MAT 515 Homework 9 - Fall 2016

1. An isometry maps segment to congruent segments, angles to congruent angles, circles to congruent circles.

2. A reflection about a line is an isometry.

3. A symmetry about a point is an isometry. (Can you prove it without using the fact that a symmetry about a point is a special kind of rotation?)

4. A rotation about a point is an isometry.

5. Each point in the plane is uniquely determined by its distance to three non-collinear points. (Thus, for every triple of non collinear points A, B, C, if P and P_1 are points, and $PA = P_1A$, $PB = P_1B$ and $PC = P_1C$ then $P = P_1$)

6. The composition of two translations is translation and that for each triple of points A, B, C in the plane, $T_{BC} \circ T_{AB} = T_{AC}$. Also, two translations commute, that is, for each four points in the plane, $T_{AB} \circ T_{CD} = T_{CD} \circ T_{AB}$.

7. Does the composition of two symmetries about points commute? What about two re- flections about two lines? Two rotations? (Recall that two transformations S and T

8. A translation maps a line segment to a parallel line segment.

9. If α is an angle different from 0, then the only fixed point of a rotation about point P through α is P.

10. The set of fixed points of a reflection about a line is the line.

11. If a function from the plane to the plane preserves distances then it is bijective.

12. Prove that the composition of $h_A \circ h_A$ of half-turn about a point A with itself is the identity. Deduce that the inverse of a half-turn about a point is the same half-turn. about the same point. Formulate and prove analogous statements about reflections.

13. Prove that a function from the plane to the plane that preserve distances is bijective.

14. Find all the symmetries of the following figures: (that is, all the isometries that leave each of the figures invariant. A figure is invariant under an isometry if the image of the figure by the isometry coincides with the figure, not necessarily pointwise.)

    a) An isosceles triangle.

    b) An equilateral triangle.

c) A rectangle.

d) A square.

e) A circle.

f) A regular n-sided polygon.