NEW TECHNIQUES IN BIRATIONAL GEOMETRY

TITLES AND ABSTRACTS

Nick Addington. *Cubic 4-folds: Hassett's condition vs. Galkin and Shinder's*

Recent work of Galkin and Shinder suggests that if a cubic 4-fold X is rational then the variety F of lines on X is birational to $\mathrm{Hilb}^2(\mathrm{K3})$. I'll compare this to Hassett's notion of X having an associated K3 surface, which turns out to be equivalent to saying that F is birational to a moduli space of sheaves on a K3 surface. The key technical tool is the Mukai lattice that Richard Thomas and I introduced a few years ago. I'll discuss (but not resolve) the question of whether Hassett's rational cubics containing a plane all satisfy Galkin and Shinder's condition. Finally I'll suggest an approach to showing that Hassett's condition implies Kuznetsov's derived category condition – always, not just generically – using Lehn's hyperkaehler 8-fold and Bayer-Macrì.

Lev Borisov. *Equality of stringy E-functions of Pfaffian double mirrors and related results*

I will describe recent work with Anatoly Libgober on proving equality of stringy Hodge numbers of double mirror complete intersections in generalized Pfaffian varieties. In particular, I will explain the geometric construction involved in the proof that the class of affine line is a zero divisor in the Grothendieck ring of complex algebraic varieties.

Sebastian Casalaina-Martin. *Geometry of universal compactified Jacobians*

There is a close connection between the geometry of a smooth curve, and the geometry of its Jacobian. In this talk I will discuss joint work with J. Kass and F. Viviani where we investigate the connection between the geometry of a stable curve and its compactified Jacobians, constructed by Caporaso, Oda-Seshadri, Pandharipande, and Simpson. Specifically, we describe the singularities of the spaces, as well as singularities of their theta divisors in the integral case. Applications to the birational geometry of universal compactified Jacobians will also be discussed.

Ana-Maria Castravet. *Birational geometry of moduli spaces of stable rational curves*
I will discuss joint work with Tevelev and recent work of Gonzalez-Karu on the birational geometry of the Grothendieck-Knudsen moduli space of stable rational curves with n markings. The main result is that for $n > 12$ this space is not a Mori Dream Space, thus answering a question of Hu and Keel.

Ivan Cheltsov. *On quartic double solids*

I will speak about rationality problem for threefolds that are double covers of the projective space branched over nodal quartic surfaces.

Olivier Debarre. *Fano varieties and EPW sextics*

We explore a connection between smooth projective varieties X of dimension n with an ample divisor H such that $H^n = 10$ and $K_X = -(n-2)H$ and a class of sextic hypersurfaces of dimension 4 considered by Eisenbud, Popescu, and Walter (EPW sextics). This connection makes possible the construction of moduli spaces for these varieties and opens the way to the study of their period maps. This is work in progress in collaboration with Alexander Kuznetsov.

Date: April 7, 2015.

Sergey Galkin. *A tentative integer-valued birational invariant of threefolds*

I will give an experimental evidence towards the existence of an integer-valued birational invariant of (rationally connected) threefolds, that tends to distinguish most Fano threefolds with Picard number one. Also I will describe what values it should take on some known examples, and what properties should it satisfy.

Ludmil Katzarkov. *Sheaves of categories and applications*

We will consider some examples of sheaves of categories. As possible applications we will consider a connection to stable rationality.

János Kollár. *How much of the Hilbert polynomial do we need to know?*

The talk aims to describe several situations where variation of the Hilbert polynomial in families is completely controlled by the leading coefficient.

Alexander Kuznetsov. *Derived categories of Gushel–Mukai varieties and duality*

We go on with a discussion of the geometry of Fano varieties of degree 10 and coindex 3 (which we call Gushel–Mukai varieties) and of the structure of their derived categories. We define a duality relation for these varieties, closely related to projective duality of quadrics. We discuss a relation between the derived categories of dual Gushel–Mukai varieties and its relation to their birational properties. This is a joint work in progress with Olivier Debarre and Alex Perry.

John Lesieutre. *Constraints on positive entropy automorphisms of rational threefolds*

There are currently few known examples of automorphisms of rational threefolds with positive entropy, i.e. for which the induced map on $N^1(X)$ has an eigenvalue larger than 1. I'll describe some constraints on smooth threefolds X admitting such automorphisms. For example, I'll show that if X is constructed as a blow-up of $\mathbb{P}^1 \times \mathbb{P}^2$ or $\mathbb{P}^3$, any positive entropy automorphism admits an equivariant map to a surface. I'll also give a related example of a non-uniruled, terminal threefold with infinitely many K_X-negative extremal rays on the cone of curves.

Valery Lunts. *Some applications of the destackification theorem of D. Bergh*

A recent theorem of Daniel Bergh is an analogue for DM stacks of the weak factorization theorem for algebraic varieties. We will discuss two applications: 1) Geometricity of derived categories of smooth DM stacks; 2) Comparison of equivariant and usual categorical motivic measures.

Emanuele Macrì. *Stability Conditions on Threefolds, I*

I will present recent joint work with Arend Bayer and Paolo Stellari on Bridgeland stability conditions on smooth projective threefolds. After introducing the basic definitions in Bridgeland's theory, I will describe a connected component of (a slice of) the space of stability conditions for abelian threefolds and their orbifold quotients.

Alena Pirutka. *Stable rationality and quartic threefolds*

Let X be a smooth complex algebraic variety. Recall that X is rational if it is birational to a projective space, X is stably rational if a product of X with some projective space becomes rational and X is unirational if it is rationally dominated by a projective space. A classical question is to distinguish the properties of rationality and unirationality. In 1970 three examples of unirational but not rational varieties were discovered : cubic threefolds (Clemens and Griffiths), some quartic threefolds (Iskovskikh and Manin) and some conic bundles (Artin and Mumford). The example of Artin and Mumford is not stably rational, but it was not known if this property holds for other examples. In this talk we will discuss the case of quartic threefolds and show that many of them are not stably rational. This is a work in common with J.-L. Colliot-Thélène. The methods we use are based on the properties of the diagonal decomposition in the Chow groups, the universal properties of the Chow group of zero cycles as well as some specialization techniques.

Jason Starr. *Report on a theorem of Arnaud Beauville*

Unfortunately Professor Beauville cannot participate in this workshop. So Jason Starr will deliver a report on Beauville's recent article "A Very General Sextic Double Solid Is Not Stably Rational".

Paolo Stellari. *Stability Conditions on Threefolds, II*

I will explain, with some details, why a Bogomolov-Gieseker type inequality holds for abelian threefolds and some smooth projective Calabi-Yau threefolds of quotient type. From this one deduces that the corresponding spaces of Bridgeland stability conditions is not empty. This is joint work with A. Bayer and E. Macrì.

Burt Totaro. *Hypersurfaces that are not stably rational*

We show that a wide class of hypersurfaces in all dimensions are not stably rational. Namely, for all d at least about $2n/3$, a very general complex hypersurface of degree d in $\mathbb{P}^{n+1}$ is not stably rational. The result covers all the degrees in which Kollar proved that a very general hypersurface is non-rational, and a bit more. For example, very general quartic 4-folds are not stably rational, whereas it was not even known whether these varieties are rational.

Yuri Tschinkel. *K3 surfaces and their punctual Hilbert schemes*

I will discuss some aspects of birational geometry of holomorphic symplectic varieties of K3 type (joint with B. Hassett).

Claire Voisin. *Decomposition of the diagonal and stable birational invariants*

The Lüroth problem asks whether a unirational variety is rational. It has a negative answer starting from dimension 3 and can be attacked by various geometric approaches. For the stable Lüroth problem, where "rational" is replaced by "stably rational" , only the Artin-Mumford approach had been used up to now to solve the problem in dimension 3. Using the notion of decomposition of the diagonal, we exhibit many unirational threefolds which are not stably rational while their Artin-Mumford invariant is trivial.