____________
- **B.** The polynomial has no common factor other than 1.

36. Factor out the greatest common factor from the following polynomial.

$$5y^2 - 30xy^3$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $5y^2 - 30xy^3 =$ ______ (Type your answer in factored form.)
- **B.** The polynomial has no common factor other than 1.

37. The amount E of voltage in an electrical circuit is given by the formula $IR_1 + IR_2 = E$. Write an equivalent equation by factoring the expression $IR_1 + IR_2$.

The equivalent equation is ______ = E.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #11

1. Fill in the blank.

A (1) ____________ is an expression that can be written in the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$.

(1) ◯ simplified expression ◯ trinomial
◯ rational expression
◯ fraction
◯ binomial

2. Select the correct choice that completes the sentence below.

A rational expression is undefined if the denominator is (1) ____________

(1) ◯ -1.
◯ 1.
◯ 0.

3. Simplify the rational expression.

$$\frac{5x - 30x^2}{5x}$$

$\frac{5x - 30x^2}{5x} =$ ____________

4. Simplify the rational expression.

$$\frac{x^2 - 16}{4 + x}$$

$\frac{x^2 - 16}{4 + x} =$ ____________

5. Simplify the rational expression.

$$\frac{6y - 18}{5y - 15}$$

$\frac{6y - 18}{5y - 15} =$ ____________

6. Select the correct choice that completes the sentence below.

A rational expression is (1) ______________ if the numerator and denominator have no common factors other than 1 or -1.

(1)
- ○ simplified
- ○ linear
- ○ a polynomial

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #12

1. Multiply.

$$\frac{15x+15}{8x+24} \cdot \frac{4x+12}{5x^2-5}$$

$\frac{15x+15}{8x+24} \cdot \frac{4x+12}{5x^2-5} =$ __________ (Simplify your answer.)

2. Multiply and simplify.

$$\frac{18a-12a^2}{4a^2+12a+9} \cdot \frac{4a^2+12a+9}{4a^2-9}$$

$\frac{18a-12a^2}{4a^2+12a+9} \cdot \frac{4a^2+12a+9}{4a^2-9} =$ __________

3. Divide and simplify.

$$\frac{4x}{9} \div \frac{16x+32}{9x+18}$$

$\frac{4x}{9} \div \frac{16x+32}{9x+18} =$ __________

4. Divide and simplify.

$$\frac{a+b}{ab} \div \frac{a^2-b^2}{4a^3b}$$

$\frac{a+b}{ab} \div \frac{a^2-b^2}{4a^3b} =$ __________

5. Perform each indicated operation.

$$\frac{5}{x} \div \frac{4xy}{x^2} \cdot \frac{16x^3}{x^5}$$

$\frac{5}{x} \div \frac{4xy}{x^2} \cdot \frac{16x^3}{x^5} =$ __________ (Simplify your answer.)

6. Find the function value.

If $f(x) = \dfrac{x+8}{2x-1}$, find f(6), f(0), and f(− 5).

$f(6) =$ ____________ (Type an integer or a simplified fraction.)

$f(0) =$ ____________ (Type an integer or a simplified fraction.)

$f(-5) =$ ____________ (Type an integer or a simplified fraction.)

---

7. Find each function value. If $g(x) = \dfrac{x^2+8}{x^3-25x}$, find g(3), g(− 2), and g(2).

$g(3) =$ ____________
(Type an integer or a simplified fraction.)

$g(-2) =$ ____________
(Type an integer or a simplified fraction.)

$g(2) =$ ____________
(Type an integer or a simplified fraction.)

---

8. Which of the expressions are equivalent to $\dfrac{x}{7-x}$?

Select all equivalent expressions.

☐ **A.** $\dfrac{-x}{-7+x}$

☐ **B.** $\dfrac{-x}{x-7}$

☐ **C.** $\dfrac{x}{x-7}$

☐ **D.** $\dfrac{-x}{7-x}$

---

9. Fill in the blank.

The denominators must be the same before performing the operations ______.

The denominators must be the same before performing the operations (1) ____________

(1) ○ multiplication and division.
○ addition and subtraction.

10. Name the operation(s) that make the statement true.

To perform this operation, multiply the first rational expression by the reciprocal of the second rational expression.

Choose the correct answer below.

- ○ Addition
- ○ Addition, Subtraction
- ○ Subtraction
- ○ Division
- ○ Multiplication
- ○ Division, Subtraction
- ○ Division, Multiplication
- ○ Addition, Multiplication

11. Fill in the blank.

Numerator times numerator all over denominator times denominator is ________.

Numerator times numerator all over denominator times denominator is (1) ______________

(1)
- ○ addition.
- ○ subtraction.
- ○ division.
- ○ multiplication.

12. Use the example in the hint to perform the following subtraction.

$$\frac{7}{2x} - \frac{x+1}{2x} = \underline{\qquad\qquad}$$

$$\text{Hint: } \frac{8}{x+1} - \frac{x+5}{x+1} = \frac{\overset{\downarrow}{8} - (\overset{\downarrow}{x}+5)}{x+1} = \frac{3-x}{x+1}$$

$\frac{7}{2x} - \frac{x+1}{2x} =$ ______________ (Simplify your answer.)

13. Subtract fractions. Simplify the answer.

$$\frac{x-6}{6x} - \frac{x+6}{6x}$$

$\frac{x-6}{6x} - \frac{x+6}{6x} =$ ______________

14. Find the sum.

$$\frac{4}{9x} + \frac{7}{5x}$$

$\frac{4}{9x} + \frac{7}{5x} =$ ______________

(Simplify your answer.)

15. Subtract fractions. Simplify the answer.

$$\frac{7}{2y^2} - \frac{2}{5y}$$

$\frac{7}{2y^2} - \frac{2}{5y} =$ ______________

16. Perform the indicated operation.

$$\frac{x-2}{x+4} - \frac{x+7}{x-4}$$

$\frac{x-2}{x+4} - \frac{x+7}{x-4} =$ ______________ (Simplify your answer.)

17. Add.

$$\frac{9}{4x+8} + \frac{16}{3x+6}$$

$\frac{9}{4x+8} + \frac{16}{3x+6} =$ ______________ (Simplify your answer.)

Student: ____________________ **Instructor:** Deb Wertz **Assignment:** Homework #13
Date: ____________________ **Course:** MAP102 MASTER

1. Complete the following table. The first row has been completed.

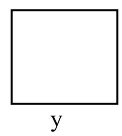
	First Integer	All Described Integers
Three consecutive integers	18	18, 19, 20
Three consecutive odd integers	23	________

	First Integer	All Described Integers
Three consecutive integers	18	18, 19, 20
Three consecutive odd integers	23	____________, ____________, ____________

(Use ascending order.)

2. Write the following as an algebraic expression. Then simplify.

The perimeter of the square with side length y.



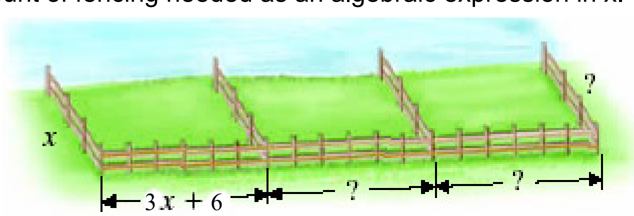
The answer is ____________. (Simplify your answer.)

3. Write the following as an algebraic expression. Then simplify.

The sum of four consecutive integers if the first integer is x.

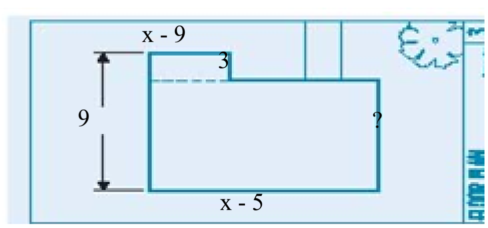
The answer is ____________. (Type a simplified expression.)

4. A piece of land is to be fenced and subdivided as shown so that each rectangle has the same dimensions. Express the total amount of fencing needed as an algebraic expression in x.



The total amount of fencing is ____________.

5. Write the perimeter of the floor plan shown as an algebraic expression in x.



The perimeter of the floor is ____________.
(Simplify your answer.)

6. Write the following as an algebraic expression. Then simplify.

The total amount of money (in cents) in x dimes, (x + 5) nickels, and 3x quarters. (Hint: The value of a dime is 10 cents, the value of a nickel is 5 cents, and the value of a quarter is 25 cents.)

The total amount of money is ____________ cents.
(Simplify your answer. Do not factor.)

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #14

1. Select the correct choice that completes the sentence below.

   A value for the variable in an equation that makes the equation a true statement is called a(n) (1) ____________ of the equation.

   (1) ◯ slope
   ◯ solution

---

2. Identify the following as an equation or an expression.

   $\frac{1}{3}x - 5$

   Choose the correct answer below.

   ◯ **A.** $\frac{1}{3}x - 5$ is an equation.

   ◯ **B.** $\frac{1}{3}x - 5$ is an expression.

---

3. Identify the following as an equation or an expression.

   $2(x-3) = 7$

   Choose the correct answer below.

   ◯ **A.** It is an equation, because it contains the difference of two terms.
   ◯ **B.** It is an expression, because it contains a variable.
   ◯ **C.** It is an expression, because it contains the difference of two terms.
   ◯ **D.** It is an equation, because it contains an equal sign.

---

4. Identify the following as an equation or an expression.

   $\frac{5}{9}x + \frac{1}{3} = \frac{2}{9} - x$

   Choose the correct answer below.

   ◯ **A.** $\frac{5}{9}x + \frac{1}{3} = \frac{2}{9} - x$ is an expression.

   ◯ **B.** $\frac{5}{9}x + \frac{1}{3} = \frac{2}{9} - x$ is an equation.

---

5. Identify the following as an equation or an expression.

$$\frac{5}{9}x + \frac{1}{3} - \frac{2}{9} - x$$

Choose the correct answer below.

- **A.** It is an expression, because it contains the sum and difference of terms, and does not contain an equal sign.
- **B.** It is an equation, because it does not contain an equal sign.
- **C.** It is an expression, because it contains a variable.
- **D.** It is an equation, because it contains the sum and difference of terms.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #15

1. Solve the equation and check.

   $-7x = -42$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ **A.** The solution is ____________ .
   (Simplify your answer.)
   ○ **B.** The solution is all real numbers.
   ○ **C.** There is no solution.

2. Solve the equation and check.

   $-18 = x + 8$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ **A.** The solution is ____________ .
   (Simplify your answer.)
   ○ **B.** The solution is all real numbers.
   ○ **C.** There is no solution.

3. Solve the equation and check.

   $7x - 4 = 6 + 5x$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ **A.** The solution is ____________ .
   (Simplify your answer.)
   ○ **B.** The solution is all real numbers.
   ○ **C.** There is no solution.

4. Solve the equation and check.

   $6y + 16 = 3y - 5$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ **A.** The solution is ____________ .
   (Simplify your answer.)
   ○ **B.** The solution is all real numbers.
   ○ **C.** There is no solution.

5. Solve the equation and check.

$$8x - 5x + 3 = x - 7 + 10$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ______.
  (Simplify your answer.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

6. Solve the equation and check.

$$17x + 10 = 4(4x + 3)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ______.
  (Simplify your answer.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

7. Solve the equation and check.

$$-3(6y - 9) - y = -3(y - 2)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ______.
  (Type an integer or a simplified fraction.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

8. Solve the following equation and check.

$$\frac{x}{4} + \frac{x}{7} = \frac{7}{8}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ______.
  (Type an integer or a simplified fraction.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

9. Solve the equation and check.

$$\frac{2x-5}{12}+x=\frac{2x+5}{2}+2$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ________.
  (Type an integer or a simplified fraction.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

---

10. Solve the equation.

$$\frac{1}{15}(a+2)=\frac{1}{6}(2-a)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ________.
  (Type an integer or a simplified fraction.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

---

11. Solve the equation.

$$6(n+4)=2(12+3n)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ________.
  (Simplify your answer.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

---

12. Solve the equation.

$$9(x+8)+2=9x+7$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution is ________.
  (Simplify your answer.)
- **B.** The solution is all real numbers.
- **C.** There is no solution.

Student: ______________________
Date: ______________________

Instructor: Deb Wertz
Course: MAP102 MASTER

Assignment: Homework #16

1. Solve the equation for y.

   $x + y = 9$

   y = ____________

2. One number is 2 times a first number. A third number is 100 more than the first number. If the sum of the three numbers is 208, find the numbers.

   The three numbers are ____________. (Use a comma to separate answers as needed.)

3. Solve the formula for the specified variable.

   $y = dg$ for d

   d = ____________

4. Solve $7x - 6y = 19$ for y.

   y = ____________ (Use integers or fractions for any numbers in the expression.)

5. Solve $P = 2G + 2M$ for G.

   G = ____________

6. A woman works at a law firm in city A which is about 70 miles from city B. She must go to the law library in city B to get a document. Find how long it takes her to drive round-trip if she averages 50 mph.

Translate the sentence into an equation. Use the distance formula, d = rt, where d = distance traveled, r = rate, and t = time. Fill in the blanks below.

Distance (round-trip)	equals	rate or speed	•	time
↓	↓	↓	↓	↓
______	=	______	•	t

What is the first step in solving the resulting equation for t?

- **A.** Add 50 to both sides of the equation.
- **B.** Multiply both sides of the equation by 50.
- **C.** Divide both sides of the equation by 50.
- **D.** Subtract 50 from both sides of the equation.

Divide both sides of the equation by 50 and simplify.

$$140 = 50t$$

______ = t

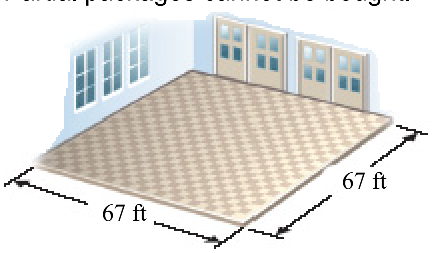
(Type an integer or a decimal.)

Interpret the result.

It takes her approximately ______ hours and ______ minutes to drive round-trip.

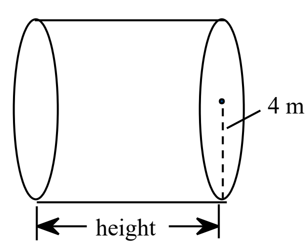
(Type a whole number.)

7. A package of floor tiles contains 26 one-foot-square tiles. Find how many packages should be bought to cover a square ballroom floor whose side measures 67 feet. Note: Partial packages cannot be bought.



______ packages should be bought to cover the floor.

8. The formula for the volume of a cylinder is $V = \pi r^2 h$. The cylinder to the right has an exact volume of 480π cubic meters. Find its height.



The height of the cylinder is ______ (1) ______

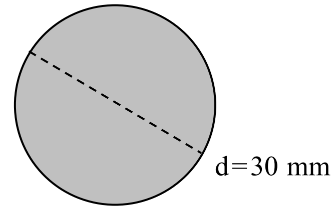
(Simplify your answer.)

(1)
- m.
- sq m.
- cu m.

9. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. The steel ball to the right is in the shape of a sphere and has a diameter of 30 millimeters.

a. Find the exact volume of the sphere.

b. Find a 2-decimal-place approximation for the volume.



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a. The exact volume of the sphere is ____________ (1) ____________

(Simplify your answer. Type an exact answer, using π as needed.)

b. The 2-decimal-place approximation for the volume is ____________ (2) ____________

(Type an integer or decimal rounded to two decimal places as needed.)

(1)		(2)	
○	mm.	○	sq mm.
○	cu mm.	○	mm.
○	sq mm.	○	cu mm.

Student: ____________________	**Instructor:** Deb Wertz	**Assignment:** Homework #17
Date: ____________________	**Course:** MAP102 MASTER	

1. Fill in the blank.

   The set $\{x \mid x \geq -1.1\}$ written in interval notation is __________.

   The set $\{x \mid x \geq -1.1\}$ written in interval notation is (1) ____________

   (1)
   - ○ $(-1.1,\infty)$.
   - ○ $[-1.1,\infty)$.
   - ○ $(-\infty,-1.1)$.
   - ○ $(-\infty,-1.1]$.

---

2. Use the choices to fill in the blank.

   The set $\{x \mid x < -2.1\}$ written in interval notation is ____.

   The set $\{x \mid x < -2.1\}$ written in interval notation is (1) ____________

   (1)
   - ○ $\left(-2.1,\infty\right)$.
   - ○ $\left(-\infty,-2.1\right)$.
   - ○ $\left[-2.1,\infty\right)$.
   - ○ $\left(-\infty,-2.1\right]$.

---

3. Fill in the blank.

   The set $\{x \mid x \leq 2.7\}$ written in interval notation is ________.

   The set $\{x \mid x \leq 2.7\}$ written in interval notation is (1) ____________

   (1)
   - ○ $\left(2.7,\infty\right)$.
   - ○ $\left(-\infty,2.7\right]$.
   - ○ $\left(-\infty,2.7\right)$.
   - ○ $\left[2.7,\infty\right)$.

---

4. Watch the section lecture video and answer the question listed below. Note: The counter in the lower right corner of the screen displays the Example number.

   Based on the lecture before Example 4, complete the following statement.

   To multiply or divide both sides of an inequality by (1) ____________ nonzero negative number(s), one must (2) ____________ the direction of the inequality symbol.

   (1)
   - ○ the same
   - ○ different

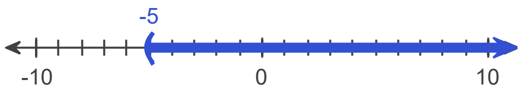
   (2)
   - ○ not change
   - ○ reverse

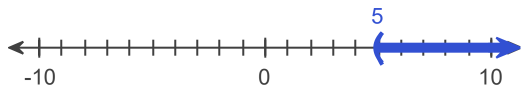
---

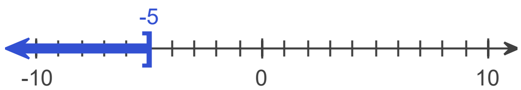
5. Graph the solution set of the inequality on a number line and then write it in interval notation.

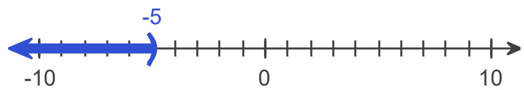
$\{x \mid x < -5\}$

Select the correct graph below.

- ◯ **A.** 

- ◯ **B.** 

- ◯ **C.** 

- ◯ **D.** 

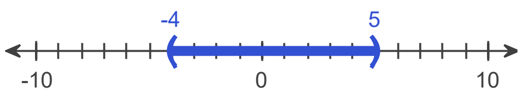
Now type the solution in interval notation.

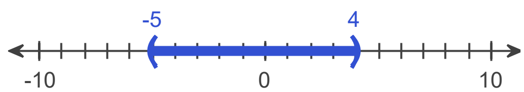
____

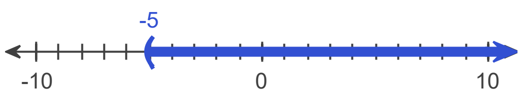
6. Graph the inequality on a number line. Then write the solution in interval notation.

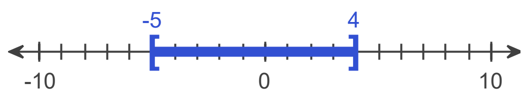
$\{x \mid -5 < x < 4\}$

Select the correct graph below.

- ◯ **A.** 

- ◯ **B.** 

- ◯ **C.** 

- ◯ **D.** 

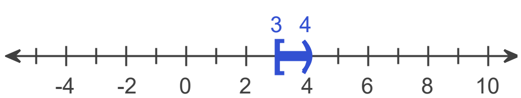
Now enter the solution in interval notation.

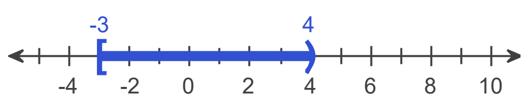
____

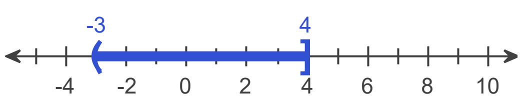
7. Graph the solution set of the inequality on a number line and then write it in interval notation.

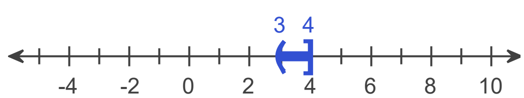
$\{x \mid 4 \geq x > -3\}$

What is the graph of the solution? Choose the correct graph below.

- ◯ **A.** 

- ◯ **B.** 

- ◯ **C.** 

- ◯ **D.** 

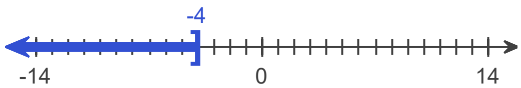
What is the solution set?

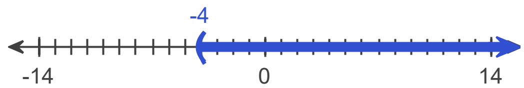
The solution set is __________. (Type your answer in interval notation.)

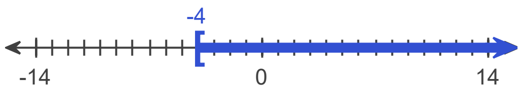
8. Solve the following inequality. Graph the solution set and write it in interval notation.

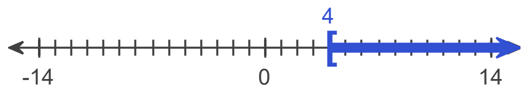
$$x - 4 \geq -8$$

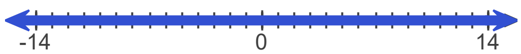
Select the correct graph below.

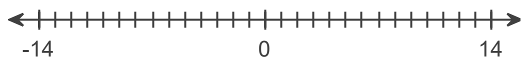
A. 

B. 

C. 

D. 

E. 

F. 

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

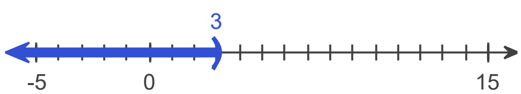
- **A.** The solution is ____________.
  (Type your answer in interval notation.)
- **B.** The solution is $\varnothing$.

---

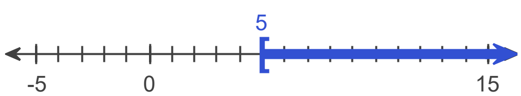
9. Solve the following inequality. Graph the solution set and write it in interval notation.

$$15x < 14x + 3$$

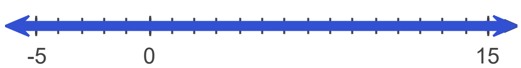
Choose the graph of the solution set.

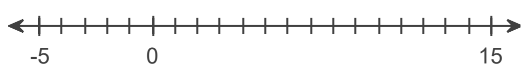
A. 

B. 

C. 

D. 

E. 

F. 

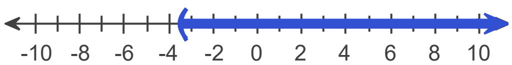
Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

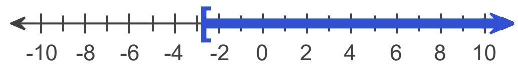
- **A.** The solution set is ____________.
  (Type your answer in interval notation.)
- **B.** The solution set is $\varnothing$.

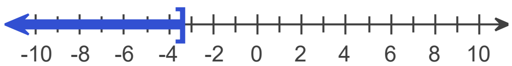
10. Solve the following inequality. Graph the solution set and write it in interval notation.

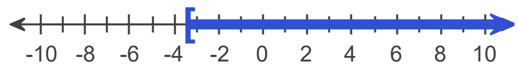
$$\frac{8}{9}x \geq -3$$

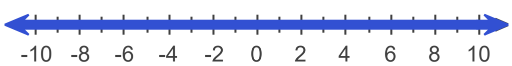
Select the correct graph below.

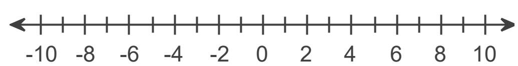
A. 

B. 

C. 

D. 

E. 

F. 

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

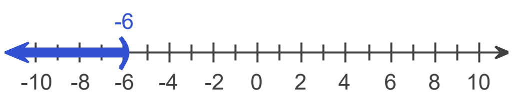
A. The solution is ________.
(Use integers or fractions for any numbers in the expression. Type your answer in interval notation.)

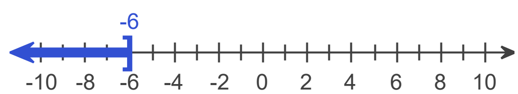
B. The solution is ∅.

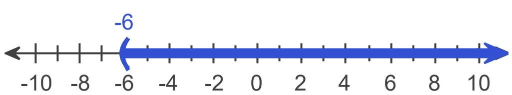
11. Solve the following inequality. Graph the solution set and then write it in interval notation.

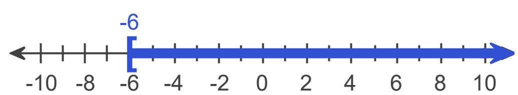
$$-4x \geq 24$$

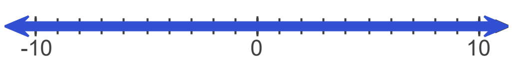
What is the graph of the solution?

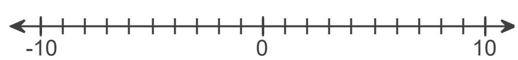
A. 

B. 

C. 

D. 

E. 

F. 

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is ________.
(Type your answer in interval notation.)

B. The solution set is ∅.

12. Solve the following inequality. Write the solution set using interval notation.

$21 + 7x \geq 3x - 7$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is ______.
  (Type your answer in interval notation. Use integers or fractions for any numbers in the expression. Simplify your answer.)
- B. The solution set is ∅.

13. Solve the following inequality. Write the solution set in interval notation.

$5(x - 6) < 3(2x - 1)$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is ______.
  (Simplify your answer. Type your answer in interval notation.).)
- B. The solution set is ∅.

14. Solve the following inequality. Write the solution set in interval notation.

$-3(2x - 1) < -2[5 + 4(x + 2)]$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is ______.
  (Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- B. The solution set is ∅.

15. Solve the following inequality. Write the solution set using interval notation.

$8 - (6x - 3) \geq -7(x + 1) - 7$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is ______.
  (Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- B. The solution set is ∅.

Student: ____________________ **Date:** ____________________

Instructor: Deb Wertz **Course:** MAP102 MASTER

Assignment: Homework #18

1. Solve the absolute value equation.

$|x| = 16$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\quad\}$.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- **B.** The solution set is $\varnothing$.

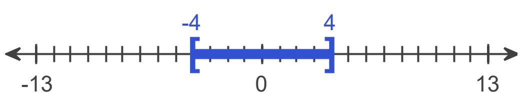
2. Solve the following inequality. Then graph the solution set.

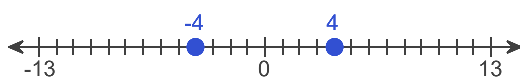
$|x| \le 4$

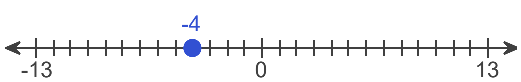
Select the correct choices below, and, if necessary, fill in the answer box to complete your choice.

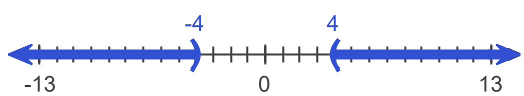
- **A.** The solution is an interval. The solution is ______.
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- **B.** The solution set is one or two points. The solution set is $\{\quad\}$.
  (Type an integer or a fraction. Use a comma to separate answers as needed.)
- **C.** The solution set is $\varnothing$.

Choose the correct graph below.

A.


B.


C.


D.


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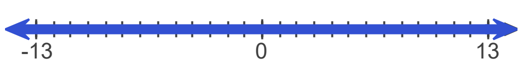
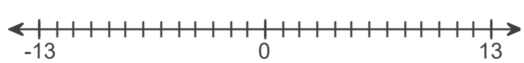


F.


3. Solve the absolute value equation.

$|2x - 11| = 17$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\quad\}$.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- **B.** The solution set is $\varnothing$.

4. Solve the absolute value equation.

$$\left|\frac{x}{4} - 3\right| = 1$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is { }.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- B. The solution set is ∅.

5. Solve the absolute value equation.

$$|7n + 2| + 15 = 5$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is { }.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- B. The solution set is ∅.

6. Solve the absolute value equation.

$$\left|\frac{2x - 5}{3}\right| = 9$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is { }.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- B. The solution set is ∅.

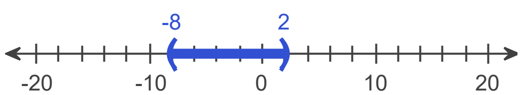
7. Solve the following inequality. Then graph the solution set.

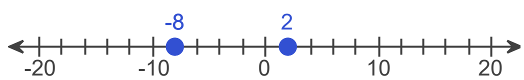
$|x+3| < 5$

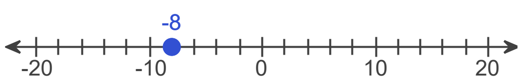
Select the correct choices below, and, if necessary, fill in the answer box to complete your choice.

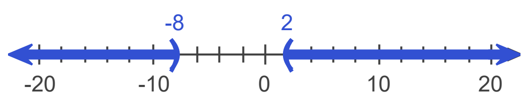
- **A.** The solution is an interval. The solution is __________.
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- **B.** The solution set is one or two points. The solution set is $\{\quad\}$.
  (Type an integer or a fraction. Use a comma to separate answers as needed.)
- **C.** The solution set is $\varnothing$.

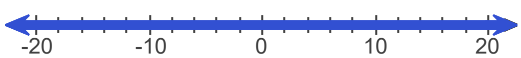
Choose the correct graph below.

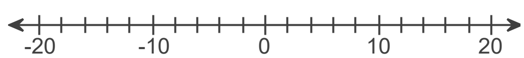
A. 

B. 

C. 

D. 

E. 

F. 

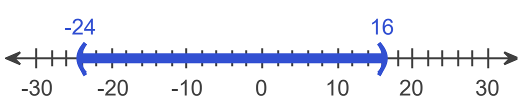
8. Solve the following inequality and graph the solution set.

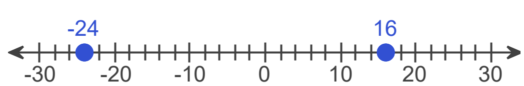
$|x+4| \geq 20$

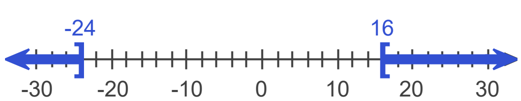
Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

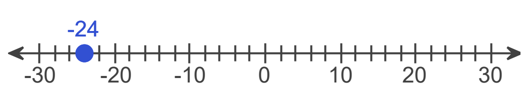
- **A.** The solution is an interval. The solution is __________.
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- **B.** The solution set is one or two points. The solution set is $\{\quad\}$.
  (Type an integer or a fraction. Use a comma to separate answers as needed.)
- **C.** The solution set is $\varnothing$.

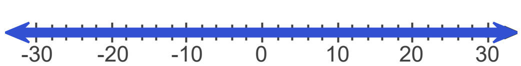
Choose the correct graph below.

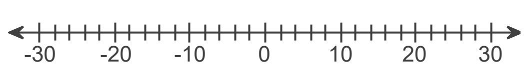
A. 

B. 

C. 

D. 

E. 

F. 

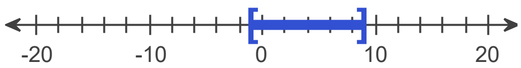
9. Solve the inequality. Then graph the solution set and write it in interval notation.

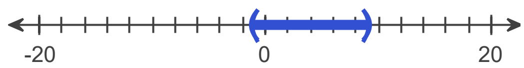
$$|x-4|-6 \le -1$$

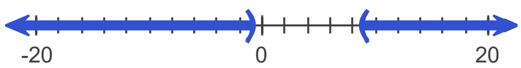
Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. The solution set is __________.
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- B. The solution set is $\varnothing$.

Choose the correct graph below.

A.


B.


C.


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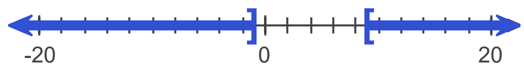

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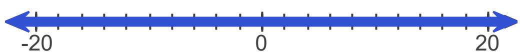

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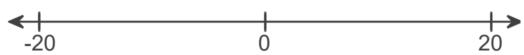
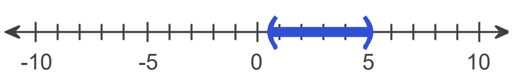


10. Solve the inequality $|6x-17|+2>15$. Graph the solution set and write it in interval notation.

Select the correct choices below and, if necessary, fill in the answer box to complete your choice.

- A. Written in interval notation, the solution is __________.
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- B. The solution is a set of points. The solution set is $\{$ __________ $\}$.
  (Type an integer or a fraction. Use a comma to separate answers as needed.)
- C. The solution set is $\varnothing$.

Graph the solution set on the number line. Choose the correct answer below.

A.


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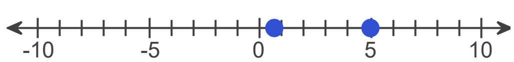

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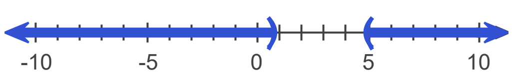

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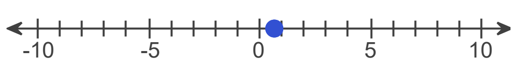

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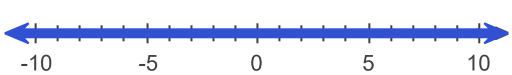

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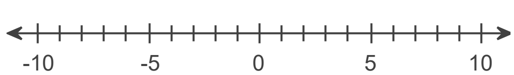


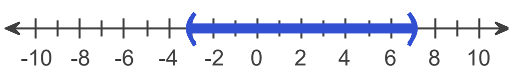
11. Solve the inequality. Graph the solution set.

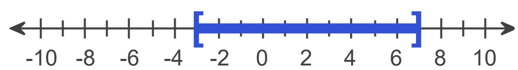
$$-18 + |2x - 4| \le -8$$

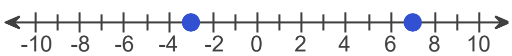
Select the correct choices below and, if necessary, fill in the answer box to complete your choice.

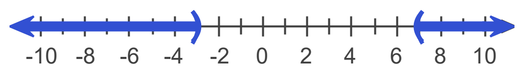
- **A.** The solution is one or more intervals. The solution is ______ .
  (Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)
- **B.** There are only one or two solutions. The solution set is { ______ }.
  (Type an integer or a fraction. Use a comma to separate answers as needed.)
- **C.** The solution set is ∅.

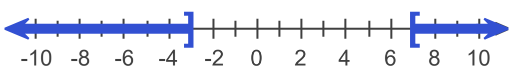
Choose the correct graph below.

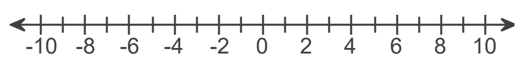
A. 

B. 

C. 

D. 

E. 

F. 

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #19-20

1. Find the slope and the y-intercept of the line.

   $y = -3x + 7$

   Select the correct choice below and fill in any answer boxes within your choice.

   ◯ **A.** The slope is ____________.
   (Simplify your answer. Type an integer or a fraction.)

   ◯ **B.** The slope is undefined.

   Select the correct choice below and fill in any answer boxes within your choice.

   ◯ **A.** The y-intercept is ____________.
   (Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)

   ◯ **B.** There is no y-intercept.

2. State the slope and the y-intercept of the line with the given equation.

   $y = 10x$

   Find the slope of the given line. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ◯ **A.** The slope is ____________.
   (Type an integer or a simplified fraction.)

   ◯ **B.** The slope of the line is undefined.

   Find the y-intercept of the given line. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ◯ **A.** The y-intercept is ____________.
   (Type an ordered pair, using integers or fractions.)

   ◯ **B.** There is no y-intercept.

3. Use the slope-intercept form of the linear equation to write the equation of the line with the given slope and y-intercept.

   Slope -4; y-intercept (0,8)

   The equation is ____________.
   (Type your answer in slope-intercept form.)

4. Use the slope-intercept form of the linear equation to write the equation of the line with the given slope and y-intercept.

   Slope $\frac{1}{5}$; y-intercept (0,0)

   The equation is ____________.
   (Type your answer in slope-intercept form.)

5. Find an equation of the line having the given slope and containing the given point.

Slope 8; through (5,1)

The equation of the line is ____________.
(Simplify your answer. Type your answer in slope-intercept form.)

6. Find an equation of the line having the given slope and containing the given point.

Slope $\frac{3}{4}$; through $(-4,4)$

The equation of the line is ____________.
(Simplify your answer. Type your answer in slope-intercept form.)

7. Find the equation of the line with the given slope and containing the given point.

Slope $-\frac{4}{5}$; through $(-3,0)$

The equation of the line is ____________.
(Simplify your answer. Type your answer in slope-intercept form.)

8. Decide whether the lines are parallel, perpendicular, or neither.

$y = 13x - 7$
$y = 13x + 9$

Are the lines parallel, perpendicular, or neither?

- Parallel
- Neither
- Perpendicular

9. Decide whether the following lines are parallel, perpendicular, or neither.

$$y = -10x + 3$$
$$y = \frac{7}{2}x - 2$$

Choose the correct answer below.

- **A.** The lines are parallel.
- **B.** The lines are perpendicular.
- **C.** The lines are neither parallel nor perpendicular.

10. Find an equation of the line passing through the given points. Use function notation to write the equation.

(3,2) and (5,8)

f(x) = ____________

11. Find an equation of the line passing through the given points. Use function notation to write the equation,

$(-2,12)$ and $(-1,7)$

$f(x) =$ ____________

12. Find an equation of the line passing through the given points. Use function notation to write the equation.

$(-4,-3)$ and $(-6,-2)$

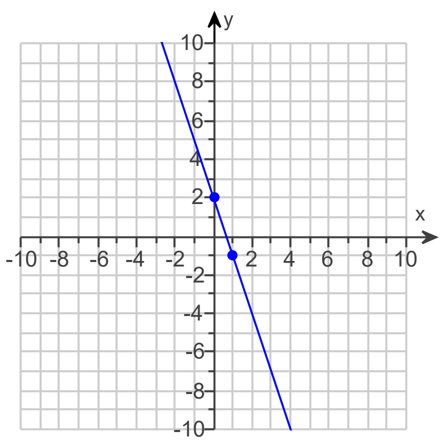
$f(x) =$ ____________

13. Find an equation of the line containing the given points. Use function notation to write the equation.

$\left(\frac{4}{7},\frac{5}{7}\right)$ and $\left(-\frac{1}{7},\frac{11}{14}\right)$

$f(x) =$ ____________
(Simplify your answer. Use integers or fractions for any numbers in the expression.)

14. Find an equation of the line graphed. Write the equation in standard form.



The equation is ____________.
(Type your answer in standard form. Simplify your answer.)

15. Find the equation of the line. Write the equation of the line in standard form.

With slope $-\frac{3}{4}$; y-intercept 3

The equation of the line in standard form is ____________.
(Type your answer in standard form. Use integers or fractions for any numbers in the equation.)

16. Find the equation of the line.

Through $(9,-1)$; parallel to the line $4x + 5y = 3$

Which of the following is the equation of the line in standard form?

- **A.** $4x + 5y = 31$
- **B.** $4x - 5y = 41$
- **C.** $5x - 4y = 49$
- **D.** $4x + 5y = 41$

17. Find an equation of the line. Write the equation using function notation.

Through $(4, -1)$; perpendicular to $8y = x - 16$

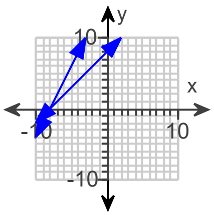
The equation of the line is $f(x) =$ __________.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #21

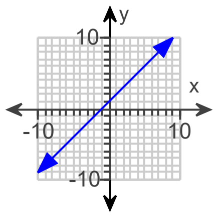
1. Determine which of the graphs is of a system of linear equations that has no solution.

Which of the following graphs is of a system that has no solution? Choose the correct graph below.

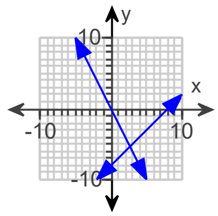
○ **A.**



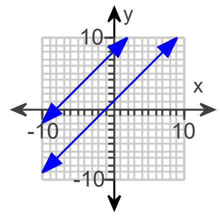
○ **B.**



○ **C.**



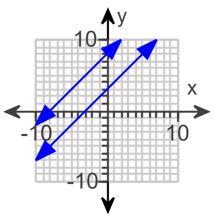
○ **D.**



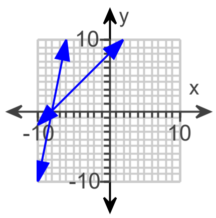
2. Determine which of the graphs is of a system of linear equations that has an infinite number of solutions.

Which of the following graphs is of a system that has infinitely many intersection points? Choose the correct graph below.

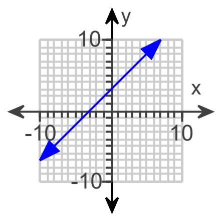
○ **A.**



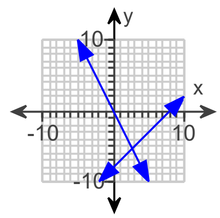
○ **B.**



○ **C.**



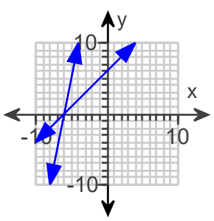
○ **D.**



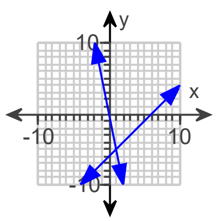
3. Determine which of the graphs is of a system of linear equations that has $(1, -5)$ as its only solution.

Which of the below graphs is of a system of linear equations that has $(1, -5)$ as its only solution? Choose the correct graph below.

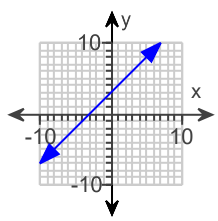
○ **A.**



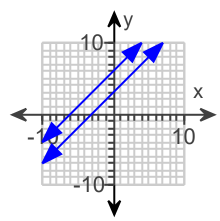
○ **B.**



○ **C.**



○ **D.**



4. Determine whether the given ordered pair is a solution of the system.

$$\begin{cases} x - y = 5 \\ 2x - 3y = 14 \end{cases} \qquad (1, -4)$$

Is $(1, -4)$ a solution of the system?

○ Yes

○ No

5. Solve the system of equations.

$$\begin{cases} 2x = 4 \\ y = 5 - x \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is __________.
  (Simplify your answer. Type an ordered pair.)
- **B.** The solution set of the system is $\{(x,y) \mid y = 5 - x\}$.
- **C.** The solution set is $\varnothing$.

---

6. Solve the system of equations.

$$\begin{cases} 4x - y = -1 \\ y = -4x \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is __________.
  (Simplify your answer. Type an ordered pair.)
- **B.** The solution set of the system is $\{(x,y) \mid 4x - y = -1\}$.
- **C.** The solution set is $\varnothing$.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #22

1. Use the substitution method to solve the following system of equations.

$$\begin{cases} x + y = 12 \\ y = 5x \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is ____________.
  (Simplify your answer. Type an ordered pair.)
- **B.** The solution set of the system is $\{(x,y) | \ x + y = 12\}$.
- **C.** The solution set is $\varnothing$.

2. Use the substitution method to solve the following system of equations.

$$\begin{cases} 5x - y = 46 \\ 2x + 3y = -2 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is ____________.
  (Simplify your answer. Type an ordered pair.)
- **B.** The solution set of the system is $\{(x,y) | \ 5x - y = 46\}$.
- **C.** The solution set is $\varnothing$.

3. Solve the system of equations by the elimination method.

$$\begin{cases} -x + 2y = 0 \\ x + 2y = 1 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is ____________.
  (Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)
- **B.** The solution set of the system is $\{(x,y) | -x + 2y = 0\}$.
- **C.** The solution set is $\varnothing$.

4. Use the elimination method to solve the following system of equations.

$$\begin{cases} 4x + y = 10 \\ x - 3y = 9 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** There is one solution. The solution of the system is ____________.
  (Simplify your answer. Type an ordered pair.)
- **B.** The solution set of the system is $\{(x,y) \ | \ 4x + y = 10\}$.
- **C.** The solution set is { }or $\varnothing$.

5. Solve the system of equations by the elimination method.

$$\begin{cases} 8x - 6y = 6 \\ 7x - 5y = 6 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. There is one solution. The solution of the system is ______.
  (Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)
- B. The solution set of the system is $\{(x,y) | 8x - 6y = 6\}$.
- C. The solution set is $\varnothing$.

6. Solve the system of equations.

$$\begin{cases} x = 2y + 3 \\ 2x - 4y = 6 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. There is one solution. The solution of the system is ______.
  (Simplify your answer. Type an ordered pair.)
- B. The solution set of the system is $\{(x,y) | \ x = 2y + 3\}$.
- C. The solution set is $\varnothing$.

7. Solve the system of equations.

$$\begin{cases} 7x - y = -5 \\ y = -7x \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- A. There is one solution. The solution of the system is ______.
  (Simplify your answer. Type an ordered pair.)
- B. The solution set of the system is $\{(x,y) | 7x - y = -5\}$.
- C. The solution set is $\varnothing$.

8. Without graphing, determine whether system has one solution, no solution, or an infinite number of solutions.

$$\begin{cases} y = 6x - 5 \\ y = 6x + 7 \end{cases}$$

Choose the correct answer below.

- A. There is one solution.
- B. There are an infinite number of solutions.
- C. There is no solution.

9. Without graphing, determine whether system has one solution, no solution, or an infinite number of solutions.

$$\begin{cases} x + y = 7 \\ 6x + 6y = 42 \end{cases}$$

Choose the correct answer below.

- **A.** There is one solution.
- **B.** There are an infinite number of solutions.
- **C.** There is no solution.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #23

1. A woman bought some large frames for \$16 each and some small frames for \$4 each at a closeout sale. If she bought 29 frames for \$236, find how many of each type she bought.

   She bought __________ large frames.

   She bought __________ small frames.

---

2. One number is nine less than a second number. Three times the first is 6 more than 4 times the second. Find the numbers.

   The value of the first number is __________.

   The value of the second number is __________.

---

3. At a concession stand, five hot dog(s) and four hamburger(s) cost \$16.50; four hot dog(s) and five hamburger(s) cost \$17.25. Find the cost of one hot dog and the cost of one hamburger.

   What is the cost of one hot dog? \$ __________

   What is the cost of one hamburger? \$ __________

---

4. Solve the system of equations.

$$\begin{cases} 9x - 2y = 65 \\ -2x + 5y = 22 \end{cases}$$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   - **A.** There is one solution. The solution of the system is __________.
     (Simplify your answer. Type an ordered pair.)
   - **B.** The solution set of the system is $\{(x,y)|\ 9x - 2y = 65\}$.
   - **C.** The solution set is $\varnothing$.

---

5. Solve the system of equations by the substitution method.

$$\begin{cases} \dfrac{x}{4} + y = -\dfrac{25}{4} \\ -x + 4y = -31 \end{cases}$$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   - **A.** There is one solution. The solution of the system is __________.
     (Simplify your answer. Type an ordered pair.)
   - **B.** The solution set of the system is $\left\{(x,y)\middle|\ \dfrac{x}{4} + y = -\dfrac{25}{4}\right\}$.
   - **C.** The solution set is $\varnothing$.

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #24

1. Find the square root.

$\sqrt{121}$

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The square root is ________ .
- **B.** The square root is not a real number.

2. Simplify.

$-\sqrt{\frac{1}{81}}$

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $-\sqrt{\frac{1}{81}} =$ ________
- **B.** The root is not a real number.

3. Find the square root.

$-\sqrt{100}$

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The square root is a real number. $-\sqrt{100} =$ ________
- **B.** The square root is not a real number.

4. Simplify. Assume that variables represent nonnegative real numbers.

$\sqrt{x^8}$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\sqrt{x^8} =$ ________
- **B.** The square root is not a real number.

5. Simplify by factoring. Assume that all variables under radicals represent nonnegative numbers.

$\sqrt{49x^6}$

Select the correct choice below and, if necessary, fill in the answer box that completes your choice.

- **A.** $\sqrt{49x^6} =$ ________
  (Type an exact answer, using radicals as needed.)
- **B.** The square root is not a real number.

6. Simplify.

$$\sqrt{(-8)^2}$$

Select the correct choice below and, if necessary, fill in the answer box that completes your choice.

- **A.** $\sqrt{(-8)^2}$ = ____________
  (Type an exact answer, using radicals as needed.)
- **B.** The square root is not a real number.

---

7. Simplify. Assume that the variable represents any real number.

$$\sqrt{100x^2}$$

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $\sqrt{100x^2}$ = ____________
- **B.** The root does not represent a real number.

---

8. Rationalize the denominator.

$$\frac{\sqrt{10}}{\sqrt{7}}$$

The answer is ____________.

---

9. Rationalize the denominator.

$$\sqrt{\frac{1}{149}}$$

$\sqrt{\frac{1}{149}}$ = ____________ (Type an exact answer, using radicals as needed.)

---

10. Rationalize the denominator. Assume that all variables represent positive real numbers.

$$\sqrt{\frac{121}{x}}$$

$\sqrt{\frac{121}{x}}$ = ____________ (Type an exact answer, using radicals as needed.)

---

11. Rationalize the denominator. Assume that all variables represent positive real numbers.

$$\frac{9}{\sqrt{28x}}$$

$\frac{9}{\sqrt{28x}}$ = ____________ (Type an exact answer, using radicals as needed.)

12. Rationalize the denominator of $\frac{7}{\sqrt{7x}}$. Assume that all variables represent positive real numbers.

$\frac{7}{\sqrt{7x}}$ = ____________ (Type an exact answer, using radicals as needed.)

13. Rationalize the denominator.

$$\frac{5\sqrt{3}}{\sqrt{2}}$$

$\frac{5\sqrt{3}}{\sqrt{2}}$ = ____________

(Type an exact answer, using radicals as needed.)

14. Rationalize the denominator.

$$\sqrt{\frac{17x}{2y}}$$

$\sqrt{\frac{17x}{2y}}$ = ____________ (Type an exact answer, using radical as needed.)

15. Rationalize the denominator. Assume that all variables represent positive real numbers.

$$\sqrt{\frac{3x}{125}}$$

$\sqrt{\frac{3x}{125}}$ = ____________ (Type an exact answer, using radicals as needed.)

16. Rationalize the denominator. Assume that all variables represent positive real numbers.

$$\frac{1}{\sqrt{27z}}$$

$\frac{1}{\sqrt{27z}}$ = ____________ (Type an exact answer, using radicals as needed.)

17. Rationalize the denominator.

$$\frac{6}{1-\sqrt{3}}$$

$\frac{6}{1-\sqrt{3}}$ = ____________

(Simplify your answer. Type an exact answer, using radicals as needed.)

18. Rationalize the denominator.

$$\frac{\sqrt{14} - \sqrt{13}}{\sqrt{14} + \sqrt{13}}$$

$\dfrac{\sqrt{14} - \sqrt{13}}{\sqrt{14} + \sqrt{13}} =$ ____________ (Type an exact answer, using radicals as needed.)

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #25

1. Use the product rule to multiply.

$$\sqrt[3]{6} \cdot \sqrt[3]{7}$$

$\sqrt[3]{6} \cdot \sqrt[3]{7} =$ ____________

(Type an exact answer, using radicals as needed. Simplify your answer.)

2. Use the product rule to multiply. Assume that all variables represent positive real numbers.

$$\sqrt[4]{2x^3} \cdot \sqrt[4]{3}$$

$\sqrt[4]{2x^3} \cdot \sqrt[4]{3} =$ ____________

(Type an exact answer, using radicals as needed. Simplify your answer.)

3. Use the quotient rule to simplify.

$$\sqrt[3]{\frac{7}{27}}$$

$\sqrt[3]{\frac{7}{27}} =$ ____________

(Type an exact answer, using radicals as needed. Simplify your answer.)

4. Simplify.

$$\sqrt[3]{135}$$

$\sqrt[3]{135} =$ ____________

5. Use the quotient rule to divide. Then simplify if possible.

$$\frac{3\sqrt[4]{48}}{\sqrt[4]{3}}$$

$\frac{3\sqrt[4]{48}}{\sqrt[4]{3}} =$ ____________

(Type an exact answer, using radicals as needed. Simplify your answer.)

6. Rationalize the denominator of $\frac{7}{\sqrt[3]{6}}$.

$\frac{7}{\sqrt[3]{6}} =$ ____________ (Type an exact answer, using radicals as needed.)

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #26

1. Use the square root property to solve the equation. The equation has real number solutions.

   $x^2 - 14 = 0$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

2. Use the square root property to solve the equation. The equation has real number solutions.

   $x^2 = 20$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

3. Use the square root property to solve the equation. The equation has real number solutions.

   $2z^2 - 28 = 0$

   z = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

4. Use the square root property to solve the equation. The equation has real number solutions.

   $(x + 2)^2 = 9$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

5. Use the square root property to solve the equation.

   $x^2 - 11 = 0$

   x = ____________
   (Simplify your answer, including any radicals and i as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

6. Use the square root property to solve the equation.

   $2x^2 + 90 = 0$

   x = ____________
   (Simplify your answer, including any radicals and i as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

Student: ____________________ **Date:** ____________________	**Instructor:** Deb Wertz **Course:** MAP102 MASTER	**Assignment:** Homework #27

1. Use the quadratic formula to solve the equation.

   $m^2 - 4m + 3 = 0$

   m = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

2. Use the quadratic formula to solve the equation. The equation has real number solutions.

   $4y = 4y^2 - 8$

   y = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

3. Use the quadratic formula to solve the equation.

   $x^2 - 10x + 25 = 0$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

4. Use the quadratic formula to solve the equation.

   $x^2 + x - 4 = 0$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

5. Use the quadratic formula to solve the equation.

   $10m^2 - 2m = 9$

   m = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

6. Use the quadratic formula to solve the equation. The equation has real number solutions.

   $\frac{1}{3}x^2 + 4x + 4 = 0$

   x = ____________
   (Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

7. Use the quadratic formula to solve the equation.

$$(m-3)(3m+4)=5(m+1)+8$$

m = __________
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

8. Use the discriminant to determine the number and types of solutions of the quadratic equation.

$$x^2-6=0$$

The equation has (1) __________

(1) ◯ two real solutions.
◯ two complex but not real solutions.
◯ one real solution.

9. Use the discriminant to determine the number and types of solutions of the quadratic equation.

$$4x^2-8x=-4$$

The equation has (1) __________

(1) ◯ two complex but not real solutions.
◯ one real solution.
◯ two real solutions.

10. Use the discriminant to determine the number and types of solutions of the quadratic equation.

$$3=3x-5x^2$$

The equation has (1) __________

(1) ◯ two complex but not real solutions.
◯ one real solution.
◯ two real solutions.

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hw-sol-11.pdf	2018-07-10 07:56	41K	
hw-sol-03.pdf	2018-07-10 07:56	42K	
hw-sol-25.pdf	2018-07-10 07:56	42K	
hw-sol-14.pdf	2018-07-10 07:56	42K	
hw-sol-13.pdf	2018-07-10 07:56	43K	
hw-sol-27.pdf	2018-07-10 07:56	43K	
hw-sol-05.pdf	2018-07-10 07:56	43K	
hw-sol-09.pdf	2018-07-10 07:56	43K	
hw-sol-07.pdf	2018-07-10 07:56	44K	
hw-sol-08.pdf	2018-07-10 07:56	44K	
hw-sol-26.pdf	2018-07-10 07:56	44K	
hw-sol-01.pdf	2018-07-10 07:56	44K	
hw-sol-06.pdf	2018-07-10 07:56	45K	
hw-sol-12.pdf	2018-07-10 07:56	45K	
hw-sol-16.pdf	2018-07-10 07:56	47K	
hw-sol-04.pdf	2018-07-10 07:56	72K	
hw-sol-23.pdf	2018-07-10 07:56	73K	
hw-sol-02.pdf	2018-07-10 07:56	73K	
hw-sol-15.pdf	2018-07-10 07:56	73K	
hw-sol-20.pdf	2018-07-10 07:56	74K	
hw-sol-24.pdf	2018-07-10 07:56	75K	
hw-sol-22.pdf	2018-07-10 07:56	75K	
hw-sol-21.pdf	2018-07-10 07:56	76K	
hw-sol-18.pdf	2018-07-10 07:56	86K	
hw-sol-17.pdf	2018-07-10 07:56	89K	
hw-sol-10.pdf	2018-07-10 07:56	121K	

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Name	Last modified	Size	Description
Parent Directory		-	
hw-sol-01.pdf	2018-07-10 07:56	44K	
hw-sol-02.pdf	2018-07-10 07:56	73K	
hw-sol-03.pdf	2018-07-10 07:56	42K	
hw-sol-04.pdf	2018-07-10 07:56	72K	
hw-sol-05.pdf	2018-07-10 07:56	43K	
hw-sol-06.pdf	2018-07-10 07:56	45K	
hw-sol-07.pdf	2018-07-10 07:56	44K	
hw-sol-08.pdf	2018-07-10 07:56	44K	
hw-sol-09.pdf	2018-07-10 07:56	43K	
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hw-sol-25.pdf	2018-07-10 07:56	42K	
hw-sol-26.pdf	2018-07-10 07:56	44K	
hw-sol-27.pdf	2018-07-10 07:56	43K	

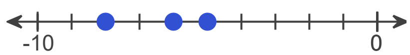
1. (1) integers

2. (1) irrational number.

3. (1) rational number.

4. 1

5.

B. 

6. $2, 0, \sqrt{25}$

1. 10

---

2. -2

---

3. $\frac{3}{2}$

---

4. -4

---

5. $-\frac{1}{3}$

---

6. A. $\frac{-8}{-4}=$ **2** (Simplify your answer.)

---

7. A. $2\left(-\frac{1}{18}\right)=$ $\mathbf{-\frac{1}{9}}$ (Type an integer or a simplified fraction.)

---

8. -10

---

9. 13

---

10. A. $\frac{-16}{8}=$ **-2** (Simplify your answer. Type an integer or a fraction.)

---

11. A. $(-6)(-8)(-1)=$ **-48**

---

12. 136

---

1. -4

2. $-\frac{5}{8}$

3. 43

4. -81

5. $-\frac{89}{150}$

6. $\frac{11}{2}$

1. $y + 13x$

2. $h \cdot g$

3. $\frac{x}{8} \cdot \frac{1}{6}$

4. $(10 \cdot 2)x$

5. $x + (9.7 + y)$

6. $22 \cdot (x \cdot y)$

7. $25n$

8. $13 + 3x$

9. 0

10. 3

11. $(12 \cdot 4) \cdot y$

12. $(3 \cdot 6) \cdot y$

13. D.
If a real number a satisfies the given condition, then $a = -a$. The only real number that satisfies this equation is **0**.

1. (1) 0

---

2. (1) reciprocal

---

3. (1) 0

   (2) undefined.

---

4. (1) $\frac{-a}{b}$

   (2) $\frac{a}{-b}$.

---

5. (1) $-a$.

---

6. (1) $\frac{1}{a}$.

---

7. (1) commutative

---

8. (1) associative

---

9. -9

---

10. $-\frac{1}{1000}$

---

11. A. $\frac{1}{-5}$, C. $\frac{-1}{5}$

---

12. A. $-\frac{8}{(p+r)}$, C. $\frac{-8}{(p+r)}$

---

13. D. $\frac{8r}{9s}$

---

14. 20

    $\frac{5}{4}$

    (1) is not

---

1. (1) distributive

2. A. They are simplified by combining like terms.

   distributive property

3. (1) terms

4. $6x + 18$

5. (1) -5

   (2) $\frac{1}{5}$

6. (1) -6

   (2) $-\frac{1}{6}$

7. $b + 10a$

8. $8x + 8$

9. $-3x - y$

10. $12x + 15y + 9z$

11. $-6x + 12y - 54$

12. $-5x + 8$

13. $9y - 6$

14. $4k + 18$

15. $-7c - 4$

16. $-28y$

17. $-x^2 + 5xy + 6$

---

18. $n - 3$

---

19. -48

---

20. $\frac{59}{45}b - \frac{8}{15}$

---

21. $4x + \frac{3}{4}y - 6$

---

22. $c(f + d)$

$cf + cd$

---

1. -6

---

2. 8

---

3. x

---

4. (1) exponent

---

5. (1) exponent

   (2) base.

---

6. -81

---

7. 4

---

8. $-\frac{1}{1000}$

---

9. $\frac{4}{au^3}$

---

10. $\frac{a^3}{bc^9}$

---

11. $\frac{q^7}{p^5}$

---

12. 1

---

13. -5

---

14. 8

---

15. $\frac{1}{81}$

---

16. $-\frac{1}{27}$

---

17. $\frac{9}{x^2}$

---

18. -2

---

19. $\frac{7}{12}$

---

20. $\frac{y}{25}$

---

1. y^{13}

2. $-2z^3$

3. x^4

4. $\frac{1}{x^4}$

5. $5r^{10}$

6. 4

7. $\frac{b^8}{5a^8}$

8. $-\frac{8}{x^4}$

9. $40x^8y^5$

10. $\frac{3}{x^{11}z^8}$

11. x^{6a+6}

12. x^{8t-3}

13. n^{12}

14. g^{56}

15. $\frac{1}{27}$

16. $125x^{24}y^{27}$

---

17. $\frac{c^{18}}{64a^{6}b^{3}}$

---

18. $\frac{y^{14}}{x^{4}z^{2}}$

---

19. $\frac{125}{64}$

---

20. $\frac{x^{6}}{8}$

---

21. $\frac{1}{x^{28}b^{5}c^{5}}$

---

22. $\frac{25}{4x^{5}y^{4}}$

---

23. $x^{6a + 14}$

---

24. $-13x + 3$

---

1. (1) coefficient.

2. (1) polynomial

3. (1) binomial

4. (1) monomial

5. (1) trinomial

6. (1) degree

7. (1) Like

8. 0

9. binomial

   1

10. Trinomial

    2

11. $17y - 9y^2$

12. $-11x^2y + 4x - \frac{1}{3}$

1. 59

2. 399

3. $-\frac{10}{81}$

4. 106

5. $15y^2 - 13$

6. $15x^3y + 3x + 3$

7. $-2y^2 - 5$

8. $20x^3 + 20x^2 - 7x + 5$

9. $4x^2 + 3x + 10$

10. $14ab + 8a^2b - 18a^2 + 4b^2$

11. 0

12. $\frac{1}{2}x^2 - \frac{13}{14}x + \frac{5}{6}$

13. $6x^2 - 4x + 7$

14. $3a - 7$

    $-3x - 7$

    $3x + 3h - 7$

15. $6a - 7$

    $-6x - 7$

    $6x + 6h - 7$

16. D. $(x + 18)^2 = x^2 + 36x + 324$

---

17. A. $x^2 - 9$

---

18. (1) 3.

---

19. (1) $(a + 4)^2 + 9$

---

20. (1) $[x + (2y + 1)]\ [x + (2y + 1)]$

---

21. $-18x^2y - 6xy^2$

---

22. $3xa^3b + 3ya^8b + 15ab$

---

23. $2a^2 - 1a - 15$

---

24. $-6x^4 + 2x^3 + 6x^2 + 28x - 10$

---

25. $x^2 + 6x + 9$

---

26. $36x^2 - 49$

---

27. $64x^2 - 16xy + y^2$

---

28. $9x^2 - \frac{1}{4}$

---

29. $35x^5 + 15x^4 + 25x^3 + 21x^2 + 9x + 15$

---

30. $a^2 + 2ah + h^2 - 15a - 15h$

---

31. $b^2 - 23b + 126$

---

32. x^3

---

33. $y^2 \cdot z^3$

---

34. $21xy^3$

---

35. A. $12x - 18 =$ **$6(2x - 3)$**

---

36. A. $5y^2 - 30xy^3 =$ **$5y^2(1 - 6xy)$** (Type your answer in factored form.)

---

37. $I\left(R_1 + R_2\right)$

---

1. (1) rational expression

---

2. (1) 0.

---

3. $1 - 6x$

---

4. $x - 4$

---

5. $\frac{6}{5}$

---

6. (1) simplified

---

1. $\frac{3}{2(x-1)}$

2. $-\frac{6a}{2a+3}$

3. $\frac{x}{4}$

4. $\frac{4a^2}{a-b}$

5. $\frac{20}{x^2y}$

6. $\frac{14}{11}$

-8

$-\frac{3}{11}$

7. $-\frac{17}{48}$

$\frac{2}{7}$

$-\frac{2}{7}$

8. A. $\frac{-x}{-7+x}$, B. $\frac{-x}{x-7}$

9. (1) addition and subtraction.

10. Division

11. (1) multiplication.

12. $\frac{-x+6}{2x}$

13. $-\frac{2}{x}$

---

14. $\frac{83}{45x}$

---

15. $\frac{35-4y}{10y^2}$

---

16. $\frac{-17x-20}{(x-4)(x+4)}$

---

17. $\frac{91}{12(x+2)}$

---

1. 23

   25

   27

2. $4y$

3. $4x + 6$

4. $13x + 18$

5. $2x + 8$

6. $90x + 25$

1. (1) solution

---

2. B. $\frac{1}{3}x - 5$ is an expression.

---

3. D. It is an equation, because it contains an equal sign.

---

4. B. $\frac{5}{9}x + \frac{1}{3} = \frac{2}{9} - x$ is an equation.

---

5. A. It is an expression, because it contains the sum and difference of terms, and does not contain an equal sign.

---

1. A. The solution is **6** .(Simplify your answer.)

2. A. The solution is $\mathbf{-26}$.(Simplify your answer.)

3. A. The solution is **5** .(Simplify your answer.)

4. A. The solution is $\mathbf{-7}$.(Simplify your answer.)

5. A. The solution is **0** .(Simplify your answer.)

6. A. The solution is **2** .(Simplify your answer.)

7. A. The solution is $\frac{21}{16}$.(Type an integer or a simplified fraction.)

8. A. The solution is $\frac{49}{22}$.(Type an integer or a simplified fraction.)

9. A. The solution is $\frac{59}{2}$. (Type an integer or a simplified fraction.)

10. A. The solution is $\frac{6}{7}$. (Type an integer or a simplified fraction.)

11. B. The solution is all real numbers.

12. C. There is no solution.

1. $9 - x$

---

2. 54,27,127

---

3. $\frac{y}{g}$

---

4. $\frac{7x - 19}{6}$

---

5. $\frac{P - 2M}{2}$

---

6. 140

   50

   C. Divide both sides of the equation by 50.

   2.8

   2

   48

---

7. 173

---

8. 30

   (1) m.

---

9. 4500π

   (1) cu mm.

   14,137.17

   (2) cu mm.

---

1. (1) $[-1.1,\infty)$.

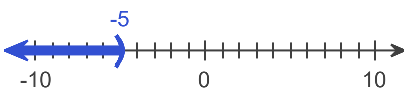
2. (1) $(-\infty,-2.1)$.

3. (1) $(-\infty,2.7]$.

4. (1) the same

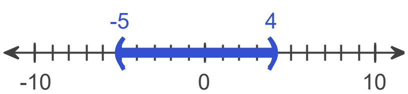
(2) reverse

5.

D. 

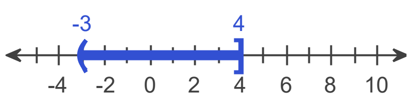
$(-\infty, \ -5)$

6.

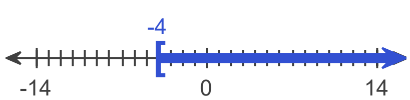
B. 

$(-5,4)$

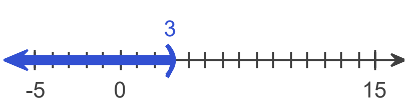
7.

C. 

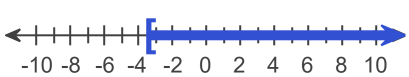
$(-3,4]$

8. C. 

A. The solution is $\underline{\ \mathbf{[-4, \ \infty)}\ }$.(Type your answer in interval notation.)

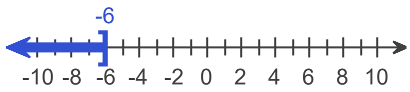
9. A. 

A. The solution set is $\underline{\ \mathbf{(-\infty,3)}\ }$.(Type your answer in interval notation.)

10. D. 

A. The solution is $\underline{\left[-\frac{27}{8}, \infty\right)}$.

(Use integers or fractions for any numbers in the expression. Type your answer in interval notation.)

11. B. 

A. The solution set is $(-\infty,-6]$. (Type your answer in interval notation.)

12. A. The solution set is $[-7,\infty)$.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression. Simplify your answer.)

13. A. The solution set is $(-27,\infty)$.(Simplify your answer. Type your answer in interval notation.).)

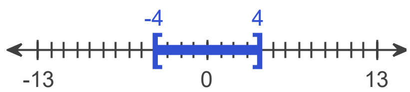
14. A. The solution set is $\left(-\infty,-\frac{29}{2}\right)$.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

15. A. The solution set is $[-25,\infty)$.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

1. A. The solution set is $\{16, -16\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

---

2. A. The solution is an interval. The solution is $[-4,4]$.
(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

A. 

---

3. A. The solution set is $\{14, -3\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

---

4. A. The solution set is $\{8,16\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

---

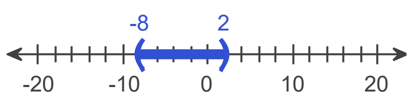
5. B. The solution set is $\varnothing$.

---

6. A. The solution set is $\{16, -11\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

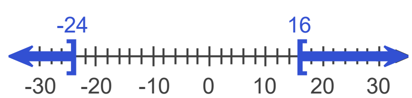
---

7. A. The solution is an interval. The solution is $(-8,2)$.
(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

A. 

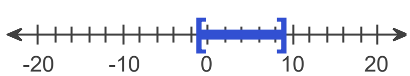
---

8. A. The solution is an interval. The solution is $(-\infty,-24]\cup[16,\infty)$.
(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

C. 

---

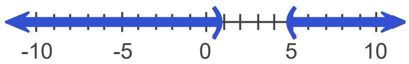
9. A. The solution set is $[-1,9]$.
(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

A. 

---

10. A. Written in interval notation, the solution is $\left(-\infty,\frac{2}{3}\right)\cup(5,\infty)$.

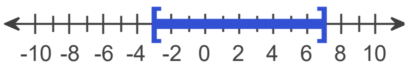
(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

C. 

---

11. A. The solution is one or more intervals. The solution is $[-3,7]$.

(Simplify your answer. Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

B. 

---

1. A. The slope is **-3**. (Simplify your answer. Type an integer or a fraction.)

A. The y-intercept is **(0,7)**.
(Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)

---

2. A. The slope is **10**. (Type an integer or a simplified fraction.)

A. The y-intercept is **(0,0)**.(Type an ordered pair, using integers or fractions.)

---

3. $y = -4x + 8$

---

4. $y = \frac{1}{5}x$

---

5. $y = 8x - 39$

---

6. $y = \frac{3}{4}x + 7$

---

7. $y = -\frac{4}{5}x - \frac{12}{5}$

---

8. Parallel

---

9. C. The lines are neither parallel nor perpendicular.

---

10. $3x - 7$

---

11. $-5x + 2$

---

12. $-\frac{1}{2}x - 5$

---

13. $-\frac{1}{10}x + \frac{27}{35}$

---

14. $3x + y = 2$

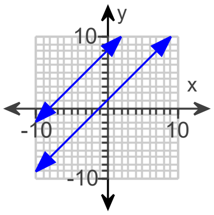
---

15. $3x + 4y = 12$

---

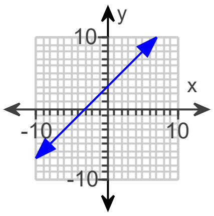
16. A. $4x + 5y = 31$

17. $-8x + 31$

1. 

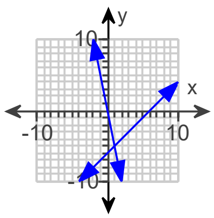
D.

---

2. 

C.

---

3. 

B.

---

4. Yes

---

5. A. There is one solution. The solution of the system is **(2,3)**. (Simplify your answer. Type an ordered pair.)

---

6. A. There is one solution. The solution of the system is $\left(-\frac{1}{8},\frac{1}{2}\right)$. (Simplify your answer. Type an ordered pair.)

---

1. A. There is one solution. The solution of the system is **(2,10)**. (Simplify your answer. Type an ordered pair.)

---

2. A. There is one solution. The solution of the system is $\mathbf{(8,-6)}$. (Simplify your answer. Type an ordered pair.)

---

3. A. There is one solution. The solution of the system is $\left(\frac{1}{2},\frac{1}{4}\right)$.
(Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)

---

4. A. There is one solution. The solution of the system is $\mathbf{(3,-2)}$.(Simplify your answer. Type an ordered pair.)

---

5. A. There is one solution. The solution of the system is **(3,3)**.
(Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)

---

6. B. The solution set of the system is $\{(x,y)|\ x = 2y + 3\}$.

---

7. A. There is one solution. The solution of the system is $\left(-\frac{5}{14},\frac{5}{2}\right)$. (Simplify your answer. Type an ordered pair.)

---

8. C. There is no solution.

---

9. B. There are an infinite number of solutions.

---

1. 10

   19

---

2. -42

   -33

---

3. 1.50

   2.25

---

4. A. There is one solution. The solution of the system is **(9,8)**. (Simplify your answer. Type an ordered pair.)

---

5. A. There is one solution. The solution of the system is **$(3,-7)$**.(Simplify your answer. Type an ordered pair.)

---

1. A. The square root is **11** .

---

2. A. $-\sqrt{\frac{1}{81}} =$ $\mathbf{-\frac{1}{9}}$

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3. A. The square root is a real number. $-\sqrt{100} =$ $\mathbf{-10}$

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4. A. $\sqrt{x^8} =$ $\mathbf{x^4}$

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5. A. $\sqrt{49x^6} =$ $\mathbf{7x^3}$ (Type an exact answer, using radicals as needed.)

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6. A. $\sqrt{(-8)^2} =$ **8** (Type an exact answer, using radicals as needed.)

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7. A. $\sqrt{100x^2} =$ $\mathbf{10|x|}$

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8. $\frac{\sqrt{70}}{7}$

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9. $\frac{\sqrt{149}}{149}$

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10. $\frac{11\sqrt{x}}{x}$

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11. $\frac{9\sqrt{7x}}{14x}$

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12. $\frac{\sqrt{7x}}{x}$

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13. $\frac{5\sqrt{6}}{2}$

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14. $\frac{\sqrt{34xy}}{2y}$

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15. $\dfrac{\sqrt{15x}}{25}$

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16. $\dfrac{\sqrt{3z}}{9z}$

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17. $-3\left(1+\sqrt{3}\right)$

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18. $27-2\sqrt{182}$

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1. $\sqrt[3]{42}$

2. $\sqrt[4]{6x^3}$

3. $\dfrac{\sqrt[3]{7}}{3}$

4. $3\sqrt[3]{5}$

5. 6

6. $\dfrac{7\sqrt[3]{36}}{6}$

1. $\sqrt{14}, -\sqrt{14}$

2. $2\sqrt{5}, -2\sqrt{5}$

3. $\sqrt{14}, -\sqrt{14}$

4. $1, -5$

5. $\sqrt{11}, -\sqrt{11}$

6. $3i\sqrt{5}, -3i\sqrt{5}$

1. 1,3

2. 2, -1

3. 5

4. $\dfrac{-1-\sqrt{17}}{2}, \dfrac{-1+\sqrt{17}}{2}$

5. $\dfrac{1-\sqrt{91}}{10}, \dfrac{1+\sqrt{91}}{10}$

6. $-6+2\sqrt{6}, -6-2\sqrt{6}$

7. $5, -\dfrac{5}{3}$

8. (1) two real solutions.

9. (1) one real solution.

10. (1) two complex but not real solutions.