Math 331, Fall 2004: Problems Set 6, Due Oct 25

1. Have `Maple` find analytic solutions to the following system of differential equations,

$$\begin{cases} y''(t) - z(t) &= e^t, \\ z'(t) - y(t) &= 0, \end{cases}$$

   with initial conditions: $y(0) = 1,\ y'(0) = 0,\ z(0) = k$. Let us denote the solutions by $y_k(t),\ z_k(t)$ (since they depend on the parameter k).

   For k taking all integer values from -10 to 10, and $t \in [-4, 2]$, plot the functions y_k in blue, and the functions z_k in red, all on the same graph. (Yes, you will then have 42 functions plotted on the same graph.) [*This is certainly a case when you don't want to retype the functions that* `Maple` *finds. You will almost certainly need to read the help page for* `dsolve`*. I also found* `subs`*,* `unapply`*, and* `seq` *useful.*]

2. For the functions $y_k(t)$ and $z_k(t)$ found in the previous problem plot the parametric curves $\varphi_k(t) = [y_k(t), z_k(t)]$ for integer values of k between -5 and 5 and $-6 < t < 4$ on the same graph. Use the `view` option of plot to only show what lies in the region $-10 < y < 10, -10 < z < 10$, and use a sequence of colors so that each solution is a different color. [*HINT: you might find something like* `seq(COLOR(HUE,i/11),i=0..10)` *useful for the latter.*]

3. Consider the differential equation $\dot{\mathbf{z}}(t) = \mathbf{F}(\mathbf{z}(t))$, where the vector $\mathbf{z}(t) = (x(t), y(t))$ and the field $\mathbf{F}(x, y) = (-y, x - y)$. Plot a few solutions. What happens to them when $t \to +\infty$? Give a "`Maple`-proof" that this is a general fact for *every* solution. [*A "*`Maple`*-proof" is an argument that is rigorous once we accept* `Maple` *results as incontrovertibly true.*]

4. For the system of differential equations of problem set 5,

$$\begin{cases} \dot{x} &= x^2 + y, \\ \dot{y} &= x(y^2 - 1), \end{cases}$$

   find the eigenvalues and eigenvectors of the Jacobian at the fixed points.

5. Consider the equations of the glider with no drag term ($R = 0$). Use `dsolve, type=numeric` to solve them numerically with initial conditions $\theta(0) = 0$, $v(0) = 0.8$. Then solve exactly the linearized system around the fixed point $(\theta_0, v_0) = (0, 1)$, with the same initial conditions. Graph the two functions for $0 \le t \le 5$, and give a good estimate of their maximum difference. What happens if we take a larger t-range?