MAT 515 Homework 10 - Fall 2016

1. In class discussed the theorem "Every Euclidean isometry is either a rotation, a translation or a glide reflection". The proof uses the fact that every isometry can be expressed as the composition of two or three reflections. The case of two reflections is straightforward (If you do not see it, stop and think about it). In this problem, you are asked to complete the proof in the case of three reflections across lines l, m and n. This proof is divided in four cases:

    (a) The lines l, m and n are parallel.

    (b) The lines l, m and n are concurrent.

    (c) The lines l and m intersect at a point P and P is not in n .

    (d) The lines n and m intersect at a point P and P is not in l .

    A big chunk of the proof of (c) was done in class, in the geogebra activity. You have to follow those steps to write done the composition of reflections $r_n r_m r_l$ as composition of reflections across (possibly new) lines, satisfying the definition glide reflection given in class (that is three lines a, b, c such that b is perpendicular to a and c is perpendicular to c.

2. Prove that the composition of two half-turns in is a translation. Determine the direction of this translation.

3. Prove that the composition of two translations commute, that is $T_{AB}T_{CD} = T_{CD}T_{AB}$ for all points A, B, C, D. Is it true that the composition of two reflections commute? And the composition of two rotations?

4. Consider two isometries, S and T. In the table below, study what type of isometry one gets when composing $S \circ T$, for each possible type of S and T assigned to you. For instance, if S is a rotation and T is a translation, the composition of S and T can be a rotation or a translation.(you need to prove your statements).

Samantha: S is a translation, T is a reflection, rotation, translation, or glide reflection.

Julie: S is a glide reflection, T is a reflection, rotation, translation, or glide reflection.

John: S is a reflection, T is a reflection, rotation, translation, or glide reflection.

Akkapol: S is a rotation, T is a reflection, rotation, translation, or glide reflection.

Rob: T is a glide reflection, S is a reflection, rotation, translation, or glide reflection.

Jennifer: T is a reflection, S is a reflection, rotation, translation, or glide reflection.