Compute the volume of a sphere of radius r using an integral.

SOLUTION. The sphere of radius r can be obtained rotating the half circle graph of the function

$$y = \sqrt{r - x^2}, \qquad x \in [-r, r].$$

about the x-axis.
The volume V is obtained as follows:

$$V = \int_{-r}^{r} \pi(\sqrt{r^2 - x^2})^2 \, dx = 2 \int_{0}^{r} \pi(r^2 - x^2) \, dx = (4/3)\pi r^3.$$