Problem #1: Find the derivative of each function.

a) \( f(x) = 4x^3 + x - 7 \)
   \[ f'(x) = 12x^2 + 1 \]

b) \( f(x) = \frac{x + x^2}{4x - 11} \)
   \[ f'(x) = \frac{(4x - 11)(1 + 2x) - (x + x^2) \cdot 4}{(4x - 11)^2} \]

c) \( f(x) = (5x^2 - x)(11 + \sqrt{x}) \)
   \[ f'(x) = (5x^2 - x) \left( 2x + \frac{1}{2\sqrt{x}} \right) + (11 + \sqrt{x}) \left( 10x - 1 \right) \]
d) \( f(x) = \tan x - 3 \csc x \)
\[
\begin{align*}
\frac{df}{dx} &= \sec^2 x - 3(-\csc x \cot x) \\
&= \sec^2 x + 3 \csc x \cot x
\end{align*}
\]

e) \( f(x) = \ln \frac{(2x+5)^4}{(x-3)^2} \)
\[
\begin{align*}
\frac{df}{dx} &= 4 \ln (2x+5) - 2 \ln (x-3) \\
&= 4 \ln (2x+5) - 2 \ln (x-3)
\end{align*}
\]
\[
\frac{df}{dx} = 4 \left( \frac{2}{2x+5} \right) - 2 \left( \frac{1}{x-3} \right)
\]
Problem #2: Find the equation of the tangent line to \( y = 7x^2 - \frac{9}{x} \) at \( x = 1 \).

\[
y' = 14x + \frac{9}{x^2}
\]

at \( x=1 \):
\[
14(1) + \frac{9}{1^2} = 23
\]

\[
y - 2 = 23(x - 1)
\]
Problem #3. Find all values of \( x \) where \( y = x^3 - 3x^2 - 24x + 2 \) has an absolute maximum or minimum on the interval \([-3, 10]\).

\[
f'(x) = 3x^2 - 6x - 24 = 3(x-4)(x+2)
\]

So, there are critical points at \( x=-2 \) and \( x=4 \)
(that is, when \( y' = 0 \))

These, and the two endpoints, are possible extreme values. Let's see which is which:

\[
\begin{align*}
f(-3) &= -27 - 3*9 - 24*3 + 2 = 20 \\
f(-2) &= -8 - 12 + 48/2 = 30 \\
f(4) &= 64 - 48 - 24*4 + 2 = -78 \\
f(10) &= 1000 - 300 - 240 + 2 = 462
\end{align*}
\]

So, the absolute maximum on \([-3,10]\) occurs at \( x=10 \) and \( y=462 \)
and the absolute minimum on \([-3,10]\) occurs at \( x=-2 \) and \( y=30 \).
Problem #4: Find \( \frac{dy}{dx} \) if \( 3x^2 + xy - y^4 = 1 \).

\[
6x + \left( x \frac{dy}{dx} + y \right) - 4y^3 \frac{dy}{dx} = 0
\]
\[
6x + x \frac{dy}{dx} + y - 4y^3 \frac{dy}{dx} = 0
\]
\[
6x + y = 4y^3 \frac{dy}{dx} - x \frac{dy}{dx}
\]
\[
6x + y = \frac{dy}{dx} \left( 4y^3 - x \right)
\]
\[
\frac{6x + y}{4y^3 - x} = \frac{dy}{dx}
\]
Problem #5: Find the equation of the tangent line to \(2\sin x - \cos y = \sqrt{2}\) at \(\left(\frac{\pi}{4}, \frac{\pi}{2}\right)\).

\[
2\cos x - 2\sin y \frac{dy}{dx} = 0
\]

\[
2\cos \frac{\pi}{4} + \sin \frac{\pi}{2} \frac{dy}{dx} = 0
\]

\[
2 \left( \frac{\pi}{4} \right) + (1) \frac{dy}{dx} = 0
\]

\[
\sqrt{2} = -1 \frac{dy}{dx}
\]

\[
-\sqrt{2} = \frac{dy}{dx}
\]

\[
y - \frac{\pi}{2} = -\sqrt{2} \left( x - \frac{\pi}{4} \right)
\]
Problem #6: Find $\frac{dy}{dx}$ if $y = \tan^{-1} (x-1)$

$$\frac{dy}{dx} = \frac{1}{1 + (x-1)^2} \quad (1) = \frac{1}{1 + (x-1)^2}$$
Problem #7. Find all x-values of \( f(x) = x^{1/3} - \frac{x^{4/3}}{8} \) for which either \( f'(x) = 0 \) or \( f'(x) \) is not defined.

\[
f'(x) = \frac{1}{3} x^{-\frac{2}{3}} - \frac{4}{8} x^{\frac{1}{3}}
\]

\[
= \frac{1}{3 \sqrt[3]{x^2}} - \frac{3\sqrt{x}}{6} = 0
\]

Cross multiply: \( 6 = 3\sqrt{x} \)

\( x = 2 \) is where \( f'(x) = 0 \)

\( f'(x) \) is undefined when \( x = 0 \)
Problem #8: Find \( \frac{dy}{dx} \)

a) \( x^3y^2 - 4y^3 = 1 \)

\[
x^2 \left( 2y \frac{dy}{dx} \right) + 2xy^2 - 12y^2 \frac{dy}{dx} = 0
\]

\[
2xy \frac{dy}{dx} - 12y^2 \frac{dy}{dx} = -2xy^2
\]

\[
\frac{dy}{dx} \left( 2xy^2 - 12y^2 \right) = -2xy^2
\]

\[
\frac{dy}{dx} = \frac{-2xy^2}{2xy^2 - 12y^2} = \frac{2xy^2}{12y^2 - 2x^2y}
\]

b) \( y = xe^x \)

\[
\frac{dy}{dx} = xe^x + e^x = xe^x + e^x = e^x(x+1)
\]