There are 7 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** You may use any Necco Sweethearts™ you might have with messages about calculus on them. Please turn them in with your exam, rather than eating them or giving them to your paramour. No telepathic ducks are allowed for this exam, sorry.

**Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.**

Use **non-erasable pen** (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, do not approximate \( \pi \), square roots, and so on.)

You have 90 minutes to complete this exam.
1. For each of the functions $f(x)$ given below, find $f'(x)$.

(a) 4 points

$$ f(x) = \frac{1 + 2x^2}{1 + x^5} $$

(b) 4 points

$$ f(x) = \sin(3x) \tan(x) $$

(c) 4 points

$$ f(x) = \arctan \left( \sqrt{1 + 4x} \right) $$
2. Compute each of the following derivatives as indicated:

(a) 4 points \[ \frac{d}{du} \left( \frac{u^3}{2} + \frac{2}{u^3} \right) \]

(b) 4 points \[ \frac{d}{dx} \left( e^x - \pi^4 \right) \]

(c) 4 points \[ \frac{d}{dw} \left( \sqrt{1 + \sqrt{1 + w}} \right) \]
3. The set of points \((x, y)\) which satisfy the relationship

\[ y^2(y^2 - 4) = x^2(x^2 - 5) \]

lie on what is known as a “devil’s curve”, shown at right. Write the equation of the line tangent to the given devil’s curve at the point \((\sqrt{5}, 2)\).
4. Let \( f(x) = x \ln(3x) \)

4 points
(a) Calculate \( f'(x) \)

4 points
(b) Calculate \( f''(x) \)

3 points
(c) For what values of \( x \) is \( f(x) \) increasing?

3 points
(d) For what values of \( x \) is \( f(x) \) concave down?
5. Give the $x$ and $y$ coordinates of the (absolute) maximum and minimum values of the function

$$y = x^4 - 8x^2 - 2$$

where $-1 \leq x \leq 3$. 

6. Calvin’s family is visiting a winery in Cutchogue, and he wanders off into the fermenting room and dives into one of the large cylindrical wine vats. The vat has a diameter of 6 feet and is 8 feet tall. The vinter hears the splash and quickly opens the taps to drain the vat, which drains at a rate of 5 cubic feet per minute. How quickly is the height of wine in the tank dropping when the wine is 4 feet deep?

†The volume of a cylinder of height \( h \) and radius \( r \) is \( \pi r^2 h \) and its surface area (excluding top and bottom) is \( 2\pi rh \). The density of the wine is about .98 kg/L or 61 pounds per cubic foot. 5 cubic feet is about 38 gallons or 142 liters. The wine is a rather sweet Riesling, but is probably less sweet after Calvin has been in it.
7. For each of the 4 functions graphed in the left column, find the corresponding derivative function among any of the 8 choices on the right (not just on the same row) and put its letter in the corresponding box. If the graph does not occur, use the letter X.